A simple method to simultaneously achieve synchronization and anti-synchronization in chaotic systems*

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In this paper, a novel adaptive control approach is presented to simultaneously achieve synchronization and anti-synchronization in partially linear chaotic systems. Through appropriately separating state vectors of such systems, synchronization and anti-synchronization could be simultaneously realized in different subspaces, which may be strictly proven theoretically. Simulation results for a Lorenz chaotic system and a new hyper-chaotic system are provided to illustrate the effectiveness of the proposed method. Finally, a new secure communication scheme based on such a synchronization phenomenon of the hyper-chaotic system is demonstrated. Numerical results show success in transmitting a periodic signal with high security.

Keywords: synchronization and anti-synchronization, adaptive control, secure communication

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1. Introduction

Chaos, a very interesting phenomenon, has been found in many nonlinear dynamical systems. Characterized by extreme sensitivity to initial conditions and having noise-like behaviours, chaotic systems can only achieve synchronization with difficulty. In 1990, Pecora and Carroll[1] introduced a method to synchronize two identical chaotic dynamical systems with different initial conditions and showed that it was possible for some chaotic systems to be synchronized. Since then, the synchronization of chaotic oscillators has become a hot topic of research due to its potential applications in secure communications, chemical reactions, biomedical science, and many other fields.

So far, many types of synchronization phenomena have been presented, such as complete synchronization (CS),[2,3] phase synchronization (PS),[4−6] lag synchronization (LS) or anticipated synchronization (AS),[7,8] generalized synchronization (GS),[9−11] etc. Recently, projective synchronization[12,13] and its extension[14−16] have attracted plenty of attention, owing to their complex and interesting dynamical behaviours. When the scaling factor is taken to be $(-1)^n$, which means that complete synchronization and anti-synchronization coexist in a chaotic system, they display a good application prospect in digital communication.[17,18] We have given the synchronization coexistence scheme in a symmetrical chaotic system,[19] but the controller proposed is complex and only adapted to several dynamical systems, which reduces the practicality.

Motivated by the above discussion, in this paper, we continue to investigate the coexistence of synchronization and anti-synchronization for a kind of chaotic system. Based on Huang's work,[20] a simple adaptive synchronization strategy is proposed based on the invariance principle of differential equations. Illustrative examples about the Lorenz system and a new 4D hyper-chaotic system are given to show the effectiveness of the above-mentioned approach. Finally, a novel information transmission method with good secure performance based on the adaptive synchronization and anti-synchronization is proposed.

2. The synchronization coexistence scheme

In this paper, we consider a general continuous-time nonlinear chaotic system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}, t), \quad \boldsymbol{x}(t_0) = \boldsymbol{x}_0 \in \boldsymbol{R}^n. \tag{1}$$

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Here, $\boldsymbol{x} \in \boldsymbol{R}^n$ is the state vector, and $\boldsymbol{f}(\boldsymbol{x}, t) \in \boldsymbol{R}^n$ is a continuous vector function. Let $\boldsymbol{\Omega} \subset \boldsymbol{R}^n$ be a bounded set including the whole attractor of Eq. (1), which is usually called the drive system, and the response system with controller $\boldsymbol{U} \in \boldsymbol{R}^n$ is introduced as follows:

$$\dot{\boldsymbol{y}} = \boldsymbol{f}(\boldsymbol{y}, t) + \boldsymbol{U}, \quad \boldsymbol{y}(t_0) = \boldsymbol{y}_0 \in \boldsymbol{R}^n. \tag{2}$$

Suppose that $\boldsymbol{y}$ is defined on a set $\boldsymbol{S}$, and $\boldsymbol{S} \supset \boldsymbol{\Omega}$.

Definition 1 If the state vector of drive system (1) can be broken into two parts $(\boldsymbol{u}, \boldsymbol{v})$, which are defined as the following differential equations:

$$\dot{\boldsymbol{u}} = \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{v}) = \boldsymbol{M}(\boldsymbol{v}) \cdot \boldsymbol{u}, \quad \dot{\boldsymbol{v}} = \boldsymbol{h}(\boldsymbol{u}, \boldsymbol{v}), \tag{3}$$

and the expressions $\boldsymbol{u}$, $\boldsymbol{v}$, $\boldsymbol{M}(\boldsymbol{v})$ match the following condition:

