Reliable impulsive synchronization for a class of nonlinear chaotic systems*

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The problem of reliable impulsive synchronization for a class of nonlinear chaotic systems has been investigated in this paper. Firstly a reliable impulsive controller is designed by using the impulsive control theory. Then by the uniform asymptotic stability criteria of systems with impulsive effects, some sufficient conditions for reliable impulsive synchronization between the drive system and the response system are obtained. Numerical simulations are given to show the effectiveness of the proposed method.

Keywords: chaotic systems, synchronization, reliable impulsive synchronization
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1. Introduction

Synchronization is a kind of typical collective behaviour and basic motion in nature. After the pioneering work of Pecora and Carroll[1], chaos synchronization as an important topic in nonlinear science has been widely investigated by a variely of methods, such as linear or nonlinear controllers,[2,3] adaptive controllers[4−10] and so on.

In the real world, there are natural phenomena that characteristically change their state abruptly at certain moments. The perturbations are usually assumed to act instantaneously, that is, in the form of impulses.[11−14] It has been shown that the impulsive control or synchronization approach is effective and robust in control or synchronization of chaotic systems.[15−23]

There is a fully reliable assumption that all actuators of the systems are in good working condition. However, in practical situations, failure of actuators often occurs. When unpredictable faults occur in actuators, the stability and performance of the system may be damaged.[24−31] Thus, it is important to design a reliable controller such that the stability and performance of the system can tolerate actuator failures. Very recently, there have been many related works on reliable control reported in the literature.[28−31] The reliable synchronization problem for a general class of chaotic systems was addressed in Ref. [30]. In Ref. [31], the authors considered the problem of impulsive robust fault-tolerant feedback control for chaotic Lur'e systems. In Refs. [24]–[31], the reliable controllers designed are all linear or nonlinear feedback controllers. However, to the best of our knowledge, there are few results on designing a reliable impulsive controller.

The problem of reliable impulsive synchronization for a class of nonlinear chaotic systems is investigated in this paper. Firstly a reliable impulsive controller is designed by using the impulsive control theory. Then by the uniform asymptotic stability criteria of systems with impulsive effects, some sufficient conditions for uniform asymptotic synchronization between the drive system and the response system are obtained. Numerical simulations are given to show the effectiveness of the proposed method.

The remainder of this paper is organized as follows: In Section 2, the problem to be investigated is given and some useful lemmas are also presented. In Section 3, some sufficient conditions for uniform asymptotic synchronization between the drive system and the response system are obtained. In Section 4, two examples are given to demonstrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

Notation Throughout this paper, the superscripts '–1' and 'T' stand for the inverse and transpose

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of a matrix, respectively; $\mathbb{R}^n$ denotes n-dimensional Euclidean space. Let $\mathbb{R}_+ = [0, \infty)$, $\mathbb{N} = \{0, 1, 2, \dots\}$, $\mathbb{N}_+ = \{1, 2, \dots\}$; $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. For real symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, the notation $\boldsymbol{X} \geq \boldsymbol{Y}$ (respectively, $\boldsymbol{X} > \boldsymbol{Y}$) means that the matrix $\boldsymbol{X} - \boldsymbol{Y}$ is positive semi-definite (respectively, positive definite); $\boldsymbol{I}_n \in \mathbb{R}^{n\times n}$ is an identity matrix; $\lambda_{\min}(\boldsymbol{P})$ ($\lambda_{\max}(\boldsymbol{P})$) denotes the smallest (largest) eigenvalue of $\boldsymbol{P}$. For a vector $\boldsymbol{x} \in \mathbb{R}^n$, let $||\boldsymbol{x}||$ denote the Euclidean vector norm, i.e., $||\boldsymbol{x}|| = \sqrt{\boldsymbol{x}^{\mathrm{T}}\boldsymbol{x}}$, and for $\boldsymbol{A} \in \mathbb{R}^{n\times n}$, let $||\boldsymbol{A}||$ indicate the norm of $\boldsymbol{A}$ induced by the Euclidean vector norm, i.e., $||\boldsymbol{A}|| = \sqrt{\lambda_{\max}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})}$. For a real-valued vector function $\boldsymbol{f} : \mathbb{R}^n \to \mathbb{R}^n$, we define $\Delta\boldsymbol{f}(\boldsymbol{a}) = \boldsymbol{f}(\boldsymbol{a}^+) - \boldsymbol{f}(\boldsymbol{a}^-)$, where $\boldsymbol{f}(\boldsymbol{a}^+) = \lim_{\boldsymbol{x}\to\boldsymbol{a}^+} \boldsymbol{f}(\boldsymbol{x})$, $\boldsymbol{f}(\boldsymbol{a}^-) = \lim_{\boldsymbol{x}\to\boldsymbol{a}^-} \boldsymbol{f}(\boldsymbol{x})$. The K class function is defined as $K = \{\varphi \in C(\mathbb{R}_+, \mathbb{R}_+) : \varphi(t)$ is strictly increasing and $\varphi(0) = 0\}$.

