Excitation of defect modes from the extended photonic band-gap structures of 1D photonic lattices*

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This paper stuides numerically the model equation in a one dimensional defective photonic lattice by modifying the potential function to a periodic function. It is found that defect modes (DMs) can be regarded as Bloch modes which are excited from the extended photonic band-gap structure at Bloch wave-numbers with $k_x = 0$. The DMs for both positive and negative defects are considered in this method.

Keywords: optical-induced photonic lattices, photonic band-gaps, defect modes
PACC: 4225B, 4265S, 4270Q

1. Introduction

Light propagation in periodic optical media such as photonic crystals,[1−3] waveguide arrays[4,5] and optical-induced photonic lattices[6] has attracted much attention for its significance in light-guiding applications. It is well known that the unique feature of such periodic systems is the existence of band gap structures[3,7] in their linear spectrum. Bloch waves exist in the bands and wave propagation is forbidden inside the gaps because of the repeated Bragg reflections. In photonic crystals, band-gaps are associated with the time-domain frequency. On the other hand, analyses of how the light field distributes in optical induced photonic lattices often focus on the band-gaps of the spatial frequency modes (i.e. propagation constant vs. transverse wave vector).

Linearly localized modes in defective periodic structures, which are called defect modes (DMs), have been investigated intensively in recent years.[1,2] In the proposed one dimensional (1D) waveguides with structured defects, DMs have been reported and analysed.[8−10] Furthermore, 1D and 2D DMs in optical induced periodic lattices have also been investigated both theoretically[11−13] and experimentally.[14,15] Using an asymptotic method, one finds analytically that DMs bifurcate out from Bloch-band edges with anomalous diffraction coefficients in weak negative defects. The situation is opposite for positive defects. When the defect is not weak, DMs have been solved numerically and discussed comprehensively for different parameters. However, the behaviours of photonic band-gap (PBG) structure in the photonic lattice with the periodic refractive index distribution destroyed by the defect and the relationship between the DMs and the modified PBG structure are unknown.

In this paper, we modify the potential function of the model equation in a 1D defective optical induced lattice and obtain profiles of DMs by solving the Bloch modes of the modified model equation (MME). It is shown that DMs correspond to Bloch modes of certain sub-bands of the MME, which are excited out from the original Bloch bands. The excitation of DMs from the extended PBG structure of MME has never been reported before.

2. Model and method

The physical situation we consider here is that an ordinarily-polarized or extraordinarily-polarized lattice beam with a single-site defect is imaged into a biased photorefractive crystal, such as a strontium barium niobate crystal. The defective lattice beam is assumed to be uniform along the direction of propagation, and forms a corresponding refractive index

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change through the electro-optic effect. Meanwhile another probe beam with a very low intensity is launched into the defect site, being incoherent with the lattice beam, thus it feels the refractive index change but has little effect on the lattice. We consider the 1D model equation[11,12] as follows:

$$\mathrm{i}\,U_z + U_{xx} - \frac{E_0}{1+I_{\mathrm{L}}(x)}U = 0. \tag{1}$$

where U is the slowly-varying amplitude of the probe beam, x is the transverse distance (in units of T/π), z is the propagation distance (in units of $2n_{\mathrm{e}}k_0T^2/\pi^2$), E_0 is the applied DC field (in units of $\pi^2/(k_0^2n_{\mathrm{e}}^4T^2r_{33})$), and

$$I_{\mathrm{L}}(x) = I_0\cos^2 x[1+\epsilon f_{\mathrm{D}}(x)] \tag{2}$$

is the intensity function of the photorefractive lattice (normalized by $I_{\mathrm{d}}+I_{\mathrm{b}}$, where I_{d} is the dark irradiance of the crystal and I_{b} is the background illumination), I_0 is the dark irradiance of the otherwise uniform photonic lattice, $f_D(x) = \exp(-x^8/128)$ is a localized function describing the shape of the defect, ϵ controls the strength of the defect, T is the lattice spacing, $k_0 = 2\pi/\lambda_0$ is the wave-number (λ_0 is the wavelength), n_{e} is the unperturbed refractive index for extraordinary polarized beams, and r_{33} is the electro-optic coefficient of the crystal.

