Entropy squeezing of an atom with a k-photon in the Jaynes–Cummings model*

Kang Dong-Peng(康冬鹏)[a], Liao Qing-Hong(廖庆洪)[a], Ahamd Muhammad Ashfaq[b], Wang Yue-Yuan(王月媛)[a], and Liu Shu-Tian(刘树田)[a]†

[a] *Department of Physics, Harbin Institute of Technology, Harbin* 150001, *China*
[b] *COMSATS Institute of Information Technology, Department of Physics, Lahore* 54000, *Pakistan*

(Received 21 March 2009; revised manuscript received 1 June 2009)

The entropy squeezing of an atom with a k-photon in the Jaynes–Cummings model is investigated. For comparison, we also study the corresponding variance squeezing and atomic inversion. Analytical results show that entropy squeezing is preferable to variance squeezing for zero atomic inversion. Moreover, for initial conditions of the system the relation between squeezing and photon transition number is also discussed. This provides a theoretical approach to finding out the optimal entropy squeezing.

Keywords: Jaynes–Cumming model, entropy squeezing, variance squeezing, atomic inversion
PACC: 4250

1. Introduction

The Jaynes–Cummings model (JCM)[1] is the most fundamental model in which the interaction between the atom and the quantized electromagnetic field is discussed. JCM reveals many important nonclassical properties such as sub-Poissonian statistics, anti-bunching, squeezing and the collapse and revival phenomenon.[2,3] The applications of highly excited Rydberg atoms and the high-Q cavity make it possible to observe the phenomena predicted by quantum theory. With the development of experimental techniques, more attention has been paid to multi-photon JCM. For example, in the two-photon JCM, the revival can be more complicated than in the single photon case.[4] Moreover, the multi-photon JCM presents the theoretical foundation of a multi-photon laser.[5,6] So far, nonclassical effects, such as decoherence, squeezing and high order squeezing in the multi-photon JCM have been investigated.[7−12]

On the other hand, much attention has been paid to atomic squeezing due to its broad applications in high-resolution spectroscopy, high precision atomic fountain clock, highly-accurate spin polarization measurement and generation of squeezed light.[13−21] In all these cases the atomic squeezing has been treated in the framework of the Heisenberg uncertainty relation (HUR). Nevertheless, the HUR cannot provide sufficient information on the atomic squeezing. The HUR is formulated in terms of variances of the system observables. However, the variances, containing only second-order statistical moments, are not an appropriate measure of uncertainties in certain circumstances, particularly in the case where the atomic inversion is zero.[22] This difficulty has been overcome by using the entropy uncertainty relation (EUR).[23,24] The entropy squeezing effect of atoms has been studied in many other systems, such as the two-photon JCM,[25] JCM in a nonlinear medium,[26,27] and two-atom JCM.[28] More recently, much interest has been aroused in entropy squeezing and entanglement in solid-state circuit QED,[29−31] in which a nano-resonator serves as a cavity. Such systems possess great potential applications in quantum information processing.[32−34]

In the present paper, we consider the entropy squeezing of the atom interacting with the quantized electromagnetic field through k-photon transition, that is, in the k-photon JCM. We compare entropy squeezing with variance squeezing and discuss the dependence on transition number k, the initial states of the atom and the field, i.e. phase angles and average photon number. These discussions provide a theoretical foundation to find out the optimal entropy squeezing.

*Project supported by the National Natural Science Foundation of China (Grant Nos. 10674038 and 10604042), and the National Basic Research Program of China (Grant No. 2006CB302901).
†Corresponding author. E-mail: stliu@hit.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

2. Reduced density operator of the atom in the k-photon JCM

We consider a two-level atom interacting with the field through the k-photon process. In the rotating wave approximation, the Hamiltonian of the system is

$$\mathcal{H} = \hbar\nu a^{\dagger}a + \frac{1}{2}\hbar\omega\sigma_z + \hbar g\left(\sigma_+ a^k + a^{\dagger k}\sigma_-\right), \quad (1)$$

where ω is the transition frequency of the atom; ν is the frequency of the field; $a^{\dagger}$ and a are the creation and the annihilation operators of the field respectively; g is the coupling constant between the field and the atom; σ_z and $\sigma_{\pm}$ are the Pauli spin operators of the atom. Now if the atom is in the coherent superposition state of the excited state $|a\rangle$ and the ground state $|b\rangle$ at time $t = 0$, then the initial state vector of the atom is

$$|\theta, \varphi\rangle = \cos(\theta/2)\,|a\rangle + \mathrm{e}^{\mathrm{i}\varphi}\sin(\theta/2)\,|b\rangle, \quad (2)$$

while the field is initially in coherent state

$$|\alpha\rangle = \sum_{n=0}^{\infty} b_n\,|n\rangle$$

with

$$b_n = \exp(\mathrm{i}\,n\beta)\exp(-\bar{n}/2)\frac{\overline{n}^{n/2}}{\sqrt{n!}}, \quad (3)$$

where $0 \le \theta \le \pi$, $0 \le \varphi \le 2\pi$, indicating the polarization of the atom, and $\bar{n}$ and β are the average photon number and the phase angle of the field respectively. For simplicity, we consider the resonant case ($\omega = k\nu$). The state vector of the system at any time t is a linear superposition of $|n, a\rangle$ and $|n + k, b\rangle$ given by

$$|\psi(t)\rangle = \sum_{n=0}^{\infty}\left[A_n(t)\,|n, a\rangle + B_{n+k}(t)\,|n + k, b\rangle\right]. \quad (4)$$