2. Problem formulation

Consider a nonlinear chaotic system, which is described by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t), \tag{1}$$

where $\boldsymbol{x}(t) = (x_1(t), x_2(t), \dots, x_n(t))^{\mathrm{T}} \in \Omega \subseteq \mathbb{R}^n$ is the state of the system, Ω is a bounded set, $\boldsymbol{f} = (f_1, f_2, \dots, f_n)^{\mathrm{T}} \in \mathbb{R}^n$ is the nonlinear vector field function of the system. The system (1) is considered as a drive system.

Assumption 1 For any $\boldsymbol{x}(t), \boldsymbol{y}(t) \in \Omega$, there exists a positive constant $L = L(\Omega) > 0$, such that $||\boldsymbol{f}(\boldsymbol{x}(t), t) - \boldsymbol{f}(\boldsymbol{y}(t), t)|| \leq L||\boldsymbol{x}(t) - \boldsymbol{y}(t)||$.

A response system is constructed as follows:

$$\begin{aligned} &\dot{\boldsymbol{y}}(t) = \boldsymbol{f}(\boldsymbol{y}(t), t), \quad t \neq t_k, \\ &\Delta\boldsymbol{y}(t) = \boldsymbol{U}_k^F(\boldsymbol{x}(t), \quad \boldsymbol{y}(t), t), \quad t = t_k, k \in \mathbb{N}_+, \end{aligned} \tag{2}$$

where $\boldsymbol{y} = (y_1, y_2, \dots, y_n)^{\mathrm{T}} \in \mathbb{R}^n$ is the state of the response system, $\boldsymbol{U}_k^F(\boldsymbol{x}(t), \boldsymbol{y}(t), t) \in \mathbb{R}^n$ is an impulsive controller subject to actuator failure, $\Delta\boldsymbol{y}(t_k) = \boldsymbol{y}(t_k^+) - \boldsymbol{y}(t_k^-)$, $\boldsymbol{y}(t_k^+) = \lim_{h\to 0+} \boldsymbol{y}(t_k + h)$, $\boldsymbol{y}(t_k^-) = \lim_{h\to 0-} \boldsymbol{y}(t_k + h)$, the impulsive time instants t_k satisfy $0 \leq t_0 < t_1 < t_2 < \dots < t_{k-1} < t_k < \dots$, and $\lim_{k\to+\infty} t_k = +\infty$. Without loss of generality, we assume that $\boldsymbol{y}(t_k^-) = \boldsymbol{y}(t_k)$, that is, the solution $\boldsymbol{y}(t)$ is left continuous at the time t_k.

From systems (1) and (2), we obtain the following error dynamical system

$$\begin{aligned} &\dot{\boldsymbol{e}}(t) = \boldsymbol{f}(\boldsymbol{y}(t), t) - \boldsymbol{f}(\boldsymbol{x}(t), t), \quad t \neq t_k, \\ &\Delta\boldsymbol{e}(t) = \boldsymbol{U}_k^F(\boldsymbol{x}(t), \boldsymbol{y}(t), t), \quad t = t_k, \quad k \in \mathbb{N}_+, \end{aligned} \tag{3}$$

where $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{x}(t)$, $\Delta\boldsymbol{e}(t_k) = \boldsymbol{e}(t_k^+) - \boldsymbol{e}(t_k^-)$.