In order to find DMs for Eq. (1), we modify the function $I_{\mathrm{L}}(x)$ to $I'_{\mathrm{L}}(x)$, and get such a periodic function with a longer period $N_{\mathrm{p}}\pi$:

$$I'_L(x) = I_0\cos^2(x')[1+\epsilon f_{\mathrm{D}}(x')], \tag{3}$$

where $x' = \mathrm{mod}(x+N_{\mathrm{p}}\pi/2, N_{\mathrm{p}}\pi) - N_{\mathrm{p}}\pi/2$. Then Eq. (1) is changed to the following MME:

$$\mathrm{i}\,U'_z + U'_{xx} - \frac{E_0}{1+I'_{\mathrm{L}}(x)}U' = 0. \tag{4}$$

As the above MME has a periodic potential function, we can get its PBG structure and corresponding Bloch modes by using the numerical method. The Bloch mode for the MME can be written as

$$U'_{\mathrm{B}}(x) = u_{k_x,\mu}(x)\mathrm{e}^{\mathrm{i}\,k_x x}\mathrm{e}^{-\mathrm{i}\,\mu z}, \tag{5}$$

where k_x is the Bloch wave-number, μ is the eigenvalue of the Bloch mode. The $u_{k_x,\mu}(x)$ is a periodic function with period $N_{\mathrm{p}}\pi$. We choose an appropriate N_{p} so that some of the Bloch modes at $k_x = 0$ are localized around the defect and cover fewer sites of the lattice than N_{p}, thus the following function can be regarded as a DM of the original defective lattice, and μ becomes the propagation constant of this DM:

$$U_{\mathrm{d}}(x) = U'_{\mathrm{B}}(x)|_{k_x=0} = u_{0,\mu}(x)\mathrm{e}^{-\mathrm{i}\,\mu z}. \tag{6}$$

The extended PBG structure of the MME is similar to the original PBG structure (situation for $\epsilon = 0$); with each original Bloch band splitting into N_{p} sub-bands, as shown in Figs. 1(a) and 1(b). (For comparison, we make the regions between the 1st and 18th sub-bands shaded.) It is found that in a positive defect ($\epsilon > 0$), only the 1st sub-bands are excited from the original Bloch bands, while in negative defects ($\epsilon < 0$) either the last sub-bands or second-last sub-bands are excited. All the excited sub-bands whose eigenvalues fall into original gaps result in localized solutions. Detailed discussions about the excitation of DMs from the extended PBG structure are outlined in the following sections.

3. Situation for positive defects ($\epsilon > 0$)

For a positive defect, the intensity around the defect is larger than other sites. We take $E_0 = 10$, $I_0 = 3$, $\epsilon = +0.5$ and $N_{\mathrm{p}} = 18$ as an example. The original PBG structure of the photonic lattice is shown in Fig. 1(a). There are infinite numbers of Bloch bands here, which are

$$\mu \in [4.00,\ 4.20]\cup[6.98,\ 8.27]\cup[10.18,\ 13.77]\cdots. \tag{7}$$

Only the first three bands are shown in the shaded regions of Fig. 1(a), and the extended PBG structure of the MME is depicted in Fig. 1(b) for comparison. As can be seen, the last 17 sub-bands stay in the original Bloch band, connecting with each other end to end, only the 1st sub-band is excited from the original Bloch band, as shown in Fig. 1(b). We can conclude that it is the introduction of the defect which leads to the 1st sub-band excitation.

We solved Bloch modes for points at $k_x = 0$, and found that all other modes are non-zero along the transverse direction, this is expected for those in the 1st sub-bands, which are localized around the defect sites as shown in Figs. 1(c) and 1(d) with propagation constants $\mu_{\mathrm{c}} = 3.33$ and $\mu_{\mathrm{d}} = 6.55$. As mentioned in Section 2, these two modes can be regarded as DMs of the defective photonic lattice which bifurcate from the first and second Bloch bands respectively.