(i) $\boldsymbol{u} = (x_{n_1}, x_{n_2}, \ldots, x_{n_r})$, $\boldsymbol{v} = (x_{m_1}, x_{m_2}, \ldots, x_{m_s})$; n_i $(i = 1, \ldots, r)$, m_j $(j = 1, \ldots, s) \in \{1, 2, \ldots, n\}$,

$$\forall i, j, \quad n_i \neq m_j, \quad r + s = n;$$

(ii) $\boldsymbol{M}(\boldsymbol{v})$ is an $r \times r$ matrix with components $\boldsymbol{M}_{ij}(\boldsymbol{v})$which are smooth functions and have no $\boldsymbol{u}$ dependence;

then, one could label this special chaotic system as a partially linear chaotic system.

Remark 1 The above definition can be considered as the extension of a partially linear system in the case of low-dimensional chaotic systems.[12]

Remark 2 Many known chaotic systems have the form of system (3) by an appropriate partition, such as the Lorenz-family systems[21] and their derivative system,[22−24], the hyper-chaotic Lorenz system[25], the hyper-chaotic Chen system,[26] the hyper-chaotic Lü system,[27] the generalized Lorenz system,[28] the Chen–Lia system,[29] the new 4-D systems,[30−32] etc.

Our aim is to design the simple and practical controller $\boldsymbol{U}$, with which the synchronization and anti-synchronization can coexist in one chaotic system. For this purpose, let the error state vector be

$$\begin{aligned} &e_i = y_i + x_i, \quad i = 1, 2, \ldots, r; \\ &e_j = y_j - x_j, \quad j = 1, 2, \ldots, s. \end{aligned} \tag{4}$$

Assumption 1 For any $\boldsymbol{y} = (y_1, \ldots, y_s)^{\mathrm{T}}$, $\boldsymbol{z} = (z_1, \ldots, z_s)^{\mathrm{T}} \in \boldsymbol{S}$, there exists a positive constant L such that

$$\begin{aligned} &|h_i(y) - h_i(z)| \leq L \max_{1 \leq j \leq s} |y_j - z_j|, \\ &i = 1,\ 2, \cdots, s, \quad \forall \boldsymbol{y}, \boldsymbol{z} \in \boldsymbol{S}. \end{aligned}$$

Theorem 1 For drive system (1) and response system (2), if the controller $\boldsymbol{U}$ is given by

$$\begin{aligned} &\text{if} \quad x_i \in \boldsymbol{u}, \quad \boldsymbol{U}_1 = \alpha_i (y_i + x_i), \quad i = 1, 2, \ldots, r \\ &\text{and if} \quad x_j \in \boldsymbol{v}, \boldsymbol{U}_2 = \varepsilon_j (y_j - x_j), \quad j = 1, 2, \ldots, s, \end{aligned} \tag{5}$$

where adaptive control parameters α_i, ε_j are updated by the following laws:

$$\begin{aligned} &\dot{\alpha}_i = -\beta_i e_i^2 = -\beta_i (y_i + x_i)^2, \quad i = 1, 2, \ldots, r; \\ &\dot{\varepsilon}_j = -\gamma_j e_j^2 = -\gamma_j (y_j - x_j)^2, \quad j = 1, 2, \ldots, s. \end{aligned} \tag{6}$$

Here, β_i, γ_j are arbitrary positive constants. Then asymptotically stable anti-synchronization can be achieved in the subspace $\boldsymbol{u}$ and asymptotically stable synchronization can be achieved in the subspace $\boldsymbol{v}$.

Proof According to dynamic equations (1) and (1), one can obtain

$$\begin{aligned} &\dot{e}_i = g_i(y) + g_i(x) + \alpha_i (y_i + x_i), \quad i = 1, 2, \ldots, r; \\ &\dot{e}_j = h_j(y) - h_j(x) + \varepsilon_j (y_j - x_j), \quad j = 1, 2, \ldots, s. \end{aligned} \tag{7}$$

Since $\boldsymbol{g}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{u}, \boldsymbol{v}) = \boldsymbol{M}(\boldsymbol{v})\boldsymbol{u}$ and $\boldsymbol{v}$ is the chaotic vector, then a positive constant K exists such that

$$\begin{aligned} &|g_i(y) + g_i(x)| \leq K \max_{1 \leq j \leq r} |y_j + x_j|, \\ &i = 1,\ 2, \ldots, r, \quad \forall \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{S}. \end{aligned}$$

For system (7), construct the Lyapunov function in the form of

$$\begin{aligned} V = &\frac{1}{2} \sum_{i=1}^{r} e_i^2 + \frac{1}{2} \sum_{j=1}^{s} e_j^2 + \frac{1}{2} \sum_{i=1}^{r} \frac{1}{\beta_i} (\alpha_i + K_1)^2 \\ &+ \frac{1}{2} \sum_{j=1}^{s} \frac{1}{\gamma_j} (\varepsilon_j + K_2)^2, \end{aligned} \tag{8}$$

where $K_1 > rK$, $K_2 > sL$.