For the reliable impulsive synchronization problem, the following actuator fault model is used.

$$\boldsymbol{U}_k^F(\boldsymbol{x}(t), \boldsymbol{y}(t), t) = \boldsymbol{M}_k\boldsymbol{U}_k(\boldsymbol{x}(t), \boldsymbol{y}(t), t), \tag{4}$$

where $\boldsymbol{U}_k(\boldsymbol{x}(t), \boldsymbol{y}(t), t)$ is the reliable impulsive controller and $\boldsymbol{M}_k = \mathrm{diag}\{m_k^1, m_k^2, \dots, m_k^n\}$ is the actuator failure matrix with the following property[28]

$$0 \leq m_k^i \leq 1, \quad i = 1, 2, \dots, n. \tag{5}$$

Here $m_k^i = 0$ means that the ith impulsive actuator is invalid, $m_k^i \in (0, 1)$ implies that the ith impulsive actuator is at fault to some extent and $m_k^i = 1$ denotes that the ith system actuator operates properly. For a given diagonal matrix $\boldsymbol{M}_0 \geq 0$, the set $\Theta = \{\boldsymbol{M}_k | \boldsymbol{M}_k \geq \boldsymbol{M}_0\}$ is called an admissible set of actuator fault. The reliable impulsive synchronization problem is to design a reliable impulsive controller such that the response system (2) and the drive system (1) are uniformly asymptotically synchronized not only when all impulsive controller components operate well, but also in the case of any failure $\boldsymbol{M}_k \in \Theta$ occurring.

The reliable impulsive controller is chosen as follows:

$$\boldsymbol{U}_k(\boldsymbol{x}(t), \boldsymbol{y}(t), t) = b_k\boldsymbol{e}(t), \quad k \in \mathbb{N}_+, \tag{6}$$

where $b_k \in \mathbb{R}$ are constants to be determined later. This type of controller is called an impulsive proportional control law.

Therefore, the error dynamical system (3) is transformed into

$$\begin{aligned} &\dot{\boldsymbol{e}}(t) = \boldsymbol{f}(\boldsymbol{y}(t), t) - \boldsymbol{f}(\boldsymbol{x}(t), t), \quad t \neq t_k, \\ &\Delta\boldsymbol{e}(t) = \boldsymbol{M}_k(b_k\boldsymbol{e}(t)), \ t = t_k, \ k \in \mathbb{N}_+. \end{aligned} \tag{7}$$

Before proceeding further, we recall some preliminaries which will be used throughout the proofs of our main results.

Definition 1[13,17] Let $V : \mathbb{R}^n \times \mathbb{R}_+ \to \mathbb{R}_+$, then V is said to belong to the class ν_0 if

(i) V is continuous in each of the sets $\mathbb{R}^n \times (t_{k-1}, t_k]$, and for each $\boldsymbol{x} \in \mathbb{R}^n$, $k \in \mathbb{N}_+$, $\lim_{(\boldsymbol{y},t)\to(\boldsymbol{x},t_k^-)} V(y, t) = V(\boldsymbol{x}, t_k^-)$ exists;

(ii) V is locally Lipschitzian in $\boldsymbol{x} \in \mathbb{R}^n$.

Let

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), t), \ t \neq t_k, \\ &\Delta\boldsymbol{x}(t) = \boldsymbol{I}_k(\boldsymbol{x}(t), t), \ t = t_k, \ k \in \mathbb{N}_+, \end{aligned}$$

then we have the following generalized upper right Dini derivative of a Lyapunov function $V(\boldsymbol{x}(t),t)$.

