The correlation between stochastic resonance and the average phase-synchronization time of a bistable system driven by colour-correlated noises

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This paper investigates the correlation between stochastic resonance (SR) and the average phase-synchronization time which is between the input signal and the output signal in a bistable system driven by colour-correlated noises. The results show that the output signal-to-noise ratio can reach a maximum with the increase of the average phase-synchronization time, which may be helpful for understanding the principle of SR from the point of synchronization; however, SR and the maximum of the average phase-synchronization time appear at different optimal noise level, moreover, the effects on them of additive and multiplicative noise are different.

Keywords: stochastic resonance, phase-synchronization, signal-to-noise ratio, average phase-synchronization time

PACC: 0540, 0250

1. Introduction

Stochastic resonance (SR) has been an active topic in statistical and nonlinear physics[1−14] since it was discovered in 1981, and one recognises the advantage of noise. Generally, the aim of investigating SR is to detect the role of noise, because noise is a sort of energy, it is a pleasure to translate the energy of noise into the energy of the output signal which is beneficial to us. In the process of studying SR, people always try their best to investigate and explain the mechanism of SR.

Recently, many people have paid attention to relating SR to phase synchronization, which is another important phenomenon in nonlinear dynamics. At first, we will give the definition of phase synchronization:[1−3] given a signal $x(t)$, with its fluctuation, we can define a corresponding phase variable $\phi_x(t)$. Set $\phi_x(0) = 0$ at $t = 0$ and monitor the evolution of $x(t)$. Wherever $x(t)$ completes one cycle of fluctuation, $\phi_x(t)$ is increased by 2π, so $\phi_x(t)$ can be defined as a nondecreasing function with time t, determined by the fluctuation of $x(t)$. We can write $\phi_x(t) = \omega_x t + \theta_x(t)$, where ω_x is the average frequency of $x(t)$ and $\theta_x(t)$ models the random fluctuation of the phase, and $|\theta_x(t)| < 2\pi$. For another signal $y(t)$, a phase variable $\phi_y(t)$ can be defined in a similar way: $\phi_y(t) = \omega_y t + \theta_y(t)$. Now consider a time interval $[t_1, t_2]$ that contains many cycles of oscillation. There is phase synchronization if the phase variables satisfy $\Delta\phi(t) = |\phi_x(t) - \phi_y(t)| < 2\pi$ for all t in this interval. For the existence of noise, the interval $\Delta t = t_2 - t_1$ can not be arbitrarily long, and one can consider an ensemble of the identical SR system and define the average phase synchronization time as the ensemble average: $\tau \equiv \langle \Delta t \rangle$, where $\langle . \rangle$ means the ensemble average;[3] of course, τ is related to the input signal and the noise. Latterly, Park *et al.*[2] have shown that aperiodic stochastic resonance can be understood as a phenomenon of phase synchronization. This activates our interest to investigate the relationship between SR and the average phase-synchronization time which is between the input signal and the output signal in a bistable system. On the other hand, people hope to enhance the output power spectrum at expectant time by utilizing the principle of SR. So if we know the relationship between SR and the average phase-synchronization time τ, we can alter the parameters of the system according to the character of the average phase-synchronization time τ, and make the systems respond to external excitations for learning and adaptation in terms of their abilities.[1,3]

In 2007, Park *et al.*[1,3] investigated the dependence of the averaged phase-synchronization time on the frequency of the input signal and the noise in-

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tensity; their results showed that the averaged phase-synchronization time is sensitive to the frequency of the input signal and noise, in other words, with the change of the frequency of the input signal and the noise intensity, the averaged phase-synchronization time can arrive at a maximum value. Moreover, there are investigations on the averaged phase-synchronization time for a bistable system driven by only one noise, but in fact, a system is driven by several noises simultaneously, and there may be relations between them.[5] This provides motivation to investigate the effects of the correlated noises on the average phase-synchronization time in a bistable system.

2. The expressions of the output signal-to-nose ratio (SNR) and the average phase-synchronization time in a bistable system

In this section, we will use the two-state theory[4] to obtain the expressions of the output SNR and the average phase-synchronization time between the input signal and the output signal in a bistable system.