Taking the time derivation of V along the trajectories of the error system leads to

$$\begin{aligned}\dot{V} &= \sum_{i=1}^{r} e_i(\dot{y}_i + \dot{x}_i) + \sum_{i=1}^{r} \frac{1}{\beta_i}(\alpha_i + K_1)\,\dot{\alpha}_i + \sum_{j=1}^{s} e_j(\dot{y}_i - \dot{x}_i) + \sum_{j=1}^{s} \frac{1}{\gamma_j}(\varepsilon_j + K_2)\,\dot{\varepsilon}_j \\ &= \sum_{i=1}^{r} e_i(g_i(y) + g_i(x) + \alpha_i e_i) - \sum_{i=1}^{r} (\alpha_i + K_1)e_i^2 + \sum_{j=1}^{s} e_j(h_j(y) - h_j(x) + \varepsilon_j e_j) - \sum_{j=1}^{s} (\varepsilon_j + K_2)e_j^2 \\ &\le \sum_{i=1}^{r} e_i(K \max_{1\le l\le r} |e_l|\,) - K_1 \sum_{i=1}^{r} e_i^2 + L \sum_{j=1}^{s} e_j(\max_{1\le m\le s} |y_m - x_m|\,) - K_2 \sum_{j=1}^{s} e_j^2 \\ &\le K \sum_{i=1}^{r} e_i \sum_{l=1}^{r} |e_l| - K_1 \sum_{i=1}^{r} e_i^2 + L \sum_{j=1}^{s} e_j \sum_{m=1}^{s} |e_m| - K_2 \sum_{j=1}^{s} e_j^2 \\ &\le (rK - K_1) \sum_{i=1}^{r} e_i^2 + (sL - K_2) \sum_{j=1}^{s} e_j^2.\end{aligned}$$

It is obvious that $\dot{V} = 0$ if and only if $e_i = e_j = 0$ $(i = 1, \ldots, r; j = 1, \ldots, s)$, then from Eq. (6), one can obtain the largest invariant set $\boldsymbol{E}$ which is described as

$$\boldsymbol{E} = \{\boldsymbol{e} \in \boldsymbol{R}^n, \boldsymbol{\alpha} \in \boldsymbol{R}^r, \boldsymbol{\varepsilon} \in \boldsymbol{R}^s \,|\, e_i = e_j = 0, \alpha = \alpha^*, \varepsilon = \varepsilon^*\},$$

where α^*, ε^* are constant vectors depending on dynamical systems and their initial conditions. So, based on the LaSalle invariance principle,[33] starting with arbitrary initial conditions in set $\boldsymbol{S}$, the orbit of the error system will converge asymptotically to the set $\boldsymbol{E}$, namely anti-synchronization and synchronization could coexist in one chaotic system.

Remark 3 For a chaotic system, the choice of vector $(\boldsymbol{u}, \boldsymbol{v})$ is not unique.

Consequently, the synchronization coexistence mode is diverse, which can be used in digital communication for achieving high security.

Remark 4 Compared with the results in Ref. [19], the adaptive technique is used in our new controllers which will lead to fast convergence.

Remark 5 In the proposed synchronization method (5), it is not necessary to exert controllers on all the variables, which will be illustrated in the following example.

3. Numerical simulations

In this section, a series of numerical experiments are given to demonstrate the synchronization coexistence phenomenon and verify the performance of the proposed control method. In what follows, the fourth-order Runge–Kutta integration algorithm is applied to the simulations with time step size 0.001.

Example 1

The well-known Lorenz system is given by

$$\begin{cases} \dot{x}_1 = p_1(x_2 - x_1), \\ \dot{x}_2 = p_2 x_1 - x_1 x_3 - x_2, \\ \dot{x}_3 = x_1 x_2 - p_3 x_3, \end{cases} \tag{9}$$

which has a chaotic attractor with $p_1 = 10$, $p_2 = 28$, $p_3 = 8/3$.