Definition 2[13,17] For $V \in \nu_0$, $(\boldsymbol{x}(t),t) \in \mathbb{R}^n \times (t_{k-1},t_k]$, we define

$$D^+V(\boldsymbol{x}(t),t) \triangleq \lim_{h\to 0^+} \sup \frac{1}{h}[V(\boldsymbol{x}(t) + h\boldsymbol{f}(\boldsymbol{x}(t),t),t+h) - V(\boldsymbol{x}(t),t)].$$

The class of systems with impulse effects under investigation can be described by equations of the form

$$\begin{aligned} &\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t),t), \quad t \neq t_k, \\ &\Delta\boldsymbol{x}(t) = \boldsymbol{I}_k(\boldsymbol{x}(t)), \quad t = t_k, \quad k \in \mathbb{N}_+, \end{aligned} \tag{8}$$

where $\boldsymbol{x}(t) \in X \subseteq \mathbb{R}^n$ denotes the state, $f \in C[\mathbb{R}^n \times \mathbb{R}, \mathbb{R}^n]$ satisfying a Lipschitz condition with respect to $\boldsymbol{x}$, which guarantees the existence and uniqueness of solutions of system (8) for given initial conditions. $\boldsymbol{I}_k \in \mathbb{R}^n \to \mathbb{R}^n$ denotes the increment change of the state at time t_k.

Definition 3[11] The trivial solution $\boldsymbol{x}(t) = 0$ of systems (8) is said to be uniformly asymptotically stable if (i) for every $\delta > 0$ and for every $t_0 > 0$ there exists an $\varepsilon = \varepsilon(\delta)$ independent of t_0 such that for any the solution $||\boldsymbol{x}(t_0)|| < \delta$ of system (8) satisfies $||\boldsymbol{x}(t)|| < \varepsilon$ for all $t \geq t_0$; (ii) for every $\eta > 0$ and for every $t_0 > 0$ there exists a $T(\eta)$ independent of t_0 such that $||\boldsymbol{x}(t)|| < \eta$, $t \geq t_0 + T(\eta)$.

Lemma 1[32] Assume that for system (8) $\boldsymbol{f}(\boldsymbol{0},t) = \boldsymbol{0}$ and $\boldsymbol{I}_k(\boldsymbol{0}) = \boldsymbol{0}$ for all $t \in \mathbb{R}_+$ and $k \in \mathbb{N}$, there exists an $h \in C[\mathbb{R}_+,\mathbb{R}_+]$ such that $h(0) = 0$ and a $V : X \times \mathbb{R}_+ \to \mathbb{R}_+$ and ϕ_1, $\phi_2 \in K$ such that $\phi_1(||\boldsymbol{x}(t)||) \leq V(\boldsymbol{x}(t),t) \leq \phi_2(||\boldsymbol{x}(t)||)$ for all $(\boldsymbol{x}(t),t) \in X \times \mathbb{R}_+$.

1) Assume that for any solution $\boldsymbol{x}(t)$ of system (8) which is defined on $[t_0,+\infty)$, $V(\boldsymbol{x}(t),t)$ is left continuous on $[t_0,+\infty)$ and is differentiable everywhere on $(t_0,+\infty)$ except on an unbounded closed discrete set $E = \{t_1,t_2,\dots\}$, where E is the set of times when jumps occur for $\boldsymbol{x}(t)$ and that $V(\boldsymbol{x}(t_k^+),t_k^+)$ (which is actually $\lim_{t\to t_k^+} V(\boldsymbol{x}(t),t)$) is non-increasing for $k \in \mathbb{N}$. Furthermore, assume that $V(\boldsymbol{x}(t),t) \leq h(V(\boldsymbol{x}(t_k^+),t_k^+))$ is true for all $t \in (t_k,t_{k+1}]$, $k \in \mathbb{N}$. Then the equilibrium $\boldsymbol{x} = 0$ of system (8) is uniformly stable.