Consider the following Langevin equations:[4−6]

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\partial U_0(x)}{\partial x} + x\xi(t) + \eta(t) + F(t), \tag{1}$$

$$U_0(x) = -\frac{1}{2}x^2 + \frac{1}{4}x^4, \tag{2}$$

where $U_0(x)$ denotes a symmetric bistable potential function, $\xi(t)$ and $\eta(t)$ are coloured cross-correlated white noises, and their statistical properties are characterized by their mean and variance:

$$\begin{aligned}
&\langle \xi(t)\rangle = \langle \eta(t)\rangle = 0,\\
&\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t'),\\
&\langle \eta(t)\eta(t')\rangle = 2\alpha\delta(t-t'),\\
&\langle \xi(t)\eta(t')\rangle = \langle \eta(t)\xi(t')\rangle = \frac{\lambda\sqrt{\alpha D}}{\tau_1}\exp\left[-\frac{1}{\tau_1}|t-t'|\right]\\
&\to 2\lambda\sqrt{\alpha D}\delta(t-t') \quad \text{as} \quad \tau_1 \to 0,
\end{aligned} \tag{3}$$

where α and D are the additive and multiplicative noise intensities respectively. The parameter λ ($0 \le \lambda < 1$) measures the strength of correlation between multiplicative and additive noise terms, and τ_1 is the correlation time of the correlation between the noises.[6] The $F(t)$ is a rectangular periodic input signal with a period of T_0, mathematically, the forcing function can be written as $F(t) = (-1)^{n(t)}A$, where $n(t) = \lfloor 2t/T_0 \rfloor$ and $\lfloor . \rfloor$ denotes the floor function. That is, $F(t) = A(-A)$ if $t \in [nT_0/2, (n+1)T_0/2]$ for n even (odd). The frequency of the input signal is $\Omega_{\mathrm{in}} = 2\pi/T_0$.[3,4]

By using the Novikov theorem,[7] Fox's approach[8] and the unified coloured-noise approximation,[9] the Fokker–Planck equation corresponding to Eqs. (1)–(3) can be written as[6]

$$\begin{aligned}
\frac{\partial P(x,t)}{\partial t} = &-\frac{\partial}{\partial x}[x - x^3 + Dx + a - F(t)]P(x,t)\\
&+ \frac{\partial^2}{\partial x^2}[Dx^2 + 2ax + \alpha]P(x,t)
\end{aligned}$$

with

$$a = \frac{\lambda\sqrt{\alpha D}}{1+2\tau_1}. \tag{4}$$

The stationary probability distribution of Eq. (4) can be written as

$$\rho_{st}(x) = \frac{N}{\sqrt{Dx^2 + 2ax + \alpha}}\exp\left[-\frac{\tilde{U}(x)}{D}\right],$$

where N is the normalization constant, and the modified potential $\tilde{U}(x)$ is

$$\begin{aligned}
\tilde{U}(x) = &\frac{x^2}{2} + bx + c\ln\left|Dx^2 + 2ax + \alpha\right|\\
&- d\arctan\left(x + \frac{a}{D}\right),
\end{aligned} \tag{5}$$

where

$$b = \frac{-2a}{D}, \quad c = \frac{4a^2 - aD - D^2 - D^3}{2D^2},$$

$$d = \frac{4a^3 - aD^2 - a^2D - 2Da\alpha + F(t)D^3}{D^2\sqrt{D\alpha - a^2}}.$$

For the modified potential $\tilde{U}(x)$, when $\alpha \ll 1$, $D \ll 1$, and $A < A_{\mathrm{th}} = \sqrt{4/27}$, the potential $\tilde{U}(x)$ has a 'double-well' shape in the sense that it has two minima, located at $x_-(t) < 0$ and $x_+(t) > 0$ respectively, and a maximum at $x_0(t)$ for all t. By the symmetry of the normalization potential, these three positions satisfy the following conditions:

$$\begin{aligned}
&x_0(t) = (-1)^{n(t)}x_0(0),\\
&x_-(t) + x_+(t) + x_0(t) = 0,\\
&x_+(t) = \frac{\Delta x(0)}{2} - (-1)^{n(t)}\frac{x_0(0)}{2},\\
&x_-(t) = \frac{-\Delta x(0)}{2} - (-1)^{n(t)}\frac{x_0(0)}{2},
\end{aligned}$$

where $\Delta x(0) = x_+(0) - x_-(0)$.[3,4]

