Impulsive control for permanent magnet synchronous motors with uncertainties: LMI approach*

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A permanent magnet synchronous motor (PMSM) may have chaotic behaviours under certain working conditions, especially for uncertain values of parameters, which threatens the security and stability of motor-driven operation. Hence, it is important to study methods of controlling or suppressing chaos in PMSMs. In this paper, the stability of a PMSM with parameter uncertainties is investigated. After uncertain matrices which represent the variable system parameters are formulated through matrix analysis, a novel asymptotical stability criterion is established by employing the method of Lyapunov functions and linear matrix inequality technology. An example is also given to illustrate the effectiveness of our results.

Keywords: permanent magnet synchronous motors, impulsive control, uncertainty, linear matrix inequality

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1. Introduction

Permanent magnet synchronous motors (PMSMs) are of great interest, particularly for industrial applications in the low-medium power range, since they have superior features such as compact size, high torque/weight ratio, high torque/inertia ratio and absence of rotor losses.[1] Moreover, compared with induction motors, PMSMs have the advantages of higher efficiency, due to the absence of rotor losses and lower no-load current below the rated speed.[2]

The control technique analysis and systematic design for PMSMs are among the most important issues because the performance of the PMSMs is very sensitive to external load disturbances and parameter variations. Thus, a considerable amount of control techniques have been developed to overcome these problems. Many approaches such as feed-forward control, nonlinear control, optimal control, variable structure system control, adaptive control, hybrid control and robust control have been developed for the PMSMs to deal with uncertainties under various operating conditions.[1−12]

Besides this, we must point out that a PMSM is a nonlinear system. The control techniques for nonlinear systems have been widely studied, such as impulsive control strategy[13−20] and fuzzy control strategy.[21] The impulsive control strategy has become an important control method for nonlinear systems recently. From the references cited above, we find that they usually used a special Lyapunov function $\frac{1}{2}x^{\mathrm{T}}x$ or $\frac{1}{2}x^{\mathrm{T}}\boldsymbol{\Gamma}x$ (where $\boldsymbol{\Gamma}$ is a special positive definite matrix). But the construction of the Lyapunov function itself is one of the important parts of the stability theory. The linear matrix inequality technique can be used for solving the problem. Moreover, the PMSM is a special nonlinear system, which is more rigorous than ordinary nonlinear systems. So a stability study on PMSMs is important. We cannot regard it as a simple application of the nonlinear system.

The control schemes for PMSMs (or nonlinear systems) described above have been widely used in industry. However, the values of parameters of the PMSM model are not constant, but span some interval. The main reason for this is that the permanent magnetic-flux will vary with the following factors: the temperature rise in the permanent-magnet

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material, the variation of rotary inertia due to the attrition of the device, the modeling errors, and the measurement inaccuracy, etc. However, only a few studies on the nonlinear systems with variable parameters exist.[21−23] Therefore, the control strategy of PMSMs with parameter variation is quite valuable. Impulsive control has the advantages of higher reliability and stronger anti-jamming ability. If applying analog voltage to control the motor, the control voltage will rise to the limit rapidly when a blunder in wiring topology or some element breakage happens. But this kind of incident will not happen if applying impulse as control signal to control the motor. So impulsive control for PMSMs with parameter variation will be discussed in this paper. To the best of our knowledge, few reports exist on stability results for uncertain impulsive dynamical systems, and far fewer for PMSMs with parameter variation have been presented in the literature.

The organization of the paper is as follows. In the next section, the problems investigated in this paper are formulated, and some preliminaries are presented. In Section 3, the asymptotic stability criteria for the equilibrium points of PMSMs with parameter variation are derived. A numerical example and simulations are presented in Section 4. Finally, some conclusions are drawn in Section 5.

2. Problem formulation and preliminaries

The machine model of a PMSM with parameter uncertainties can be described in the rotor rotating reference frame as follows:

$$\begin{cases} \dfrac{\mathrm{d}i_d}{\mathrm{d}t} = \left(-Ri_d + \omega L_q i_q\right)/L_d, \\ \dfrac{\mathrm{d}i_q}{\mathrm{d}t} = \left(-Ri_q - \omega L_d i_d - \omega\psi_r\right)/L_q, \\ \dfrac{d\omega}{dt} = \left[n_p\psi_r i_q + n_p(L_d - L_q)i_d i_q - T_L - \beta\omega\right]/J, \end{cases} \tag{1}$$

where i_d and i_q are respectively d, q axis stator currents, ω is rotor speed, L_d and L_q are d, q axis stator inductance; R is stator resistance, T_L the external load torque, ψ_r the maximum flux induced by the rotor magnet n_p the number of pole-pairs, β the damping coefficient, and J the moment of inertia of the rotor. Because there exist some uncertainties such as modeling errors and measurement inaccuracy, in this uncertain system, $R \in [R_1, R_2]$, $\beta \in [\beta_1, \beta_2]$, $\psi_r \in [\psi_1, \psi_2]$ and $J \in [J_1, J_2]$ are unknown parameters which span their own intervals.