2) If in addition to 1), we assume that there exists a $\phi_3 \in K$ such that $DV(\boldsymbol{x}(t_k^+)),t_k^+) \leq -\phi_3(||\boldsymbol{x}(t_k^+)||)$ is true for all $k \in \mathbb{N}$, where

$$DV(\boldsymbol{x}(t_k^+),t_k^+) \triangleq \frac{1}{t_{k+1}-t_k}[V(\boldsymbol{x}(t_{k+1}^+),t_{k+1}^+) - V(\boldsymbol{x}(t_k^+),t_k^+)],$$

then the equilibrium $\boldsymbol{x} = \boldsymbol{0}$ of system (8) is uniformly asymptotically stable.

Remark 1 As shown in Ref. [32], Lemma 1 is less conservative than Theorem 13.1, 13.2 in Ref. [11]. Lemma 1 requires that the right limits of $V(\boldsymbol{x}(t),t)$ at times t_k when jumps occur be non-increasing and that at all other times between t_k and t_{k+1} the Lyapunov function $V(\boldsymbol{x}(t),t)$ be bounded by the combination of a pre-specified bounded function and the right limit of $V(\boldsymbol{x}(t),t)$ at t_k.

3. Main results

Theorem 1 Assume that the drive system (1) satisfies Assumption 1 and the reliable impulsive controller in the response system (2) is chosen as Eq. (6). If the following conditions are satisfied

(i) $\inf_{k\in\mathbb{N}_+}\{t_k - t_{k-1}\} > 0$,

(ii) $g_k \mathrm{e}^{2L\lambda} \leq q < 1$, $k \in \mathbb{N}_+$,

where $\lambda = \sup_{k\in\mathbb{N}_+}\{t_k - t_{k-1}\}$, $g_k = \max\{(1+b_k)^2, ||I + b_k M_0||^2\}$, then the error dynamical system (7) is uniformly asymptotically stable at the origin, which implies that the drive system (1) and the response system (2) are uniformly asymptotically synchronized for all $\boldsymbol{M}_k \in \Theta$, $k \in \mathbb{N}_+$.

Proof Consider the following Lyapunov candidate

$$V(\boldsymbol{e}(t),t) = \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t).$$

Taking the Dini derivative of $V(\boldsymbol{e}(t),t)$ for $t \in (t_k,t_{k+1}]$, $k \in \mathbb{N}$, we obtain,

$$\begin{aligned} D^+V(\boldsymbol{e}(t),t) &= \boldsymbol{e}^{\mathrm{T}}(t)\dot{\boldsymbol{e}}(t) + \dot{\boldsymbol{e}}^{\mathrm{T}}(t)\boldsymbol{e}(t) \\ &= 2\boldsymbol{e}^{\mathrm{T}}(t)(\boldsymbol{f}(\boldsymbol{y}(t),t) - \boldsymbol{f}(\boldsymbol{x}(t),t)) \\ &\leq 2L\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{e}(t) \\ &= 2LV(\boldsymbol{e}(t),t). \end{aligned}$$

Then

$$\begin{aligned} V(\boldsymbol{e}(t),t) &\leq \mathrm{e}^{2L(t-t_k)}V(\boldsymbol{e}(t_k^+),t_k^+) \\ &\leq \mathrm{e}^{2L(t_{k+1}-t_k)}V(\boldsymbol{e}(t_k^+),t_k^+) \\ &\leq \mathrm{e}^{2L\lambda}V(\boldsymbol{e}(t_k^+),t_k^+), \end{aligned}$$

where $\lambda = \sup_{k\in\mathbb{N}_+}\{t_k - t_{k-1}\}$.

Letting $h(s) = \mathrm{e}^{2L\lambda}s$ yields

$$V(\boldsymbol{e}(t),t) \leq h(V(\boldsymbol{e}(t_k^+),t_k^+)).$$

Since $\boldsymbol{0} \leq \boldsymbol{M}_0 \leq \boldsymbol{M}_k \leq \boldsymbol{I}$, it follows that

$$||\boldsymbol{I} + b_k\boldsymbol{M}_k||^2 \leq \max\{(1+b_k)^2, ||\boldsymbol{I} + b_k\boldsymbol{M}_0||^2\}.$$