Using the two-state theory,[4] we can approximate the exact Langevin equation by a nonstationary, Markovian two-state description of the form

$$\begin{aligned}\dot{n}_+ &= -\dot{n}_- = W_-(t)n_- - W_+(t)n_+ \\ &= W_-(t) - [W_-(t) + W_+(t)]n_+, \end{aligned} \tag{6}$$

where n_- and n_+ denote the probabilities that the particle is in the left and in the right well respectively, and $W_\pm(t)$ is the Kramers rate of escape out of stable states $x_\pm$ at time t:[4–6]

$$W_\pm(t) = \frac{\left|U_0''(x_0)U_0''(x_\pm)\right|^{1/2}}{2\pi} \exp\left[\frac{\tilde{U}(x_\pm) - \tilde{U}(x_0)}{D}\right]. \tag{7}$$

The Kramers rate formula of Eq. (7) can be simplified by using the symmetry of $\tilde{U}(x)$. One obtains[1–3]

$$W_\pm(t) = \frac{\Gamma}{2}[1 \pm (-1)^{n(t)}\Delta P_{\rm eq}(0)], \tag{8}$$

where

$$\Gamma = W_+(0) + W_-(0) = W_+(t) + W_-(t), \tag{9}$$

$$\Delta P_{\rm eq}(0) = P_{\rm eq}(+,0) - P_{\rm eq}(-,0), \tag{10}$$

$$P_{\rm eq}(\pm,0) = [\delta_{\pm,-}W_+(0) + \delta_{\pm,+}W_-(0)]/\Gamma, \tag{11}$$

and $P_{\rm eq}\ (\pm,0)$ are the equilibrium probabilities of state $x_\pm$ at $t=0$ respectively.

So the expression of the output SNR is obtained as[4,5]

$$\mathrm{SNR} = \frac{4[4\left\langle x(0)\right\rangle_{\rm eq}^2\Gamma^2 + [x_0(0)]^2\Omega_{\rm in}^2]}{\pi\Gamma[\Delta x(0)]^2\{1 + [\Delta P_{\rm eq}(0)]^2 \times [4\tanh(T_0\Gamma/4)/T_0\Gamma - 1]\}} \tag{12}$$

with $\Delta x(0) = x_+(0) - x_-(0)$, $\left\langle x(0)\right\rangle_{\rm eq} = x_+(0)P_{\rm eq}(+,0) + x_-(0)P_{\rm eq}(-,0)$, and Γ, $\Delta P_{\rm eq}(0)$ as above.

On the other hand, we will obtain the expression of the average phase-synchronization time τ,[1–3] which can be calculated if the average frequency of the output signal $\Omega_{\rm out}$ and the effective diffusion coefficient $D_{\rm eff}$ are known. The diffusion coefficient is defined by[1,3]

$$D_{\rm eff} = \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}[\left\langle\Psi^2(t)\right\rangle - \left\langle\Psi(t)\right\rangle^2], \tag{13}$$

where $\Psi(t)$ is the phase difference between the input and the output signals, and $\left\langle\Psi(t)\right\rangle \equiv (\Omega_{\rm out} - \Omega_{\rm in})t$.[1–3]

We can use the definition of the diffusion coefficient Eq. (13) to write[1,3]

$$\left\langle\Psi^2(t)\right\rangle \approx \left\langle\dot{\Psi}(t)\right\rangle^2t^2 + 2D_{\rm eff}t, \tag{14}$$

since τ is the average time required for a 2π change in $\Psi(t)$ to occur, we have $\left\langle\Psi^2(n\tau)\right\rangle = (2n\pi)^2$, leading to $\sqrt{\left\langle\Psi^2(t)\right\rangle}\,|_{t=\tau} = 2\pi$. Using this result and $\left\langle\dot{\Psi}\right\rangle = \Omega_{\rm out} - \Omega_{\rm in}$ yield the following formula for τ:[1,3]