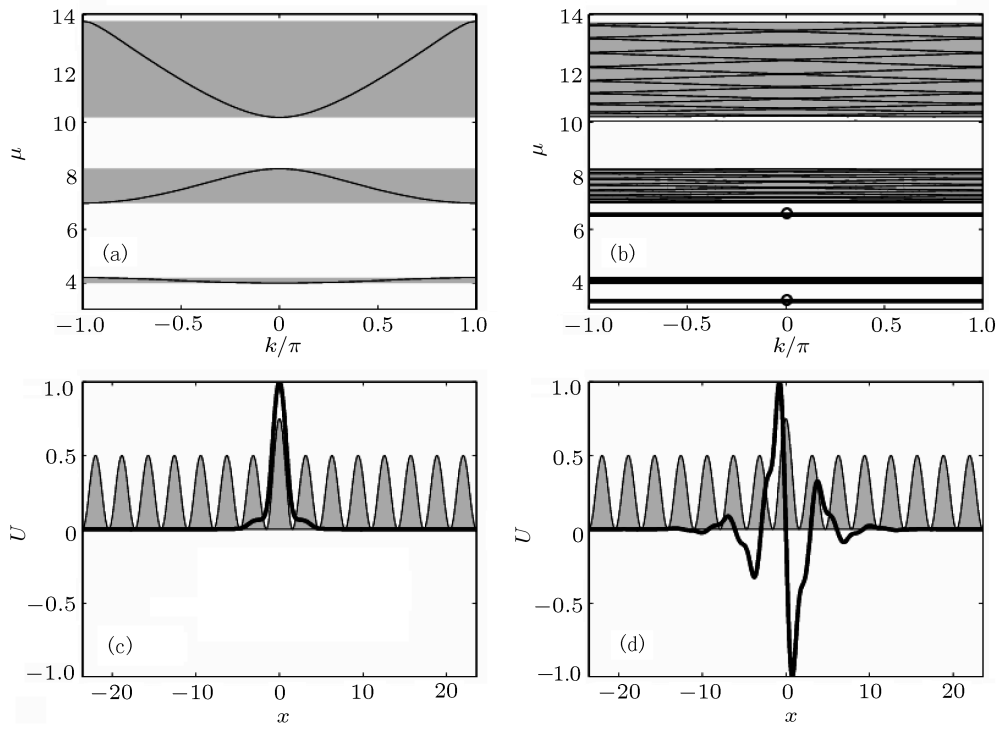


Fig. 1. PBG structures and Bloch modes for $I_0 = 3$ and $E_0 = 10$: (a) the original PBG structure with $\epsilon = 0$; (b) the extended PBG structure with $\epsilon = 0.5$ and $N_\mathrm{p} = 18$; (c) and (d) correspond to Bloch modes for the points marked in (b). The propagation constants are $\mu_\mathrm{c} = 3.33$ and $\mu_\mathrm{d} = 6.55$, respectively.

To further understand the relationship between DMs and the extended PBG structure, we fix the value of E_0, I_0 and N_p, allowing the defect parameter to vary from 0 to 1. It is found that the situation is similar to the results for $\epsilon = 0.5$. The DMs are excited down from the original Bloch modes, corresponding to every 1st sub-band in each band. The larger ϵ is, the further the 1st sub-bands excited from the original band and the DMs are more localized. It is notable that the excited sub-bands never reach the next original bands, which is different from the situation for negative defects, as will be discussed in the following section.

We also changed other parameters such as E_0, I_0 and N_p. It is found that a larger DC field or dark irradiance gets deeper modulation of the induced photonic lattices, as a result, the corresponding photonic gaps become larger and DMs are more easily excited. We chose $N_\mathrm{p} = 18$, so that it can cover enough lattice sites which are needed by the DMs considered by us.