By solving the Schrödinger equation in the interaction picture, we obtain

$$\begin{aligned} A_n(t) &= \cos(\theta/2)\,b_n\cos(\Omega_n t) \\ &\quad - \mathrm{i}\sin(\theta/2)\,\mathrm{e}^{\mathrm{i}\varphi}b_{n+k}\sin(\Omega_n t), \\ B_n(t) &= \sin(\theta/2)\,\mathrm{e}^{\mathrm{i}\varphi}b_n\cos(\Omega_{n-k}t) \\ &\quad - \mathrm{i}\cos(\theta/2)\,b_{n-k}\sin(\Omega_{n-k}t), \end{aligned} \quad (5)$$

where $n = 0, 1, 2, \ldots$, $b_{-k} = b_{-k+1} = \ldots = b_{-1} = 0$ and $\Omega_n = g\sqrt{(n+1)(n+2)\ldots(n+k)}$. After some minor algebra, the matrix elements of the reduced density operator of the atom $\rho(t)$ are

$$\rho_{11} = \sum_{n=0}^{\infty}|A_n|^2, \quad (6)$$

$$\rho_{12} = \sum_{n=0}^{\infty}A_n B_n^* = \rho_{21}^*, \quad (7)$$

$$\rho_{22} = \sum_{n=0}^{\infty}|B_n|^2. \quad (8)$$

By employing expressions (5)–(8), the entropy squeezing of the atom can be studied.

3. Entropy squeezing of the atom

For the Pauli operators $S_\alpha(\alpha \equiv x, y, z)$ of a two-level atom, the Shannon information entropies are defined as[22]

$$H(S_\alpha) = -\sum_{i=1}^{2}P_i(S_\alpha)\ln P_i(S_\alpha), \quad (9)$$

where $P_i(S_\alpha) = \langle\psi_{\alpha i}|\,\rho\,|\psi_{\alpha i}\rangle$, $i = 1, 2$, which is the probability distribution for the two possible outcomes of measurements for the operator S_α. The information entropies of the operators S_x, S_y and S_z satisfy the entropy uncertainty relation[22]

$$\delta H(S_x)\,\delta H(S_y) \ge \frac{4}{\delta H(S_z)}, \quad (10)$$

where $\delta H(S_\alpha) \equiv \exp[H(S_\alpha)]$. The fluctuations in the components $S_\alpha(\alpha = x, y)$ of the atomic dipole are said to be 'squeezed in entropy' if the information entropy $H(S_\alpha)$ of S_α satisfies the following condition:

$$E(S_\alpha) = \delta H(S_\alpha) - \frac{2}{[\delta H(S_z)]^{1/2}} < 0, \quad \alpha = x, y. \quad (11)$$

Using the reduced density operator of the atom, $\rho(t)$, which is given by expressions (5)–(8), we obtain the information entropies of the operators S_x, S_y, S_z as

$$H(S_x) = -\left[\frac{1}{2} + \mathrm{Re}\,\rho_{21}(t)\right]\ln\left[\frac{1}{2} + \mathrm{Re}\,\rho_{21}(t)\right] - \left[\frac{1}{2} - \mathrm{Re}\,\rho_{21}(t)\right]\ln\left[\frac{1}{2} - \mathrm{Re}\,\rho_{21}(t)\right], \quad (12)$$

$$H(S_y) = -\left[\frac{1}{2} + \mathrm{Im}\,\rho_{21}(t)\right]\ln\left[\frac{1}{2} + \mathrm{Im}\,\rho_{21}(t)\right] - \left[\frac{1}{2} - \mathrm{Im}\,\rho_{21}(t)\right]\ln\left[\frac{1}{2} - \mathrm{Im}\,\rho_{21}(t)\right], \quad (13)$$

$$H(S_z) = -\rho_{22}(t)\ln\rho_{22}(t) - \rho_{11}(t)\ln\rho_{11}(t). \quad (14)$$

On the other hand, the fluctuation in the component S_α of the atomic dipole moment is said to be squeezed if S_α satisfies

$$V(S_\alpha) = \Delta S_\alpha - (|\langle S_z\rangle|/2)^{1/2} < 0, \quad \alpha = x, y, \quad (15)$$

where $\Delta S_\alpha = \sqrt{\langle S_\alpha^2\rangle - \langle S_\alpha\rangle^2}$ is the variance squeezing based on the HUR. Expression (15) can be trivially satisfied if $\langle S_z\rangle = 0$. However, it fails to provide any useful information about atomic squeezing because ΔS_α contains only the second-order statistical moments of the density operator. Expression (11) is nontrivial even if $\langle S_z\rangle = 0$, thus it characterizes the fluctuation in atomic observables better.