To match Eq. (3), one can choose $\boldsymbol{u} = (x_1, x_2)$, $\boldsymbol{v} = x_3$ and

$$\begin{aligned} \boldsymbol{M}(\boldsymbol{v}) &= \begin{pmatrix} -p_1 & p_1 \\ p_2 - x_3 & -1 \end{pmatrix}, \\ \boldsymbol{h}(\boldsymbol{u}, \boldsymbol{v}) &= x_1 x_2 - p_3 x_3. \end{aligned} \tag{10}$$

Thus, the corresponding error system is

$$\begin{cases} \dot{e}_1 = p_1(e_2 - e_1) + \alpha_1 e_1, \\ \dot{e}_2 = p_2 e_1 - e_2 - e_1 e_3 - e_1 x_3 + e_3 x_1 + \alpha_2 e_2, \\ \dot{e}_3 = -p_3 e_3 + e_1 e_2 - e_1 x_2 - e_2 x_1 + \varepsilon_3 e_3 \end{cases} \tag{11}$$

with the updated law (6). According to Theorem 1, asymptotically stable anti-synchronization and synchronization can coexist in the Lorenz chaotic system.

On the other hand, by $\dot{V}(\cdot) = 0$ in Section 2, one has

$$\begin{aligned} &e_1((p_1 + p_2)e_2 - e_2 x_3 - e_3 x_2) \\ &= K_1(e_1^2 + e_2^2) + K_2 e_3^2 + p_1 e_1^2 + e_2^2 + p_3 e_3^2. \end{aligned} \tag{12}$$

It is apparent that if $e_1 = 0$ on the left-hand side of the above equation, then $e_2 = e_3 = 0$ from the

right-hand side of the above equation. Thus, we can set $\alpha_2 = \varepsilon_1 = 0$, that is only the time series of variable x_1 are selected as the driven signal, and let $\beta_1 = 1.0$. In the numerical simulations of Lorenz chaotic system, the initial conditions of the drive system and the error system are chosen to be $\boldsymbol{x}_0 = (-1.0, 2.0, 5.0)$, $\boldsymbol{e}_0 = (5.0, -6.0, 12.0)$. Figures 1(a)–1(c) show the time histories of state vector for the drive system and the response system, respectively, and the synchronization coexistence error curve is displayed in Fig. 2. As expected, one can observe that asymptotically stable synchronization and anti-synchronization are achieved simultaneously.