4. Situation for negative defects ($\epsilon < 0$)

Compared with the positive defects, we find things are opposite for negative defects: all other 17 sub-bands of the extended PBG structure remain in the original Bloch bands, only the 18th sub-bands are excited up into the gaps. We take $\epsilon = -0.5$ as an example, and depict the extended PBG structure and profiles of the first two DMs in Fig. 2. The propagation constants of the two DMs in Figs. 2(c) and 2(d) are $\mu_\mathrm{c} = 5.64$ and $\mu_\mathrm{d} = 9.05$ respectively.

As shown in Figs. 1 and 2, DMs for both positive and negative defects behave with the following features. Firstly, DMs are excited from every original Bloch band, sharing the same symmetry behaviours with the Bloch modes of the original equation at $k_x = 0$: modes from odd bands taking symmetric profiles and those from even bands taking asymmetric profiles. Secondly, as the defects get stronger (for larger $|\epsilon|$), the sub-bands are excited further from the original Bloch bands. Thirdly, the dependence on DC field and dark irradiance is all the same for both positive and negative defects.

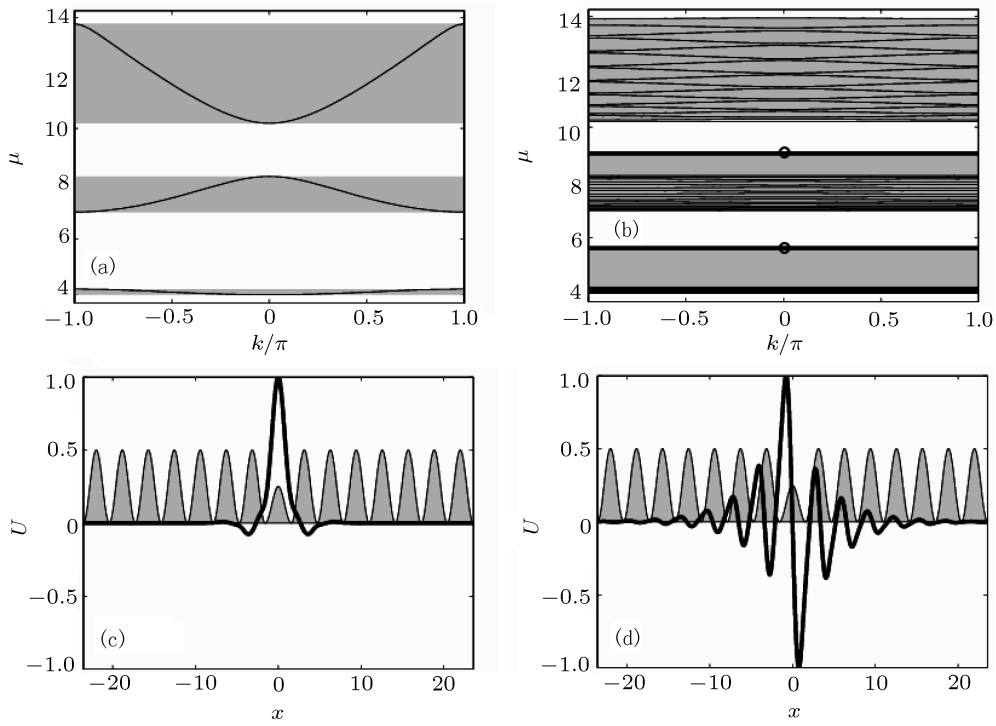


Fig. 2. PBG structures and Bloch modes for $I_0 = 3$ and $E_0 = 10$: (a) the original PBG structure with $\epsilon =$; (b) the extended PBG structure with $\epsilon = -0.5$ and $N_{\rm p} = 18$; (c) and (d) correspond to Bloch modes for the points marked in (b). The propagation constants are $\mu_{\rm c} = 5.64$ and $\mu_{\rm d} = 9.05$ respectively.