Let $g_k = \max\{(1+b_k)^2, ||\boldsymbol{I} + b_k\boldsymbol{M}_0||^2\}$, then

$$\begin{aligned} V(\boldsymbol{e}(t_k^+), t_k^+) &= ||(\boldsymbol{I} + b_k\boldsymbol{M}_k)\boldsymbol{e}(t_k)||^2 \\ &\le ||\boldsymbol{I} + b_k\boldsymbol{M}_k||^2||\boldsymbol{e}(t_k)||^2 \\ &\le g_k V(\boldsymbol{e}(t_k), t_k) \\ &\le g_k \mathrm{e}^{2L\lambda} V(\boldsymbol{e}(t_{k-1}^+), t_{k-1}^+) \\ &\le V(\boldsymbol{e}(t_{k-1}^+), t_{k-1}^+). \end{aligned}$$

Hence $V(\boldsymbol{e}(t_k^+), t_k^+)$ is non-increasing.

Furthermore,

$$\begin{aligned} DV(\boldsymbol{e}(t_k^+), t_k^+) &= \frac{V(\boldsymbol{e}(t_{k+1}^+), t_{k+1}^+) - V(\boldsymbol{e}(t_k^+), t_k^+)}{t_{k+1} - t_k} \\ &\le \frac{g_{k+1}\mathrm{e}^{2L\lambda} - 1}{t_{k+1} - t_k} V(\boldsymbol{e}(t_k^+), t_k^+) \\ &\le -\frac{1-q}{t_{k+1} - t_k} ||\boldsymbol{e}(t_k^+)||^2 \\ &\le -\frac{1-q}{\lambda} ||\boldsymbol{e}(t_k^+)||^2. \end{aligned}$$

Letting $\phi_3(s) = (1-q)s^2/\lambda$ yields

$$DV(\boldsymbol{e}(t_k^+), t_k^+) \le -\phi_3(||\boldsymbol{e}(t_k^+)||).$$

By Lemma 1, the error dynamical system (3) is uniformly asymptotically stable at the origin, which implies that the drive system (1) and the response system (2) are uniformly asymptotically synchronized for all $\boldsymbol{M}_k \in \Theta$, $k \in \mathbb{N}_+$. This completes the proof.

4. Illustrative examples

Example 1 Consider the Lorenz system[15]

$$\begin{cases} \dot{x}_1(t) = -\sigma(x_1(t) - x_2(t)), \\ \dot{x}_2(t) = \gamma x_1(t) - x_2(t) - x_1(t)x_3(t), \\ \dot{x}_3(t) = x_1(t)x_2(t) - bx_3(t), \end{cases} \tag{9}$$

where $\sigma = 10$, $\gamma = 28$, $b = 8/3$, $\boldsymbol{x}(t) = (x_1(t),\ x_2(t),\ x_3(t))^{\mathrm{T}}$.

The response system is constructed as follows

$$\begin{cases} \begin{cases} \dot{y}_1(t) = -\sigma(y_1(t) - y_2(t)), \\ \dot{y}_2(t) = \gamma y_1(t) - y_2(t) - y_1(t)y_3(t), \\ \dot{y}_3(t) = y_1(t)y_2(t) - by_3(t) \end{cases} & t \ne t_k, \\ \Delta\boldsymbol{y}(t) = \boldsymbol{M}_k(b_k\boldsymbol{e}(t)), \quad t = t_k, \quad k \in \mathbb{N}_+, \end{cases} \tag{10}$$

where $\boldsymbol{y}(t) = (y_1(t), y_2(t), y_3(t))^{\mathrm{T}}$, $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{x}(t)$, $L = 75$, $\boldsymbol{M}_0 = \mathrm{diag}(0.8, 0.8, 0.8)$, $b_k = -0.9$, $\lambda = t_k - t_{k-1} = 0.015$, $k \in \mathbb{N}_+$, $\boldsymbol{x}(0) = (-0.2,\ 0.6,\ 0.1)^{\mathrm{T}}$, $\boldsymbol{y}(0) = (0,\ 0,\ 0)^{\mathrm{T}}$. It is easy to check that $q_k = 0.7438 < 1$, $k \in \mathbb{N}_+$, so the conditions of Theorem 1 are satisfied. A fourth-order Runge–Kutta method with step size 0.005 is used. Simulated results are shown in Fig. 1. From Fig. 1, it is shown that the drive system (9) and the response system (10) are uniformly asymptotically synchronized.