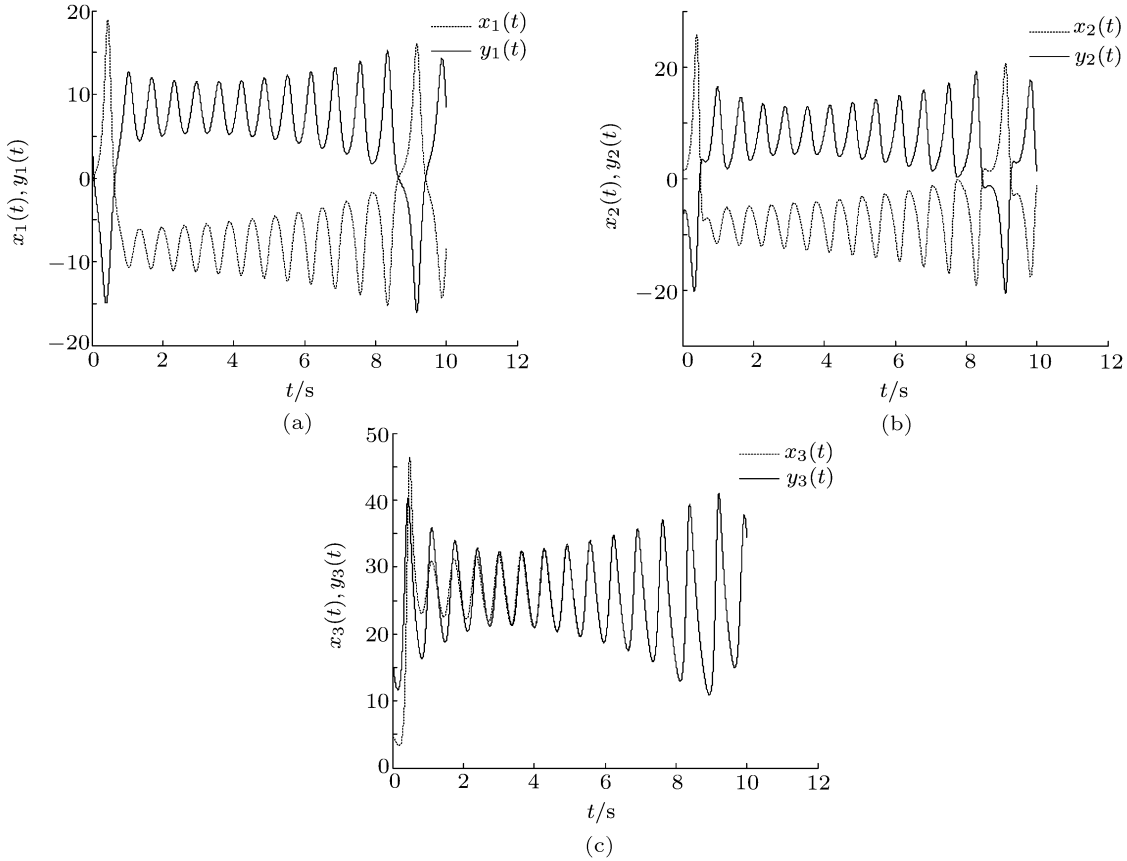


Fig. 1. Time histories of synchronization coexistence for the Lorenz chaotic system, where panel (a) shows anti-synchronization between x_1 and y_1, panel (b) exhibits anti-synchronization between x_2 and y_2, and panel (c) displays synchronization between x_3 and y_3.

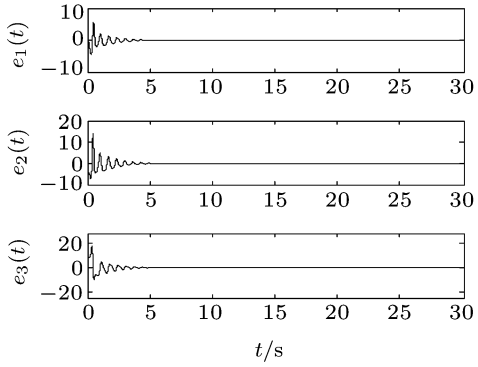


Fig. 2. Synchronization coexistence errors for the Lorenz chaotic system.

Example 2

The new 4D hyper-chaotic system is given by

$$\begin{cases} \dot{x}_1 = p_1(x_2 - x_1) + x_2 x_3, \\ \dot{x}_2 = p_2(x_1 + x_2) - x_1 x_3, \\ \dot{x}_3 = -p_3 x_3 - p_5 x_4 + x_1 x_2, \\ \dot{x}_4 = -p_4 x_4 + p_6 x_3 + x_1 x_2, \end{cases} \tag{13}$$

which has abundant chaotic and hyper-chaotic behaviours over a large range of parameter space.[32]

To match Eq. (3), one can choose $\boldsymbol{u} = (x_1, x_2)$,

$\boldsymbol{v} = (x_3, x_4)$ and

$$\boldsymbol{M}(\boldsymbol{v}) = \begin{pmatrix} -p_1 & p_1 + x_3 \\ p_2 - x_3 & p_2 \end{pmatrix},$$

$$\boldsymbol{h}(\boldsymbol{u}, \boldsymbol{v}) = \begin{pmatrix} -p_3 x_3 - p_5 x_4 + x_1 x_2 \\ -p_4 x_4 + p_6 x_3 + x_1 x_2 \end{pmatrix}. \quad (14)$$

Thus, the corresponding error system is

$$\begin{cases} \dot{e}_1 = p_1(e_2 - e_1) + e_2 e_3 + e_2 x_3 - e_3 x_2 + \alpha_1 e_1, \\ \dot{e}_2 = p_2(e_1 + e_2) - e_1 e_3 - e_1 x_3 + e_3 x_1 + \alpha_2 e_2, \\ \dot{e}_3 = -p_3 e_3 - p_5 e_4 + e_1 e_2 - e_1 x_2 - e_2 x_1 + \varepsilon_3 e_3, \\ \dot{e}_4 = -p_4 e_4 + p_6 e_3 + e_1 e_2 - e_1 x_2 - e_2 x_1 + \varepsilon_4 e_4 \end{cases} \quad (15)$$

with the updated law (6). According to Theorem 1, asymptotically stable anti-synchronization and synchronization can coexist in the 4D hyper-chaotic system.