4. Numerical simulations and discussion

In the following we discuss the entropy squeezing of the atom numerically. For comparison, we also consider the variance squeezing and atomic inversion.

We suppose that the atom is initially in an excited state and the field is in a coherent state with an average photon number $\bar{n} = 60$ and phase $\beta = 0$. Figures 1–4 show the atomic squeezing for $k = 1, 2, 3, 4$ respectively by using expressions (11) and (15), while Fig. 5 shows the corresponding atomic population inversions.

Figure 1 shows the squeezing conditions for the usual JCM, i.e. for the case of $k = 1$. It is exactly the same as what was depicted in Ref. [22]. According to Fig. 1, neither entropy nor variance displays squeezing for the component S_x, while there is a great difference between $E(S_y)$ and $V(S_y)$. Entropy squeezing exists at several periods of time, while variance squeezing occurs only at the very beginning. Especially, $E(S_y)$ in Fig. 1(b) exhibits optimal squeezing when the atom is in the eigenstate of S_y. However, at this time $\Delta S_y = 0$, thus $V(S_y)$ in Fig. 1(d) does not exhibit any variance squeezing. In this case, the information provided by the HUR is not useful, thus the EUR shows its advantages.

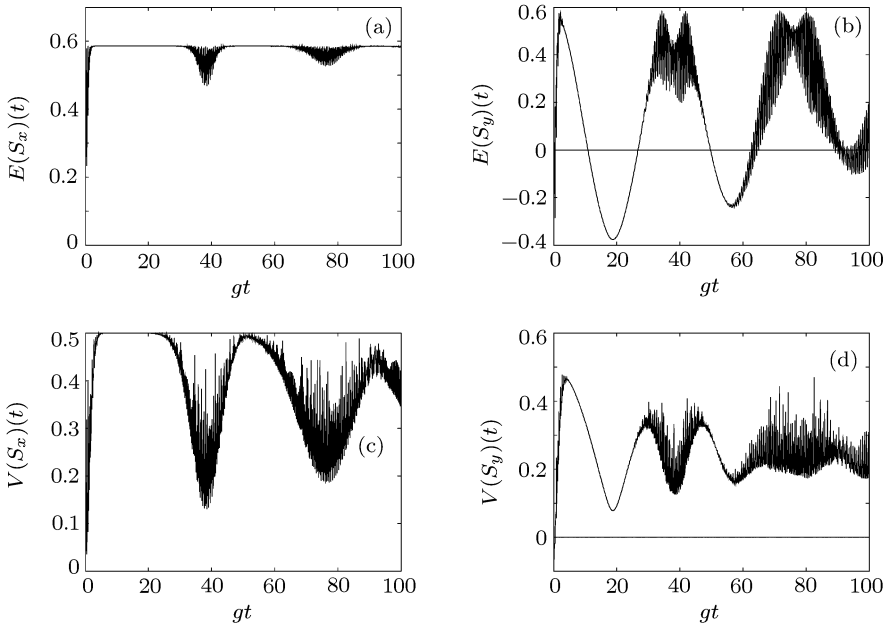


Fig. 1. Time evolutions of squeezing factors in the one-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$ and (d) $V(S_y)$. The atom is initially in an excited state, the field in a coherent state, with the average photon number $\bar{n} = 36$ and $\beta = 0$.

The entropy squeezing and the variance squeezing of the atom in the two-photon JCM are depicted in Fig. 2. Similar to the one-photon case, the entropy squeezing factor for S_y displays much longer squeezing time than the corresponding variance squeezing factor. The latter exhibits only squeezing around $\tau = gt = m\pi, (m = 0,1,2,\ldots)$. Moreover, Fig. 2 shows no squeezing for the component of S_x. A significant distinction in the two-photon case is that the atomic observables, including the atomic inversion in Fig. 5(b), display good quasi-periodicity with a period of π for the scaled time $\tau = gt$. This periodicity is due to the fact that analytical expressions (6)–(8) can be summed exactly.[35]