$$\tau \approx \frac{D_{\rm eff}}{\left\langle\dot{\Psi}\right\rangle^2}\left[\sqrt{1 + \left(\frac{2\pi\left\langle\dot{\Psi}\right\rangle}{D_{\rm eff}}\right)^2} - 1\right], \tag{15}$$

where the average frequency of the output signal $\Omega_{\rm out}$ and the effective diffusion coefficient $D_{\rm eff}$ are presented as follows:[1,4]

$$\Omega_{\rm out} = \frac{\pi}{T_0}\int_0^{T_0}[W_+(t)n_+(t) + W(t)_-n_-(t)]\mathrm{d}t = \frac{\pi\Gamma}{2}\left[1 - [\Delta P_{\rm eq}(0)]^2\left(1 - \frac{4\tanh(\Gamma T_0/4)}{\Gamma T_0}\right)\right], \tag{16}$$

$$\begin{aligned} D_{\rm eff} &= \frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}[\left\langle\Psi^2\right\rangle - \left\langle\Psi\right\rangle^2] = \Omega_{\rm out}\pi - \frac{2\pi^2}{T_0}[\Delta P_{\rm eq}(0)]^4(\tanh(\Gamma T_0/4))^3 \\ &\quad - \frac{\pi^2}{2T_0}[\Delta P_{\rm eq}(0)]^2(1 - [\Delta P_{\rm eq}(0)]^2)(12\tanh(\Gamma T_0/4) - \Gamma T_0(1 + 2[\mathrm{sech}(\Gamma T_0/4)]^2). \end{aligned} \tag{17}$$

In Eqs. (16) and (17), Γ, $\Delta P_{\rm eq}(0)$ are given by Eqs.(9)–(11). Though Eq.(15) gives the expression of the average phase-synchronization time τ qualitatively, it is enough to investigate the effects on τ of D, α, τ_1 and λ.

We all know that SR appears in a bistable system driven by colour-correlated noises, and there are papers discussing it,[10,11] so we will omit the parts of SR in this paper. We only discuss the effects on τ of D, α, τ_1 and λ firstly, then we want to find the relationship between SR and the average phase-synchronization time τ.

3. The effects on τ of D, α, τ_1 and λ

In Figs. 1(a) and 1(b), we present the curves of the average phase-synchronization time τ as a function of multiplicative noise strength D with various τ_1 and λ respectively. We can see that there is a peak in the curve of $\tau - D$, in other words, with the increase of D, the average phase-synchronization time τ can reach a maximum; at the same time, when D is fixed, we found that the effects on τ of τ_1 and λ are non-monotone, with the increase of both τ_1 and λ, τ decreases firstly, then increases.

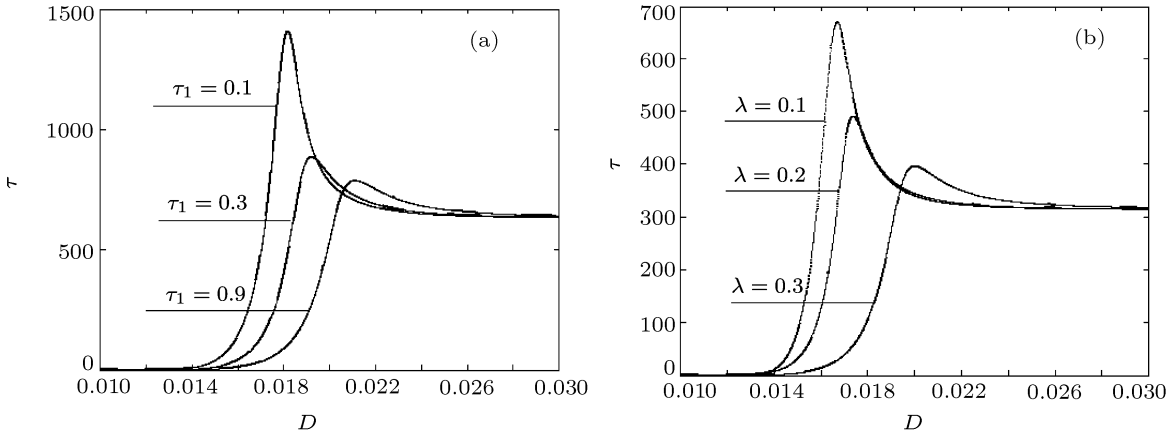


Fig. 1. The curves of τ as a function of D with various τ_1 ($\Omega_{\rm in} = 0.01$, $\alpha = 0.01$, $A = 0.15$, $\lambda = 0.2$) (a) and λ ($\Omega_{\rm in} = 0.02$, $\alpha = 0.01$, $A = 0.2$, $\tau_1 = 0.1$) (b).