Making the analysis as stated before, one has

$$\begin{aligned} & e_1[(p_1 + p_2)e_2 + e_2 e_3 + e_2 e_4 - 2e_3 x_2 - e_4 x_2] \\ & + e_4[(p_6 - p_5)e_3 - e_2 x_1] \\ & = K_1(e_1^2 + e_2^2) + K_2(e_3^2 + e_4^2) \\ & \quad + p_1 e_1^2 - p_2 e_2^2 + p_3 e_3^2 + p_4 e_4^2. \end{aligned} \quad (16)$$

It is apparent that if $e_1 = e_4 = 0$ on the left-hand side of the above equation, then $e_2 = e_3 = 0$ from the right-hand side of the above equation. Thus, we set $\alpha_2 = \varepsilon_1 = 0$, that is, only the time series of variable x_1, x_4 are selected as the driven signals, and let $\beta_1 = 0.1$, $\gamma_2 = 0.05$. In the numerical simulations of 4D hyper-chaotic system, the initial conditions of the drive system and the error system are chosen as $\boldsymbol{x}_0 = (-1.0, -4.0, 5.0, 3.0)$, $\boldsymbol{e}_0 = (9.0, -6.0, 3.0, -4.0)$. Parameters are fixed as follows: $p_1 = 50$, $p_2 = 4.025, p_3 = 13, p_4 = 8, p_5 = 33, p_6 = 30$ to ensure that system (13) has hyper-chaotic behaviour. Figures 3(a)–3(d) show the time histories of state vector for the drive system and the response system, respectively, and the synchronization coexistence error curves are displayed in Fig. 4. As expected, one can observe that asymptotically stable synchronization and anti-synchronization are achieved simultaneously.