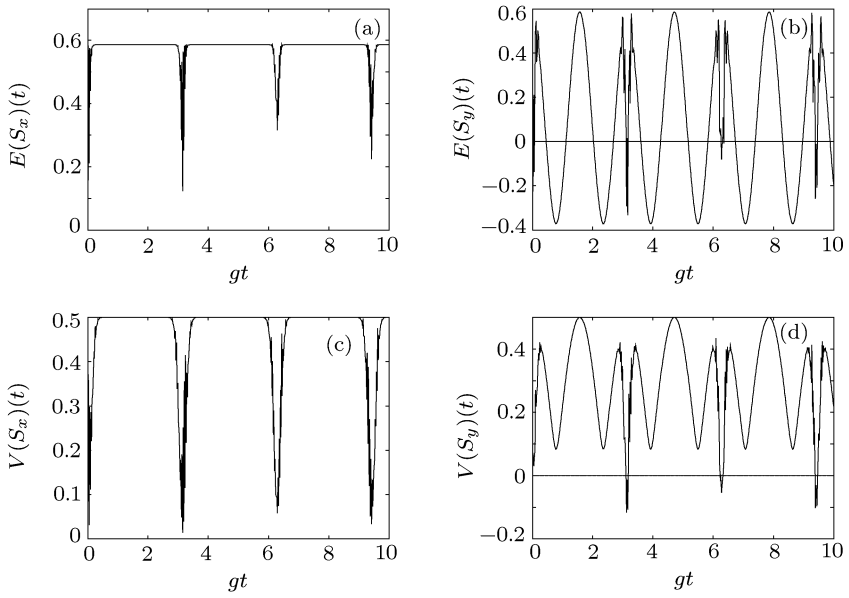


Fig. 2. Time evolutions of the squeezing factors in the two-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$ and (d) $V(S_y)$. The atom is initially in an excited state, the field in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = 0$.

The squeezing conditions for the cases of $k = 3$ and $k = 4$ corresponding to expressions (11) and (15) are illustrated in Fig. 3 and Fig. 4, respectively. The patterns of the evolution curves are similar to those in Fig. 1. They both exhibit squeezing only for the S_y component, and the entropy exhibits squeezing for a longer time period than the variance. Note that unlike the entropy squeezing of the field, the atom does not necessarily exhibit entropy squeezing when it is squeezed in variance.

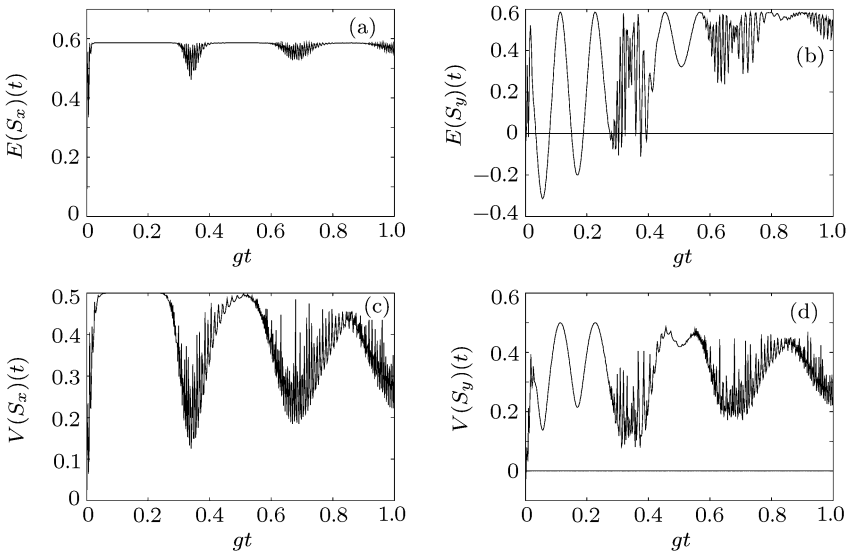


Fig. 3. Time evolutions of squeezing factors in the three-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$, (d) $V(S_y)$. The atom is initially in an excited state, the field in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = 0$.

According to Figs. 1–5, we can also conclude that the revival time in the k-photon JCM decreases with increasing k. Thus, the time evolutions of the squeezing factors fluctuate most rapidly in Fig. 4, in the four-photon process.

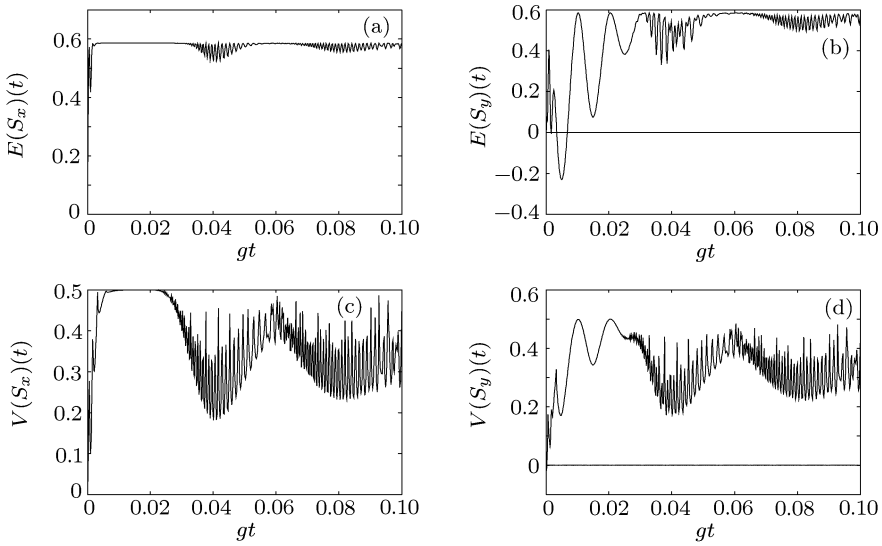


Fig. 4. Time evolutions of the squeezing factors in the four-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V\,(S_x)$ and (d) $V(S_y)$. The atom is initially in an excited state, the field in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = 0$.