Apply the affine transformation and time scale transformation to system (1) with $\boldsymbol{x} = \boldsymbol{\lambda}\tilde{\boldsymbol{x}}$, $t = \tau\tilde{t}$, where $\boldsymbol{x} = [\, i_d \;\; i_q \;\; \omega \,]^{\mathrm{T}}$, $\tilde{\boldsymbol{x}} = [\, \tilde{i}_d \;\; \tilde{i}_q \;\; \tilde{\omega} \,]^{\mathrm{T}}$, $b = L_q/L_d$, $k = \beta/(n_p\tau\psi_r)$, $\tau = L_q/R$,

$$\boldsymbol{\lambda} = \begin{bmatrix} \lambda_d & 0 & 0 \\ 0 & \lambda_q & 0 \\ 0 & 0 & \lambda_\omega \end{bmatrix} = \begin{bmatrix} bk & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1/\tau \end{bmatrix},$$

with

$$\tau = \frac{L_q}{R} \in \left[\frac{L_q}{R_2}, \frac{L_q}{R_1}\right],$$

$$k = \frac{\beta}{n_p\tau\psi_r} \in \left[\frac{\beta_1}{n_p\tau\psi_2}, \frac{\beta_2}{n_p\tau\psi_1}\right] \in \left[\frac{R_1\beta_1}{n_p L_q\psi_2}, \frac{R_2\beta_2}{n_p L_q\psi_1}\right].$$

The dimensionless mathematical model of PMSM can be described as follows:

$$\begin{cases} \dfrac{\mathrm{d}\tilde{i}_d}{\mathrm{d}t} = -\tilde{i}_d + \tilde{\omega}\tilde{i}_q, \\ \dfrac{\mathrm{d}\tilde{i}_q}{\mathrm{d}t} = -\tilde{i}_q - \tilde{\omega}\tilde{i}_d + \gamma\tilde{\omega}, \\ \dfrac{\mathrm{d}\omega}{\mathrm{d}t} = \sigma(\tilde{i}_q - \tilde{\omega}) + \varepsilon\tilde{i}_d\tilde{i}_q - \tilde{T}_L, \end{cases} \tag{2}$$

where

$$\gamma = \frac{\psi_r}{kL_q} \in \left[\frac{n_p\psi_1^2}{R_2\beta_2}, \frac{n_p\psi_2^2}{R_1\beta_1}\right],$$

$$\sigma = \frac{\beta\tau}{J} \in \left[\frac{\beta_1 L_q}{J_2 R_2}, \frac{\beta_2 L_q}{J_1 R_1}\right],$$

$$\tilde{T}_L = \frac{\tau^2}{J}T_L \in \left[\frac{L_q^2 T_1}{R_2^2 J_2}, \frac{L_q^2 T_2}{R_1^2 J_1}\right], \quad \varepsilon = \frac{n_p b\tau^2 k^2(L_d - L_q)}{J} \in \left[\frac{bR_1^2\beta_1^2(L_d - L_q)}{n_p R_2^2 J_2\psi_2^2}, \frac{bR_2^2\beta_2^2(L_d - L_q)}{n_p R_1^2 J_1\psi_1^2}\right].$$

Considering a PMSM with a smooth air gap, we have $L_d = L_q = L$, and $\varepsilon = \dfrac{n_p b\tau^2 k^2(L_d - L_q)}{J} = 0$.

Let $\boldsymbol{x} = [\, x_1 \;\; x_2 \;\; x_3 \,]^{\mathrm{T}} = [\, \tilde{i}_d \;\; \tilde{i}_q \;\; \tilde{\omega} \,]^{\mathrm{T}}$. The chaotic mathematical model of a smooth air gap PMSM with parameter variation and impulsive effects can be expressed by

$$\begin{cases} \dot{x}_1 = -x_1 + x_3 x_2, \\ \dot{x}_2 = -x_3 x_1 - x_2 + \gamma x_3, \quad t \neq \tau_k, k = 1, 2, \ldots, \\ \dot{x}_3 = \sigma(x_2 - x_3), \end{cases}$$
$$\Delta \boldsymbol{x}|_{t=t_k} \triangleq \boldsymbol{x}\left(\tau_k^+\right) - \boldsymbol{x}\left(\tau_k^-\right) = \boldsymbol{B}\boldsymbol{x}, \quad k = 1, 2, \ldots, \tag{3}$$