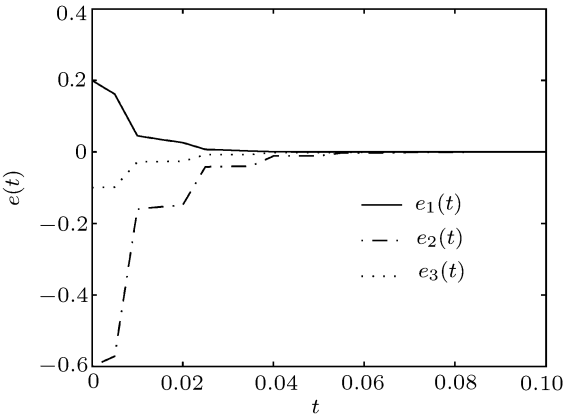


Fig. 1. The time histories of e_1, e_2 and e_3.

Example 2 Consider the Duffing system[5]

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = x_1(t) - (x_1(t))^3 - \delta x_2(t) + \gamma\cos(\omega t), \end{cases} \tag{11}$$

where $\delta = 0.25$, $\gamma = 0.4$, $\omega = 1$, $\boldsymbol{x}(t) = (x_1(t),\ x_2(t))^{\mathrm{T}}$.

The response system is constructed as follows:

$$\begin{cases} \begin{cases} \dot{y}_1(t) = y_2(t), \\ \dot{y}_2(t) = y_1(t) - (y_1(t))^3 - \delta y_2(t) + \gamma\cos(\omega t), \end{cases} & t \ne t_k, \\ \Delta\boldsymbol{y}(t) = \boldsymbol{M}_k(b_k\boldsymbol{e}(t)), \quad t = t_k, \quad k \in \mathbb{N}_+, \end{cases} \tag{12}$$

where $\boldsymbol{y}(t) = (y_1(t), y_2(t))^{\mathrm{T}}$, $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{x}(t)$, $L = 11$, $\boldsymbol{M}_0 = \mathrm{diag}(0.8, 0.8, 0.8)$, $b_k = -0.6$, $\lambda = t_k - t_{k-1} = 0.05$, $k \in \mathbb{N}_+$, $x(0) = (1,\ 1)^{\mathrm{T}}$, $y(0) = (0,\ 0)^{\mathrm{T}}$. It is easy to check that $q_k = 0.8123 < 1$, $k \in \mathbb{N}_+$, so the conditions of Theorem 1 are satisfied. A fourth-order Runge–Kutta method with step size 0.01 is used. Simulated results are shown in Fig. 2. From Fig. 2, it is shown that the drive system (11) and the response system (12) are uniformly asymptotically synchronized.

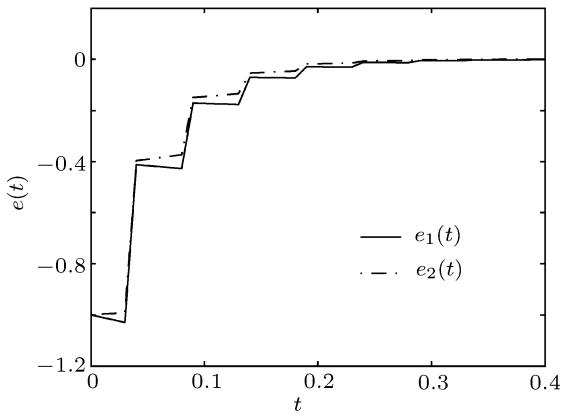


Fig. 2. The time histories of e_1 and e_2.

5. Conclusion

The problem of reliable impulsive synchronization for a class of nonlinear chaotic systems has been investigated in this paper. By the uniform asymptotic stability criteria of systems with impulsive effects, some sufficient conditions for reliable impulsive synchronization between the drive system and the response system are obtained.

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