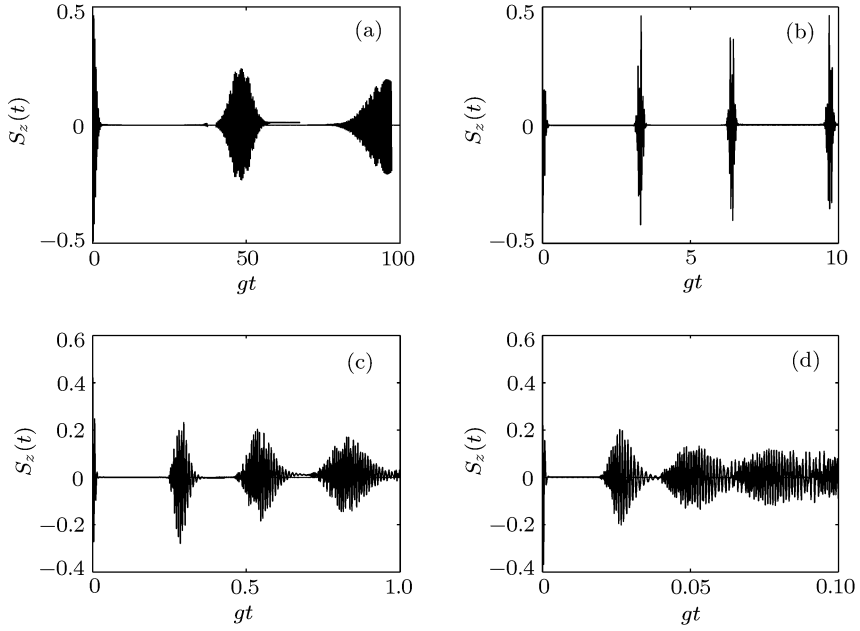


Fig. 5. Time evolutions of the atomic inversion in the k-photon JCM. (a) $k = 1$, (b) $k = 2$, (c) $k = 3$ and (d) $k = 4$. The atom is initially in an excited state, the field in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = 0$.

In Figs. 1–5, one can see that only the S_y component of the dipole moment exhibits squeezing, both in the entropy and the variance. We find that the direction of squeezing is determined by the initial phase of the field, β. For the one-photon JCM, when the atom is initially in an excited state, and $\beta = m\pi/2$ (m= 0, 2, 4, ...), squeezing occurs only for the S_y component. If we change the values of the phase angles θ and φ of the atom, squeezing for S_x can be obtained but the amount of squeezing cannot exceed that for S_y. This is because β determines the direction of squeezing. When $\beta = m\pi/2$ (m = 1,3,5,...) and other parameters are kept unchanged, squeezing occurs only for S_x component (see Fig. 6). Changing θ and φ, the amount of squeezing for S_y cannot exceed that for S_x (see Fig. 7). When β has an intermediate value, say $\pi/4$, the two components have the same fluctuation (Fig. 8).

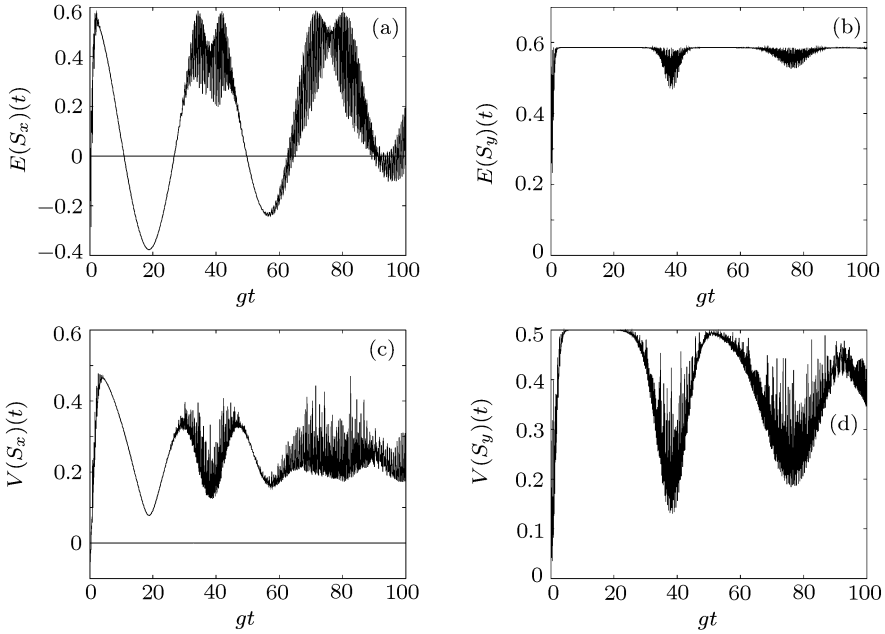


Fig. 6. Time evolutions of squeezing factors in the one-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$ and (d) $V(S_y)$. Initially $\theta = 0$, $\varphi = 0$, the field is in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = \pi/2$.