where $\boldsymbol{x} = [\, x_1 \ x_2 \ x_2 \,]^{\mathrm{T}}$ are state variables without dimension, $\boldsymbol{B} \in R^{3\times 3}$ is a diagonal matrix representing the impulsive state control in place of the external load torque. $\boldsymbol{B} \in [\boldsymbol{B}_1, \boldsymbol{B}_2]$ represents the uncertain impulsive control, where $\boldsymbol{B}_1$ and $\boldsymbol{B}_2$ are also diagonal matrices. We denote

$$\gamma = \frac{\psi_r}{kL_q} \in \left[\frac{n_p\psi_1^2}{R_2\beta_2}, \frac{n_p\psi_2^2}{R_1\beta_1}\right] = [r_1, r_2]$$

and

$$\sigma \in \left[\frac{\beta L_q}{J_2 R_2}, \frac{\beta L_q}{J_1 R_1}\right] = [l_1, l_2]\,.$$

Obviously, $\boldsymbol{x} = [\gamma - 1, \sqrt{\gamma - 1}, \sqrt{\gamma - 1}]$, $\boldsymbol{x} = [\gamma - 1, -\sqrt{\gamma - 1}, -\sqrt{\gamma - 1}]$ and $\boldsymbol{x} = [0,\ 0,\ 0]$ are the equilibrium points of system (3). We will discuss the stability of the origin of system (3).

In the following, in order to discuss the stability of system (3), we present some preliminary results with respect to impulsive differential equations initially.

3. The impulsive stability of a PMSM with uncertain parameters

Let $\boldsymbol{\Phi}(\boldsymbol{x}) = \left[\, x_3 x_2 \ -x_1 x_3 \ 0 \,\right]^{\mathrm{T}}$. Under the initial condition $x\left(t_0^+\right) = x_0 \geq 0$, system (3) becomes

$$\begin{aligned} &\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x}),\ t \neq \tau_k, \quad k \in N, \\ &\Delta \boldsymbol{x}|_{t=\tau_k} = \boldsymbol{B}\boldsymbol{x}, \quad k \in N, \\ &\boldsymbol{x}\left(t_0^+\right) = x_0 \geq 0, \end{aligned} \tag{4}$$

where

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}.$$

The matrix $\boldsymbol{A}$ is an uncertain matrix due to $\boldsymbol{\gamma} \in [r_1,\ r_2]$, $\sigma \in [l_1,\ l_2]$ and $\boldsymbol{B} \in [\boldsymbol{B}_1,\ \boldsymbol{B}_2]$.

In order to obtain the stability result for system (3), we shall establish some lemmas first.

Lemma 3.1.

(i) Let $\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & \gamma \\ 0 & \sigma & -\sigma \end{bmatrix}$, where $\gamma \in [r_1,\ r_2]$, $\sigma \in [l_1,\ l_2]$, and let $\boldsymbol{P} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & r_1 \\ 0 & l_1 & -l_2 \end{bmatrix}$, $\boldsymbol{Q} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & r_2 \\ 0 & l_2 & -l_1 \end{bmatrix}$.
Then $\boldsymbol{A}$ can be written as

$$\boldsymbol{A} = \boldsymbol{A}_0 + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F}, \tag{5}$$

where

$$\begin{aligned} &\boldsymbol{A}_0 = \frac{1}{2}(\boldsymbol{P} + \boldsymbol{Q}), \quad \boldsymbol{H} = (h_{ij})_{3\times 3} = \frac{1}{2}(\boldsymbol{Q} - \boldsymbol{P}); \\ &\boldsymbol{\Sigma} \in \boldsymbol{\Sigma}^* = \{\boldsymbol{\Sigma} \in R^{9\times 9} : \boldsymbol{\Sigma} = \mathrm{diag}(\varepsilon_{11}, \varepsilon_{12}, \ldots, \varepsilon_{13}, \ldots, \varepsilon_{31}, \ldots, \varepsilon_{33}), |\varepsilon_{ij}| \leq 1, \varepsilon_{32} = \varepsilon_{33}, i, j = 1, 2, 3\} \\ &\boldsymbol{E} = (\sqrt{h_{11}}\boldsymbol{e}_1, \sqrt{h_{12}}\boldsymbol{e}_1, \sqrt{h_{13}}\boldsymbol{e}_1, \ldots, \sqrt{h_{31}}\boldsymbol{e}_3, \ldots, \sqrt{h_{33}}\boldsymbol{e}_3) \in R^{3\times 9}, \\ &\boldsymbol{F} = (\sqrt{h_{11}}\boldsymbol{e}_1, \sqrt{h_{12}}\boldsymbol{e}_2, \sqrt{h_{13}}\boldsymbol{e}_3, \ldots, \sqrt{h_{31}}\boldsymbol{e}_1, \ldots, \sqrt{h_{33}}\boldsymbol{e}_3)^{\mathrm{T}} \in R^{9\times 3}, \\ &\boldsymbol{e}_1 = (1, 0, 0)^{\mathrm{T}}, \boldsymbol{e}_2 = (0, 1, 0)^{\mathrm{T}}, \boldsymbol{e}_3 = (0, 0, 1)^{\mathrm{T}}. \end{aligned}$$