In Figs. 2(a) and 2(b), we plot the curves of the average phase-synchronization time τ as a function of additive noise strength α with various λ and τ_1 respectively. From the figures we can see that there are also peaks in the curves of $\tau - \alpha$, namely, with the increase of α, τ can arrive at a maximum; we can also see that the effects on τ of τ_1 and λ are non-monotone.

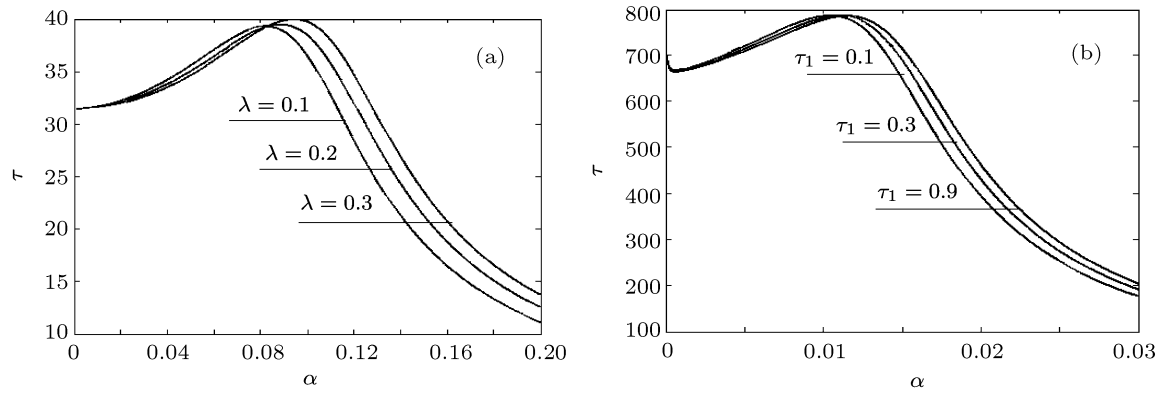


Fig. 2. The curves of τ as a function of α with various λ ($\Omega_{\rm in} = 0.2$, $D = 0.3$, $A = 0.1$, $\tau_1 = 0.1$) (a) and τ_1 ($\Omega_{\rm in} = 0.01$, $D = 0.3$, $A = 0.15$, $\tau_1 = 0.1$) (b).

In Figs. 3 and 4, we plot the curves of the average phase-synchronization time τ as a function of λ and τ_1 with various noise intensity ratios $R = D/\alpha$ respectively. Of course, we can see that there are also peaks in the $\tau - \lambda$ and $\tau - \tau_1$ curves, which are consistent with Figs. 1 and 2. On the other hand, we can see that τ arises with the increase of $R = D/\alpha$ when λ is fixed, but τ_1 is fixed; with the increase of R, τ decreases firstly, then increases.

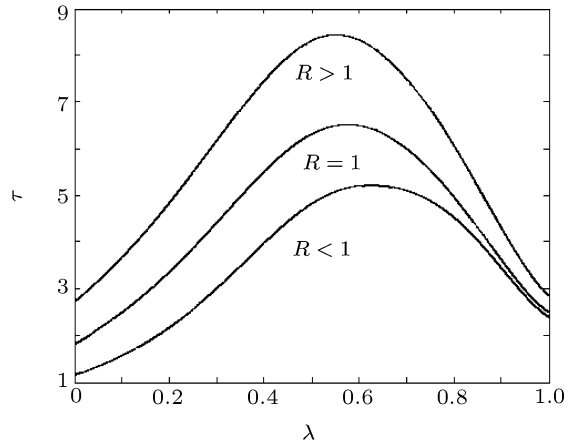


Fig. 3. The curves of τ as a function of λ with various R ($\Omega_{\rm in} = 0.2$, $R = D/\alpha$, $A = 0.1$, $\tau_1 = 0.1$).