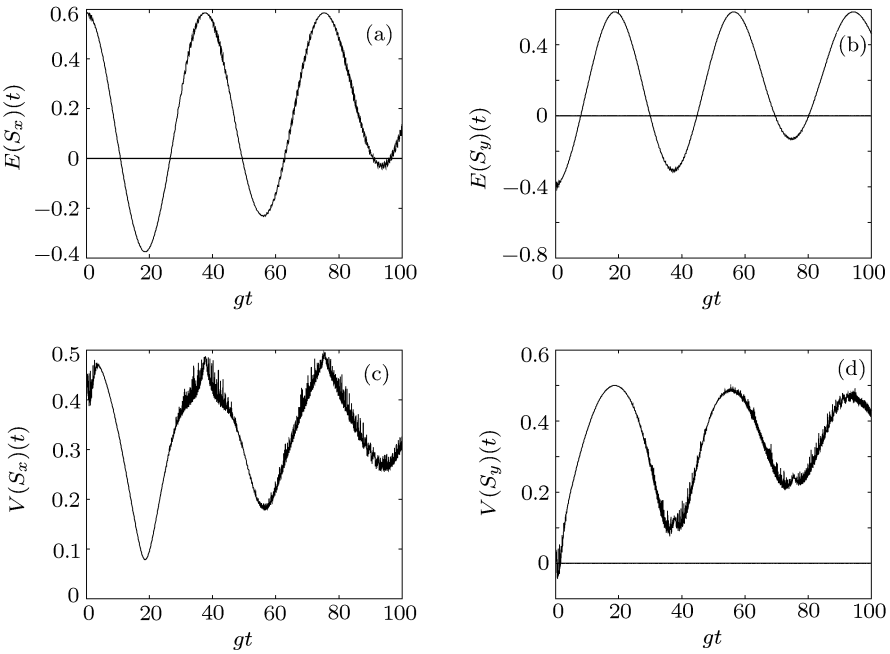
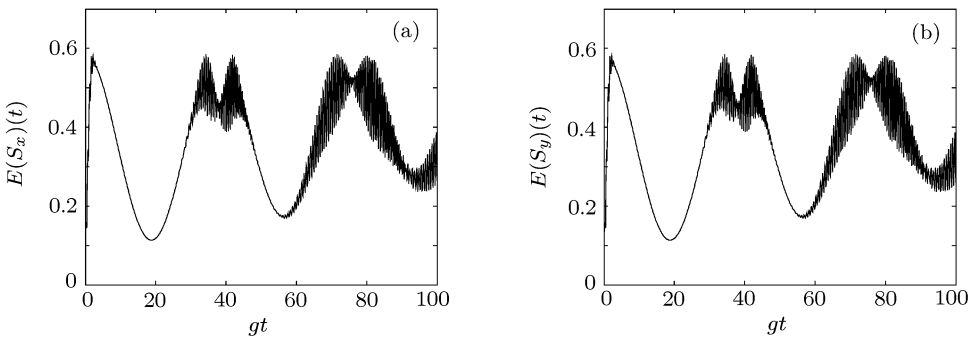


Fig. 7. Time evolutions of squeezing factors in the one-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$ and (d) $V(S_y)$. Initially $\theta = \pi/2$, $\varphi = \pi/2$, the field is in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = \pi/2$.



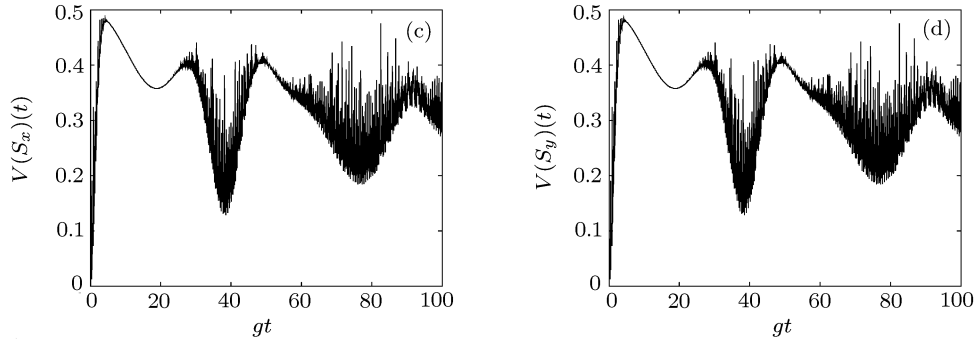


Fig. 8. Time evolutions of squeezing factors in the one-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$ and (d) $V(S_y)$. Initially $\theta = 0$, $\varphi = 0$, the field is in a coherent state, with average photon number $\bar{n} = 36$ and $\beta = \pi/4$.