(ii) Let diagonal matrix $\boldsymbol{B} \in [\boldsymbol{B}_1,\ \boldsymbol{B}_2]$; if $\boldsymbol{B}_0 = \frac{1}{2}[\boldsymbol{B}_1 + \boldsymbol{B}_2]$, $\boldsymbol{G} = \frac{1}{2}[\boldsymbol{B}_2 - \boldsymbol{B}_1] = \mathrm{diag}(g_1, g_2, g_3)$, then $\boldsymbol{B}$ can be written as

$$\boldsymbol{B} = \boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}}, \tag{6}$$

where $\tilde{\boldsymbol{\Sigma}} \in \tilde{\boldsymbol{\Sigma}}^* = \{\tilde{\boldsymbol{\Sigma}} \in R^{3\times 3} : \boldsymbol{\Sigma} = \mathrm{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3), |\varepsilon_i| \leq 1\}$,

$$\tilde{\boldsymbol{E}} = \mathrm{diag}(\sqrt{g_1}, \sqrt{g_2}, \sqrt{g_3}).$$

Remark 3.1.

Clearly, for any $\boldsymbol{\Sigma} \in \boldsymbol{\Sigma}^*$ and $\tilde{\boldsymbol{\Sigma}} \in \tilde{\boldsymbol{\Sigma}}^*$, we have

(i) $\boldsymbol{\Sigma}\boldsymbol{\Sigma}^{\mathrm{T}} = \boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{\Sigma} \leq \boldsymbol{I}$, $\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{\Sigma}}^{\mathrm{T}} = \tilde{\boldsymbol{\Sigma}}^{\mathrm{T}}\tilde{\boldsymbol{\Sigma}} \leq \boldsymbol{I}$ (3×3 identity matrix);

(ii) $\boldsymbol{E}\boldsymbol{E}^{\mathrm{T}} = \mathrm{diag}\left\{\sum\limits_{j=1}^{3} h_{1j},\ \sum\limits_{j=1}^{3} h_{2j},\ \sum\limits_{j=1}^{3} h_{3j}\right\}$, $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} = \mathrm{diag}\left\{\sum\limits_{i=1}^{3} h_{i1},\ \sum\limits_{i=1}^{3} h_{i2},\ \sum\limits_{i=1}^{3} h_{i3}\right\}$.

(iii) by Lemma 3.1, system (3) can be rewritten as

$$\begin{aligned}
&\dot{\boldsymbol{x}} = \boldsymbol{A}_0\boldsymbol{x} + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x}), \quad t \neq \tau_k, k = 1, 2, \ldots,\\
&\Delta\boldsymbol{x}|_{t=\tau_k} = \left(\boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}}\right)\boldsymbol{x}, \quad k = 1, 2, \ldots,\\
&\boldsymbol{x}\left(\tau_0^{+}\right) = \boldsymbol{x}_0 \geq 0,
\end{aligned} \tag{7}$$

where $\boldsymbol{A}_0$, $\boldsymbol{B}_0$, $\boldsymbol{E}$, $\boldsymbol{\Sigma}$, $\boldsymbol{F}$, $\tilde{\boldsymbol{E}}$ and $\tilde{\boldsymbol{\Sigma}}$ are defined as in Lemma 3.1.

Lemma 3.2[7]

If $\boldsymbol{\Sigma} \in \boldsymbol{\Sigma}^{*}$, then for any positive scalar $\lambda > 0$ and for any $\xi \in R^n$, the following inequality holds,

$$2\xi^{\mathrm{T}}\sum\boldsymbol{\eta} \leq \frac{1}{\lambda}\xi^{\mathrm{T}}\xi + \lambda\boldsymbol{\eta}^{\mathrm{T}}\boldsymbol{\eta}, \tag{8}$$

especially, the inequality holds for $\lambda = 1$.