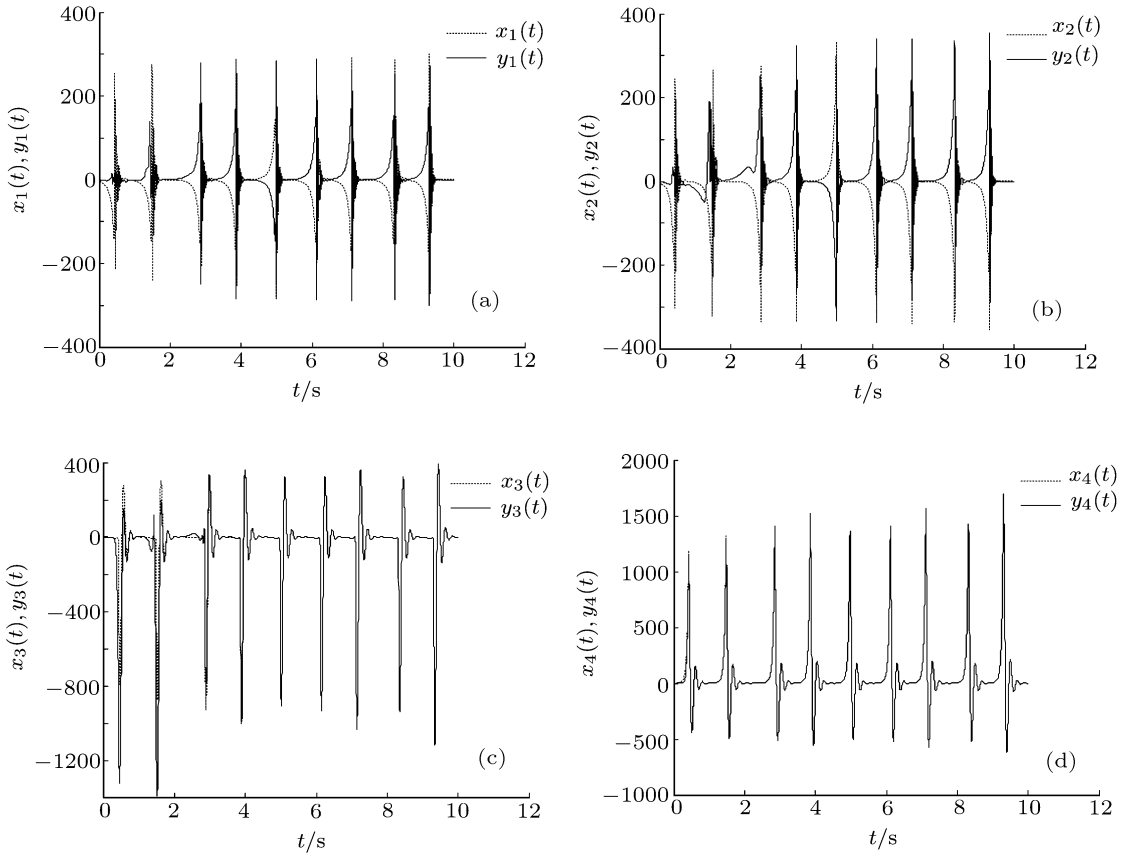


Fig. 3. Time histories of synchronization coexistence for the 4D hyper-chaotic system, where panel (a) shows anti-synchronization between x_1 and y_1, panel (b) exhibits anti-synchronization between x_2 and y_2, panel(c) indicates synchronization between x_3 and y_3, and panel (d) displays synchronization between x_4 and y_4.

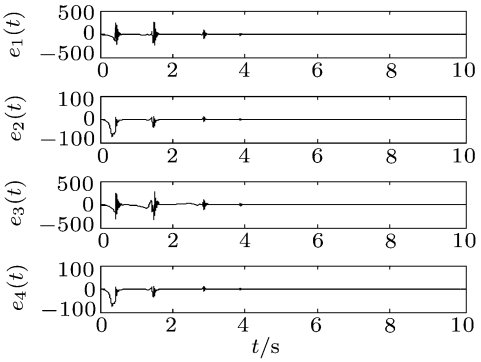


Fig. 4. Synchronization coexistence errors for the 4D hyper-chaotic system.

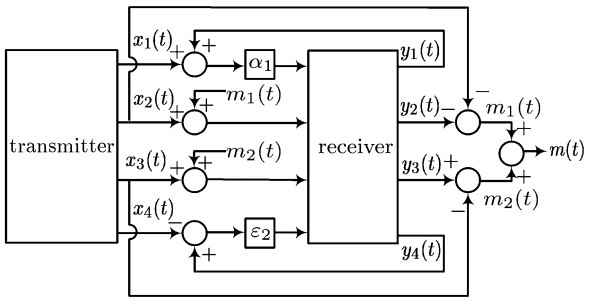


Fig. 5. Communication scheme based on synchronization and anti-synchronization coexistence in a hyper-chaotic system.

4. Application to secure communication

In this section, we apply the adaptive synchronization coexistence method to secure communication in a 4D hyper-chaotic system. The hyper-chaotic system has more than one positive Lyapunov exponent, with which the signal having larger store capacity and information processing capability can be transmitted.[34,35]

A schematic diagram for the communication scheme is shown in Fig. 5. System $\boldsymbol{X}$ is used as a transmitter and $\boldsymbol{Y}$ as a receiver. Signals $m_1(t)$ and $m_2(t)$, which are formulated by appropriately dividing message signal $m(t)$, are added to the variables $x_2(t)$ and $x_3(t)$ respectively. Both the resulting signals $s_1(t) = x_2(t) + m_1(t)$, $s_2(t) = x_3(t) + m_2(t)$ and the variables $x_1(t)$, $x_4(t)$ are transmitted to the receiver. At the receiver end, variables $x_1(t)$, $x_4(t)$ serve as the driving signals that enable system $\boldsymbol{X}$ and $\boldsymbol{Y}$ to achieve synchronization coexistence. With the occurrence of synchronization and anti-synchronization, signals can be recovered from the transformation $m_1(t) = -y_2(t) - x_2(t)$, $m_2(t) = y_3(t) - x_3(t)$.

In the general secure communication scheme, the message signal is added to the chaotic carrier and the resulting signal is used to drive the receiver, which makes the receiver unable to realize exact synchronization with the transmitter. In contrast, our method avoids such a problem, because the driving signals $x_1(t)$, $x_4(t)$ are purely chaotic. In addition, the modes for dividing message signal $m(t)$ vary, which also can improve the security.