For the two-photon JCM, when the atom is initially in an excited state and $\beta = m\pi/4$ (m= 0, 2, 4, ...), only the S_y component displays squeezing, while if $\beta = m\pi/4$ (m= 1, 3, 5, ...), only the S_x component displays squeezing. For the multi-photon JCM, a similar relation also exists but it is much more complicated.

In the above discussion, we assume that the average photon number of the field is $\bar{n} = 36$. Through numerical calculation, we also find that as $\bar{n}$ increases, the revival time of the atomic inversion for the k-photon JCM ($k \neq 2$) decreases. Therefore the corresponding squeezing factors fluctuate more rapidly, while the shapes of the curves are very similar (Fig. 9). However, the two-photon JCM is an exception, because its revival time is independent of $\bar{n}$. Thus the time evolution of the squeezing factors is almost unchanged (Fig. 10).

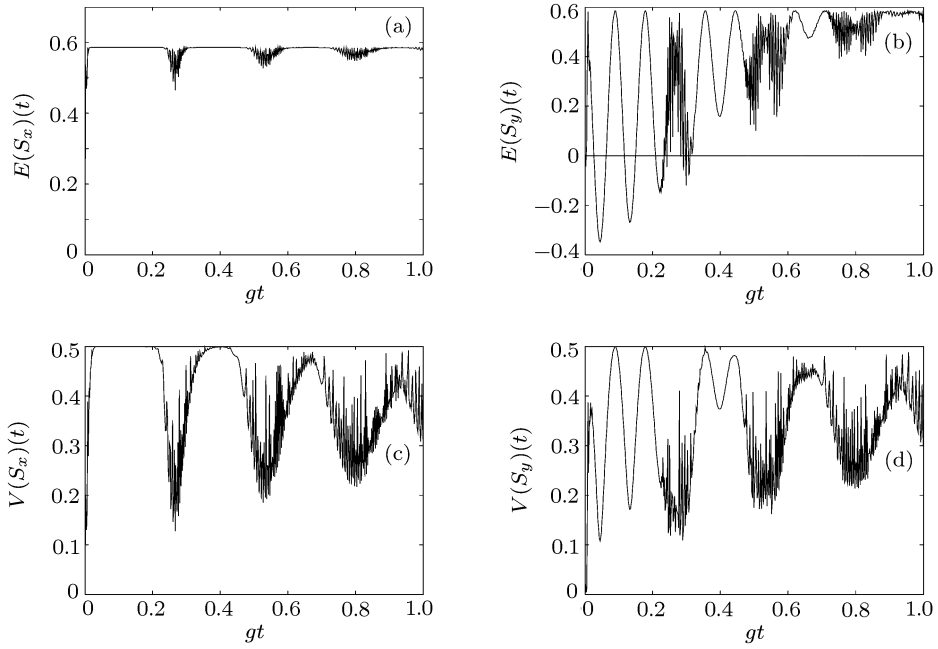
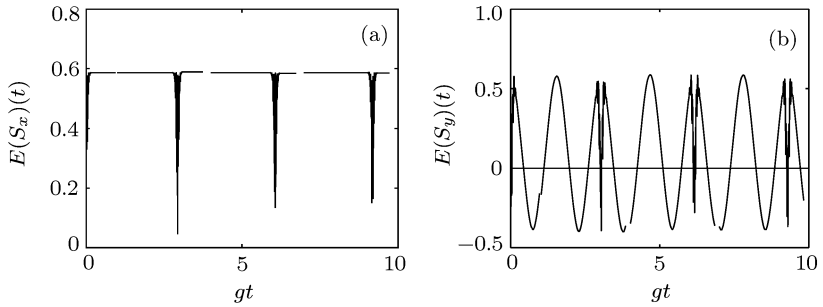


Fig. 9. Time evolutions of squeezing factors in the three-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$, (d) $V(S_y)$. The atom is initially in an excited state, the field in a coherent state, with average photon number $\bar{n} = 60$ and $\beta = 0$.



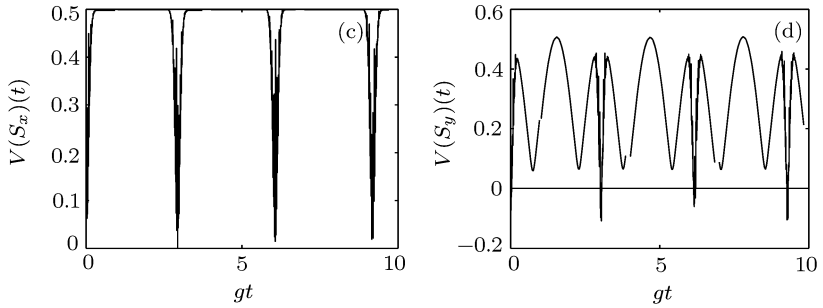


Fig. 10. Time evolutions of squeezing factors in the two-photon JCM. (a) $E(S_x)$, (b) $E(S_y)$, (c) $V(S_x)$, (d) $V(S_y)$. The atom is initially in an excited state, the field in a coherent state, with average photon number $\bar{n} = 60$ and $\beta = 0$.