Lemma 3.3[17]

Let $\boldsymbol{M}$, $\boldsymbol{E}$, $\boldsymbol{F}$ and $\boldsymbol{\Sigma}$ be real matrices of appropriate dimensions, with $\boldsymbol{\Sigma}$ satisfying $\|\boldsymbol{\Sigma}\| \leq 1$. Then we have

(i) For any scalar $\lambda > 0$,

$$\boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F} + \boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}} \leq \lambda^{-1}\boldsymbol{E}\boldsymbol{E}^{\mathrm{T}} + \lambda\boldsymbol{F}\boldsymbol{F}^{\mathrm{T}}. \tag{9}$$

(ii) For any matrix $\boldsymbol{P} > 0$ and scalar $\xi > 0$ such that $\xi\boldsymbol{I} - \boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{F} > \boldsymbol{0}$, we have

$$\begin{aligned}
&(\boldsymbol{M} + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F})^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{M} + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F})\\
&\leq \boldsymbol{M}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{M} + \boldsymbol{M}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{F}(\xi\boldsymbol{I} - \boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{F})^{-1}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{M}\\
&\quad + \xi^{-1}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{E}.
\end{aligned} \tag{10}$$

Especially, the above two inequalities hold for $\lambda = 1$ and $\xi = 1$.

Lemma 3.4[17]

The following linear matrix inequality (LMI) is positive

$$\begin{bmatrix} \boldsymbol{Q}(\boldsymbol{x}) & \boldsymbol{S}(\boldsymbol{x}) \\ \boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x}) & \boldsymbol{R}(\boldsymbol{x}) \end{bmatrix} > 0,$$

where $\boldsymbol{Q}(\boldsymbol{x}) = \boldsymbol{Q}^{\mathrm{T}}(\boldsymbol{x}), \boldsymbol{R}(\boldsymbol{x}) = \boldsymbol{R}^{\mathrm{T}}(\boldsymbol{x})$, and $\boldsymbol{S}(\boldsymbol{x})$ depend on $\boldsymbol{x}$, is equivalent to

$$\boldsymbol{R}(\boldsymbol{x}) > 0, \quad \boldsymbol{Q}(\boldsymbol{x}) - \boldsymbol{S}(\boldsymbol{x})\boldsymbol{R}^{-1}(\boldsymbol{x})\boldsymbol{S}^{\mathrm{T}}(\boldsymbol{x}) > 0. \tag{11}$$

Theorem 3.1.

Assume that there exist the positive constants $\lambda > 0$, $\beta > 0$, $\eta > 1$, and positive definite matrix $\boldsymbol{\Gamma}$, such that the following LMIs hold:

(i)

$$\begin{bmatrix} \boldsymbol{\Gamma}\boldsymbol{A}_0 + \boldsymbol{A}_0^{\mathrm{T}}\boldsymbol{\Gamma} + \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} - \lambda\boldsymbol{\Gamma} & \boldsymbol{\Gamma}\boldsymbol{E} \\ \boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Gamma} & -I \end{bmatrix} \leq 0, \tag{12}$$

(ii)

$$\begin{bmatrix} (\boldsymbol{I} + \boldsymbol{B}_0)^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I} + \boldsymbol{B}_0) + \tilde{\boldsymbol{E}}^{\mathrm{T}}\tilde{\boldsymbol{E}} - \beta\boldsymbol{\Gamma} & (\boldsymbol{I} + \boldsymbol{B}_0)^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{E}} \\ \tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I} + \boldsymbol{B}_0) & \boldsymbol{I} - (\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{E}}) \end{bmatrix} \leq 0. \tag{13}$$

If $\ln(\eta\beta) + \lambda(\tau_{k+1} - \tau_k) \leq 0$ for all k, then system (3) is robust and asymptotically stable.

Proof

Let the Lyapunov function $V(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{x}$. When $t = \tau_k (k = 1, 2, \ldots)$, we have

$$\begin{aligned}
V(\tau_k^{+}, \boldsymbol{x}) &= \frac{1}{2}(\boldsymbol{x}(\tau_k) + (\boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}})\boldsymbol{x}(\tau_k))^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{x}(\tau_k) + (\boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}})\boldsymbol{x}(\tau_k))\\
&= \frac{1}{2}\boldsymbol{x}(\tau_k)^{\mathrm{T}}[(\boldsymbol{I} + \boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}})^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I} + \boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}})]\boldsymbol{x}(\tau_k).
\end{aligned} \tag{14}$$