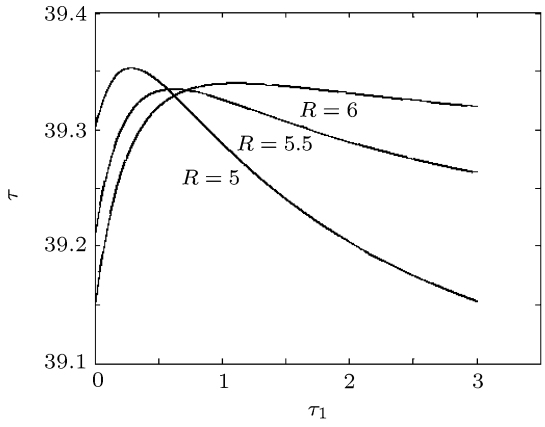


Fig. 4. The curves of τ as a function of τ_1 with various R ($\Omega_{\rm in} = 0.2$, $R = D/\alpha$, $A = 0.1$, $\lambda = 0.2$, $\tau_1 = 0.1$).

4. The correlation between SR and the average phase-synchronization time τ

4.1. Theoretical analysis

Now, we seek the correlation between SR and the average phase-synchronization time τ by the expressions of the output SNR and τ.

According to Eq.(12), we can obtain

$$[\Delta P_{\rm eq}(0)]^2 = \left(\frac{4\left[4\left\langle x(0)\right\rangle_{\rm eq}^2 \Gamma^2 + \left[x_0(0)\right]^2 \Omega_{\rm in}^2\right] - \Gamma\pi({\rm SNR})\left[\Delta x(0)\right]^2}{({\rm SNR})\left[\Delta x(0)\right]^2 \left[4\tanh\left(\frac{\Gamma T}{4}\right)/\Gamma T_0 - 1\right]} \right). \tag{18}$$

Inserting Eq. (18) into Eqs. (16) and (17), we can obtain

$$\Omega_{\rm out} = \frac{\pi\Gamma}{2}\Bigg[1 + 4\left[4\left\langle x(0)\right\rangle_{\rm eq}^2 \Gamma^2 + \left[x_0(0)\right]^2 \Omega_{\rm in}^2\right] - ({\rm SNR})\pi\Gamma\left[\Delta x(0)\right]^2 \left(4\tanh\left(\frac{\Gamma T_0}{4}\right)/\Gamma T_0 - 1\right)\Bigg], \tag{19}$$

$$\begin{aligned} D_{\rm eff} = {} & \Omega_{\rm out}\pi - \frac{2\pi^2}{T_0}\left(\frac{4\left[4\left\langle x(0)\right\rangle_{\rm eq}^2 \Gamma^2 + \left[x_0(0)\right]^2 \Omega_{\rm in}^2\right] - \Gamma\pi({\rm SNR})\left[\Delta x(0)\right]^2}{({\rm SNR})[\Delta x(0)]^2 \left[4\tanh\left(\frac{\Gamma T_0}{4}\right)/\Gamma T_0 - 1\right]} \right)^2 (\tanh(\Gamma T_0/4))^3 \\ & - \frac{\pi^2}{2T_0}\left(\frac{4\left[4\left\langle x(0)\right\rangle_{\rm eq}^2 \Gamma^2 + \left[x_0(0)\right]^2 \Omega_{\rm in}^2\right] - \Gamma\pi({\rm SNR})\left[\Delta x(0)\right]^2}{({\rm SNR})\left[\Delta x(0)\right]^2 \left[4\tanh\left(\frac{\Gamma T_0}{4}\right)/\Gamma T_0 - 1\right]} \right) \\ & \times \left(1 - \left(\frac{4\left[4\left\langle x(0)\right\rangle_{\rm eq}^2 \Gamma^2 + \left[x_0(0)\right]^2 \Omega_{\rm in}^2\right] - \Gamma\pi({\rm SNR})\left[\Delta x(0)\right]^2}{({\rm SNR})\left[\Delta x(0)\right]^2 \left[4\tanh\left(\frac{\Gamma T_0}{4}\right)/\Gamma T_0 - 1\right]} \right) (12\tanh(\Gamma T_0/4)) \right. \\ & \left. - \Gamma T_0(1 + 2[{\rm sech}(\Gamma T_0/4)]^2). \right. \end{aligned} \tag{20}$$