5. Conclusions

In this paper, the entropy squeezing of an atom in the k-photon JCM is investigated in the rotating wave approximation and compared with the variance squeezing and the atomic inversion. As photon transition number k increases, the revival time turns shorter, which shows that the squeezing factors fluctuate more rapidly. We also discuss the effect of initial state on atomic squeezing. The obtained results can be summarized as follows. The initial phase of the field, β, determines the squeezing direction and the initial phase of the atom, θ and φ, and can change the amount of squeezing for two components. For $k \neq 2$, the bigger the average photon number $\bar{n}$ is, the shorter the revival time of the atomic squeezing and the quasi-period of squeezing factor will be. For $k = 2$, the time evolution of the squeezing factor is independent of average photon number $\bar{n}$.

References

[1] Jaynes E T and Cummings F W 1963 *Proc. IEEE* **51** 89
[2] Scully M O and Zubairy M S 1997 *Quantum Optics* (UK: Cambridge University Press) p193
[3] Rempe G, Walther H and Klein N 1987 *Phys. Rev. Lett.* **58** 353
[4] Vogel W, Welsch D G and Laine L 1987 *J. Opt. Soc. Am.* B **4** 1633
[5] Singh S 1982 *Phys. Rev.* A **25** 3206
[6] Font J L, Fernandez-Soler J J, Vilaseca R and Gauthier D J 2005 *Phys. Rev.* A **72** 063810
[7] Kuang L M, Chen X and Ge M L 1995 *Phys. Rev.* A **52** 1857
[8] Gerry C C and Moyer P J 1988 *Phys. Rev.* A **38** 5665
[9] Kien F L, Kozierowski M and Quang T 1988 *Phys. Rev.* A **38** 263
[10] Mir M A and Razmi M S K 1991 *Phys. Rev.* A **44** 6071
[11] Abdel-Hafez A M and Obada A S F 1991 *Phys. Rev.* A **44** 6017
[12] Mahran M H and Obada A S F 1990 *Phys. Rev.* A **42** 1718
[13] Agarwal G S and Puri R R 1990 *Phys. Rev.* A **41** 3782
[14] Li X S, Lin D L and George T F 1989 *Phys. Rev.* A **40** 2504
[15] Ashraf M M and Razmi M S K 1992 *Phys. Rev.* A **45** 8121
[16] Wodkiewicz K, Knight P L, Buckle S J and Barnett S M 1987 *Phys. Rev.* A **35** 2567
[17] Kitagawa M and Ueda M 1993 *Phys. Rev.* A **47** 5138
[18] Wineland D J, Bollinger J J and Itano W M 1994 *Phys. Rev.* A **50** 67
[19] Sorensen A and Molmer K 1999 *Phys. Rev. Lett.* **83** 2274
[20] Sorensen J L, Hald J and Polzik E S 1998 *Phys. Rev. Lett.* **80** 3487
[21] Walls D F and Zoller P 1981 *Phys. Rev. Lett.* **47** 709
[22] Fang M -F, Zhou P and Swain S 2000 *J. Mod. Opt.* **47** 1043
[23] Sanchez-Ruiz J 1995 *Phys. Lett.* A **201** 125
[24] Sanchez-Ruiz J 1998 *Phys. Lett.* A **244** 233
[25] Liu X J, Zhou B J, Fang M F and Zhou Q P 2006 *Acta Phys. Sin.* **55** 704 (in Chinese)
[26] Abdalla M S, Hassan S S and Abdel-Aty M 2005 *Opt. Commun.* **244** 431
[27] Abdel-Aty M, Abdalla M S and Obada A S F 2002 *J. Opt.* B: *Quantum Semiclass. Opt.* **4** 134
[28] El-Orany F A A, Wahiddin M R B and Obada A S F 2008 *Opt. Commun.* **281** 2854
[29] Huo W Y and Long G L 2007 *Commun. Theor. Phys.* **48** 1029
[30] Obada A S F, Wahiddin M R B and Abdel-Aty M 2008 *Opt. Commun.* **281** 6019
[31] Huo W Y and Long G L 2008 *New. J. Phys.* **10** 013026
[32] Zipper E, Kurpas M, Szelg M, Dajka J and Szopa M 2006 *Phys. Rev.* B **74** 125426.
[33] Huo W Y and Long G L 2008 *Appl. Phys. Lett.* **92** 133102
[34] Etaki S, Poot M, Mahboob I, Onomitsu K, Yamaguchi H and Van der Zant H S J 2008 *Nat. Phys.* **4** 785
[35] Shore B W and Knight P L 1993 *J. Mod. Opt.* **40** 1195