By Lemma 3.3, we have

$$\begin{aligned}
&(\boldsymbol{I} + \boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}})^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I} + \boldsymbol{B}_0 + \tilde{\boldsymbol{E}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{E}})\\
&\leq (\boldsymbol{I} + \boldsymbol{B}_0)^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I} + \boldsymbol{B}_0) + (\boldsymbol{I} + \boldsymbol{B}_0)^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{E}}(\xi\boldsymbol{I} - \tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{E}})^{-1}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I} + \boldsymbol{B}_0) + \xi^{-1}(\tilde{\boldsymbol{E}}^{\mathrm{T}}\tilde{\boldsymbol{E}}).
\end{aligned} \tag{15}$$

Thus, Eq. (14) turns into

$$V(\tau_k^+, \boldsymbol{x}) \leq \frac{1}{2}\boldsymbol{x}(\tau_k)^{\mathrm{T}}[(\boldsymbol{I}+\boldsymbol{B}_0)^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I}+\boldsymbol{B}_0) + (\boldsymbol{I}+\boldsymbol{B}_0)^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{E}}(\boldsymbol{I}-\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{\Gamma}\tilde{\boldsymbol{E}})^{-1}\tilde{\boldsymbol{E}}^{\mathrm{T}}\boldsymbol{\Gamma}(\boldsymbol{I}+\boldsymbol{B}_0) + (\tilde{\boldsymbol{E}}^{\mathrm{T}}\tilde{\boldsymbol{E}})]\boldsymbol{x}(\tau_k). \tag{16}$$

By the condition (ii), we have

$$V(\tau_k^+, \boldsymbol{x}) \leq \beta V(\tau_k^+, \boldsymbol{x}). \tag{17}$$

When $t \neq \tau_k$, note that $\boldsymbol{\Phi}(\boldsymbol{x}) = \begin{bmatrix} x_3x_2 & -x_1x_3 & 0 \end{bmatrix}^{\mathrm{T}}$, so we obtain $\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Phi}(\boldsymbol{x}) = 0$. By the assumption and Lemmas 3.1, 3.2, 3.3 and 3.4, for all $(t, \boldsymbol{x}) \in (\tau_{k-1}, \tau_k] \times R^n (k = 1, 2, \dots)$, we have

$$\begin{aligned} D^+V(t,\boldsymbol{x}) &= \frac{1}{2}\left(\dot{\boldsymbol{x}}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Gamma}\dot{\boldsymbol{x}}\right) = \frac{1}{2}\left(\boldsymbol{A}_0\boldsymbol{x} + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x})\right)^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Gamma}\left(\boldsymbol{A}_0\boldsymbol{x} + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x})\right) \\ &= \frac{1}{2}\left(\boldsymbol{x}^{\mathrm{T}}\boldsymbol{A}_0^{\mathrm{T}} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{E}^{\mathrm{T}} + \boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x})\right)\boldsymbol{\Gamma}\boldsymbol{x} + \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Gamma}\left(\boldsymbol{A}_0\boldsymbol{x} + \boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F}\boldsymbol{x} + \boldsymbol{\Phi}(\boldsymbol{x})\right) \\ &= \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\left[\boldsymbol{A}_0^{\mathrm{T}}\boldsymbol{\Gamma} + \boldsymbol{\Gamma}\boldsymbol{A}_0 + \boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Gamma} + \boldsymbol{\Gamma}\boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F}\right]\boldsymbol{x} + \frac{1}{2}\left[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{\Gamma}\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{\Phi}(\boldsymbol{x})\right]. \end{aligned} \tag{18}$$

Note that

$$\frac{1}{2}[\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{\Gamma}\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Gamma}\boldsymbol{\Phi}(\boldsymbol{x})] \leq \frac{1}{2}\lambda_{\max}(\boldsymbol{\Gamma})\left(\boldsymbol{\Phi}^{\mathrm{T}}(\boldsymbol{x})\boldsymbol{x} + \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Phi}(\boldsymbol{x})\right) = 0,$$

and $\boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Gamma} + \boldsymbol{\Gamma}\boldsymbol{E}\boldsymbol{\Sigma}\boldsymbol{F} \leq \boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{\Sigma}\boldsymbol{F} + \boldsymbol{\Gamma}\boldsymbol{E}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Gamma} \leq \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} + \boldsymbol{\Gamma}\boldsymbol{E}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Gamma}$.