In the following, the presented secure communication scheme is demonstrated and the control parameters are selected as above. Let the message signal be $m(t) = 2(\cos(0.5t) + \sin(2t))$ and $m_1(t) = 2\cos(0.5t) + 0.5\sin(2t)$, $m_2(t) = 1.5\sin(2t)$. Signals $m_1^*(t)$ and $m_2^*(t)$ transmitted in the channel are shown in Figs. 6(a) and 6(b), from which one can see that the message signals are changed completely. Figures 7 (a) and 7(b) display the contrast between the recovered message signals $m_1(t), m_2(t)$ and the original ones $m_1'(t)$, $m_2'(t)$, from which one can see that the message signals $m_1(t)$, $m_2(t)$ are recovered exactly after a short transient.

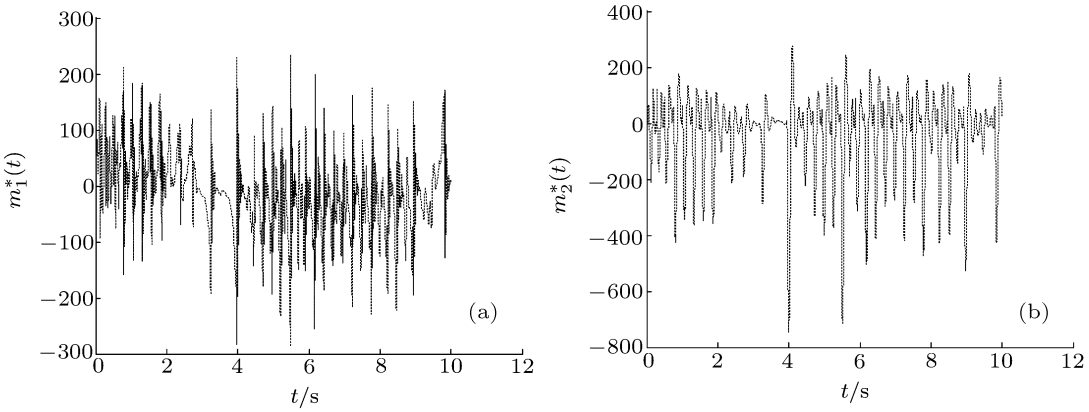


Fig. 6. Signals transmitted in the channel (a) $m_1^*(t) = x_2(t) + m_1(t)$ and (b)$m_2^*(t) = x_3(t) + m_2(t)$.

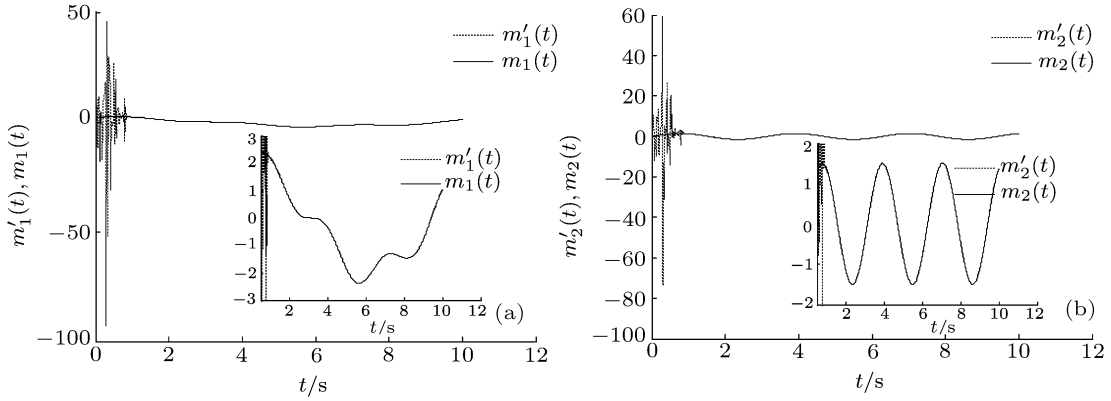


Fig. 7. Contrast between recovered signals and original signals (a) $m_1'(t) - m_1(t)$ and (b) $m_2'(t) - m_2(t)$.

5. Conclusion

Based on the Lyapunov stability theory and the invariance principle of differential equations, a new method is developed to simultaneously realize synchronization and anti-synchronization for partially linear chaotic systems. In comparison with the existing methods, the present controller is not only simple but also able to be generalized. In particular, the controller can be simplified for some chaotic systems, which has been illustrated by using the Lorenz chaotic system and a new hyper-chaotic system. Numerical simulations demonstrate the effectiveness and validity of the proposed control technique. Moreover, a novel secure communication scheme is put forward, by which the message signal transmitted between a sender and a receiver can be recovered accurately.

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