However, there are some features which are different for the two kinds of defects. As for positive defects, the excited sub-bands never reach the next original bands, while for negative defects, when ϵ changes from 0 to -1, every 18th sub-band is excited up into the photonic gaps, and the distance between the original Bloch band edge and the 18th sub-band becomes bigger for larger $|\epsilon|$. When ϵ reaches a critical value $\epsilon_{\rm cr1}$, the excited sub-bands of the first original Bloch band arrive at the bottom of the next band, and the corresponding DMs disappear. With increasing $|\epsilon|$, another new sub-band is excited from the second original Bloch band when $|\epsilon| > |\epsilon_{\rm cr2}|$, corresponding to its 17th sub-band. It is found that the Bloch mode of this sub-band is localized around the defect and can be regarded as a new branch of the DMs. As an example, we take $E_0 = 10$, $I_0 = 3$, $\epsilon = -0.9$, and the extended PBG structure is shown in Fig. 3(a). The Bloch mode for the point marked by $'\Delta'$ in Fig. 3(a) is shown in Fig. 3(b). We also mark the 18th sub-bands of the first and second Bloch bands with circles in Fig. 3(a). It can be seen that the two sub-bands are excited into the next bands, thus do not support localized DMs, and our numerical results prove this intuition.

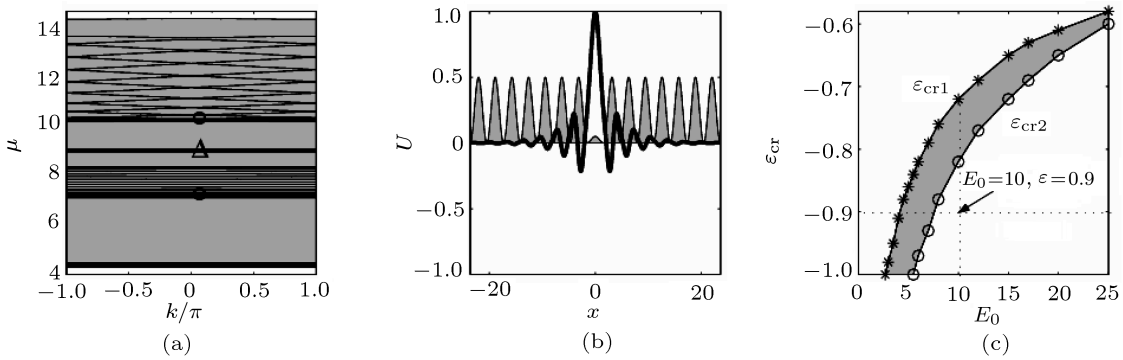


Fig. 3. (a) The extended PBG structure with $E_0 = 10$, $I_0 = 3$, $\epsilon = -0.9$ and $N_{\rm p} = 18$. (b) Bloch mode for the point marked by $'\Delta'$ in (a), with $\mu = 8.92$. (c) Critical values of ϵ vs. the applied DC field E_0.

The relationship between the two critical values of ϵ and the applied DC field E_0 is depicted in Fig. 3(c). When $\epsilon_{\rm cr1} < \epsilon < 0$ or $-1 < \epsilon < \epsilon_{\rm cr2}$, symmetric DMs are either excited from the first original bands or from the second original bands. When ϵ falls into the shaded regions of Fig. 3(c), there exist no symmetric DMs. When $-1 < \epsilon < \epsilon_{\rm cr2}$, DMs are excited from the 17th sub-band of the second original Bloch band, but take opposite symmetric characteristics with the Bloch modes (at $k_x = 0$) in the same Bloch band.

5. Conclusion

In conclusion, by modifying the potential function of the model equation in 1D defective photonic lattices and introducing the concept of sub-Bloch bands, we get profiles of the DMs in 1D defective photonic lattices for different parameters. It is found that DMs can be regarded as excited Bloch modes from the extended PBG structure. These results give a vivid picture of how DMs are excited from the Bloch bands of the photonic lattices after a single defect (either positive or negative) is introduced, and can well explain the experimental results in Ref. [14]. Our work may pave the way for studying other phenomena mediated by defects and defect dynamics in optical lattice structures.

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