Thus, by the condition (i) of the theorem, we have

$$D^+V(t,\boldsymbol{x}) \leq \frac{1}{2}\boldsymbol{x}^{\mathrm{T}}\left[\boldsymbol{A}_0^{\mathrm{T}}\boldsymbol{\Gamma} + \boldsymbol{\Gamma}\boldsymbol{A}_0 + \boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} + \boldsymbol{\Gamma}\boldsymbol{E}\boldsymbol{E}^{\mathrm{T}}\boldsymbol{\Gamma}\right]\boldsymbol{x} \leq \lambda V(t,\boldsymbol{x}). \tag{19}$$

Let $k = 1$, for all $t \in (\tau_0, \tau_1]$, by Eq. (19), we have

$$V(t,\boldsymbol{x}) \leq V(\tau_0,\boldsymbol{x})\exp\left(\lambda(t-\tau_0)\right). \tag{20}$$

Then it leads to

$$V(\tau_1,\boldsymbol{x}) \leq V(\tau_0,\boldsymbol{x})\exp\left(\lambda(\tau_1-\tau_0)\right). \tag{21}$$

From the inequality (17), we have

$$V(\tau_1^+,\boldsymbol{x}) \leq \beta V(\tau_1,\boldsymbol{x}) \leq \beta V(\tau_0,\boldsymbol{x})\exp\left(\lambda(\tau_1-\tau_0)\right). \tag{22}$$

For all $t \in (\tau_1, \tau_2]$, we have

$$V(t,\boldsymbol{x}) \leq V(\tau_1^+,\boldsymbol{x})\exp\left(\lambda(\alpha)(t-\tau_1)\right) \leq \beta V(\tau_0,\boldsymbol{x})\exp\left(\lambda(t-\tau_0)\right). \tag{23}$$

Similarly, for all k and $t \in (\tau_k, \tau_{k+1}]$, we have

$$V(t,\boldsymbol{x}) \leq \beta^k V(\tau_0,\boldsymbol{x})\exp\left(\lambda(t-\tau_0)\right). \tag{24}$$

By the condition $\ln(\eta\beta) + \lambda(\alpha)(\tau_{k+1} - \tau_k) \leq 0$, we have

$$\beta\exp(\lambda(\tau_{k+1}-\tau_k)) \leq \frac{1}{\eta}, \quad k = 1, 2, \dots. \tag{25}$$

Hence, for all $t \in (\tau_k, \tau_{k+1}]$, $(k = 1, 2, \dots)$, we have

$$\begin{aligned} V(t,\boldsymbol{x}) &\leq \beta^k V(\tau_0,\boldsymbol{x})\exp\left(\lambda(t-\tau_0)\right) \\ &= V(\tau_0,\boldsymbol{x})\left[\beta\exp\left(\lambda(\tau_k-\tau_{k-1})\right)\right]\cdots\left[\beta\exp\left(\lambda(\tau_1-\tau_0)\right)\right]\exp\left(\lambda(t-\tau_k)\right) \\ &\leq V(\tau_0,\boldsymbol{x})\frac{1}{\eta^k}\exp\left(\lambda(t-\tau_k)\right). \end{aligned} \tag{26}$$

So, if $t \to \infty$, then $k \to \infty$ and $V(t,\boldsymbol{x}) \to 0$. It follows that the conclusion holds. □

In the following, we will present the design algorithm for the stabilization of system (3) based on Theorem 3.1.

Algorithm

In order to reduce the cost of control or implementation, we hope that the impulsive distance is as large as possible. Hence, we hope that $\lambda > 0$ and $0 < \beta < 1$ are as small as possible such that the impulsive distance is larger. We have the following design algorithm for the robust stability of system (3).

1) Set a threshold T, initialize $\lambda > 0$, $\beta > 0$. For example $\lambda = 0.1$, $\beta = 0.1$;

2) Calculate $\boldsymbol{P}$ by (i) and (ii) of Theorem 3.1;

3) If $\boldsymbol{P}$ exists, stop. Otherwise let $\beta = \beta + \Delta\beta$;

4) Repeat from step 2 if $\beta < 1/\eta$. Otherwise $\theta = \theta + \Delta\theta$ and β is equal to the initial value;

5) Repeat from step 2 if $\lambda < T$. Otherwise fail.

Once the algorithm is successful, we may determine the bound of the impulsive distance by $(\tau_{k+1} - \tau_k) \leq -\ln(\eta\beta)/\lambda$. Hence, only if Algorithm 1 is slightly changed by satisfying $0 < \beta < 1/\eta$, may we obtain the asymptotical stability design algorithm of a chaotic system.

4. Example

In system (1), let n_p=1, $L_d = L_q = 0.0775$ mH, $R = 0.00256 \pm 0.00005\ \Omega$, $\Psi_r = 0.0257 \pm 0.0005$ N·m/A, $J = 0.06607 \pm 0.00005$ K·g·m^2, $\beta = 0.0121 \pm 0.0005$ N/(rad·s^{-1}). So, we have

$$\gamma \in \left[\frac{n_p\psi_1^2}{R_2\beta_2}, \frac{n_p\psi_2^2}{R_1\beta_1}\right] = [19.3103,\ 23.5760]\,,$$

$$\sigma \in \left[\frac{\beta_1 L_q}{J_2 R_2}, \frac{\beta_2 L_q}{J_1 R_1}\right] = [5.2094,\ 5.8928]\,.$$

During our experiment, we set σ and γ in their own interval randomly. The PMSM will display chaos without impulsive control. The chaotic attractor is shown in Fig. 1.

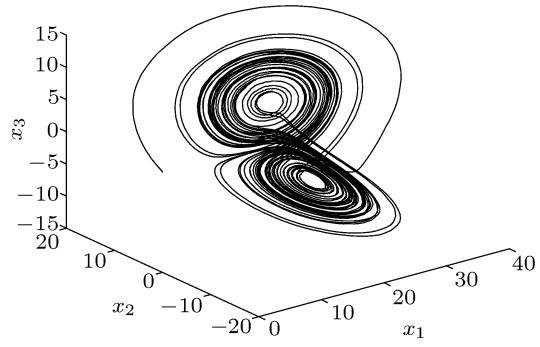


Fig. 1. The chaotic attractor in a PMSM with system parameter variation.

Let $\boldsymbol{B}_1 = \mathrm{diag}([-0.8,\ -0.5,\ -0.7])$, $\boldsymbol{B}_2 = \mathrm{diag}([-0.6,\ -0.5,\ -0.6])$. Set $\boldsymbol{x}_0 = [\,0.6\ 0.8\ 0.2\,]^{\mathrm{T}}$. Assume that the impulsive spacing is equal to, say, $\varepsilon_k \equiv \zeta$. Let $\eta = 1.2$. We choose the initial iteration values of λ and β to be 0.2 and 0.1 respectively. We obtain

$$\boldsymbol{\Gamma} = \begin{bmatrix} 2.2105 & -0.3047 & 0 \\ -0.3047 & 2.7584 & 0 \\ 0 & 0 & 1.8659 \end{bmatrix},$$

$\lambda = 27.4$ and $\beta = 0.5$

Thus, the impulsive distance for robust stability may be estimated as $(\tau_{k+1} - \tau_k) \leq -\ln(\beta)/\lambda = 0.0253$.

By Theorem 3.1 and the Algorithm that we give, the system is robust and asymptotically stable under the conditions of Theorem 3.1. Figures 2 and 3 illustrate the impulsive control state diagram and the phase diagram of a PMSM with system parameter variation for impulsive spacing $\zeta = 0.02$.

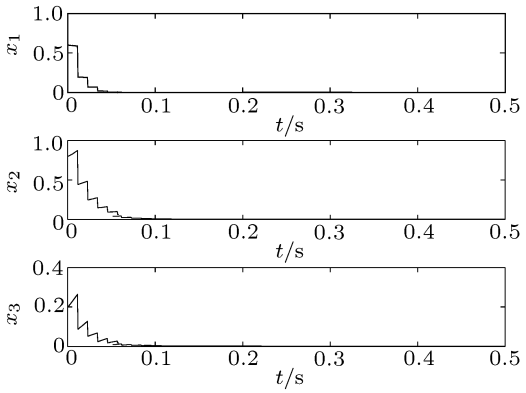


Fig. 2. The state diagram of a PMSM with impulsive control ($\zeta = 0.02$).

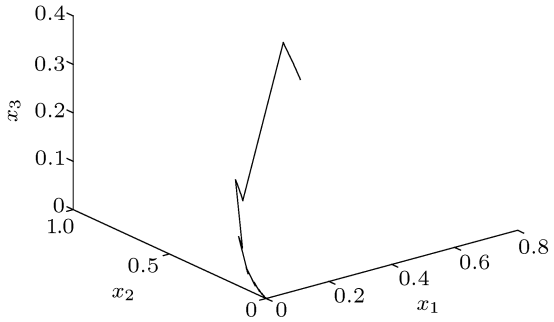


Fig. 3. The phase diagram of a PMSM with impulsive control ($\zeta = 0.02$).

In summary, according to the above simulation experiments, we think that the proposed impulsive control for a PMSM with system parameter variation and the corresponding results can provide good effectiveness with PMSM control via the LMI approach.

5. Conclusions

We have investigated the impulsive control of PMSMs. The parameter variation in the system is considered, and variable system parameters are formulated through matrix analysis. Sufficient conditions for robust asymptotic stability are established by employing the methods of Lyapunov functions and linear matrix inequality, so as to obtain more flexibility in practical application. The effectiveness of our methods has been shown by computer simulation, and the illustrated example shows that the system is robust and asymptotic stable under the conditions of Theorem 3.1.

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