The expressions of Eqs. (18)–(20) are then plugged into Eq.(15) so as to provide the correlation expressions of the output SNR and τ; because it is tedious, it is omitted here. We will discuss the relationship between them by the curve of SNR $-$ τ.

4.2. The correlation between SR and τ

In this section, our aim is to investigate the correlation between SR and the average phase-synchronization time τ, and we want to know whether the average phase-synchronization time τ can reach its maximum when SR appears in systems (1) and (2).

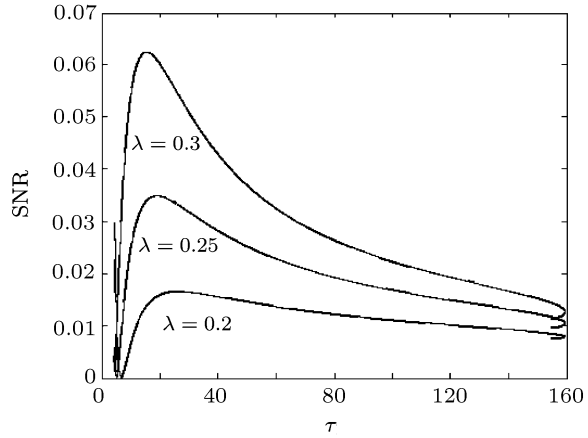


Fig. 5. The curves of SNR as a function of τ with various λ ($\Omega_{\rm in} = 0.05$, $A = 0.25$, $\tau_1 = 0.01$, $D = 0.1$).

At first, we plot the curves of the output SNR as a function of τ with various λ and τ_1 in Figs. 5 and 6 respectively. From the figures we can see that there is a restraint, then a peak in the curve of SNR $-$ τ, namely, with increasing τ, the output SNR decreases firstly, then it reaches a maximum. At the same time, the figures tell us that the output SNR can be enhanced by increasing λ, and it can be enhanced by decreasing τ_1 too; in addition, the location of the peak moves slightly. Figures 5 and

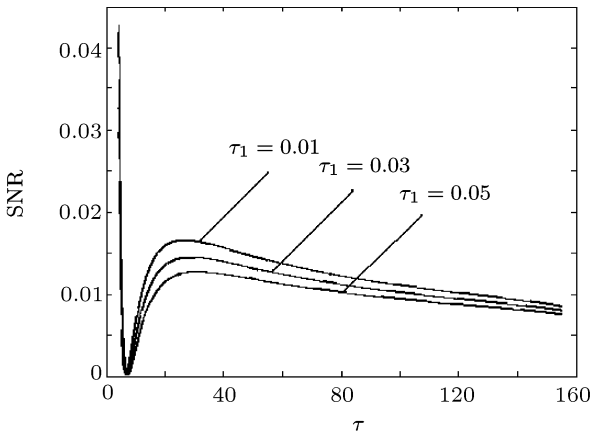


Fig. 6. The curves of SNR as a function of τ with various τ_1 ($\Omega_{\rm in} = 0.05$, $A = 0.25$, $\lambda = 0.2$, $D = 0.1$).

6 tell us that the output SNR can reach its maximum when τ arrives at an optimal level, so we can get the conclusion that the SR phenomenon is a reflection of the average phase-synchronization time τ; in other words, we may understand the principle of SR from the point of synchronization between the input and the output signal.

Next, we want to know whether the output SNR and the average phase-synchronization time τ will reach their maximum at the same noise level. So we plot the curves of SNR(τ) $-$ α and SNR(τ) $-$ D in the same figure, we use SNR $\times$ 10 and SNR $\times$ 10^3 vs. α instead of SNR $-$ α in order to see them clearly in Figs. 7 and 8 respectively.

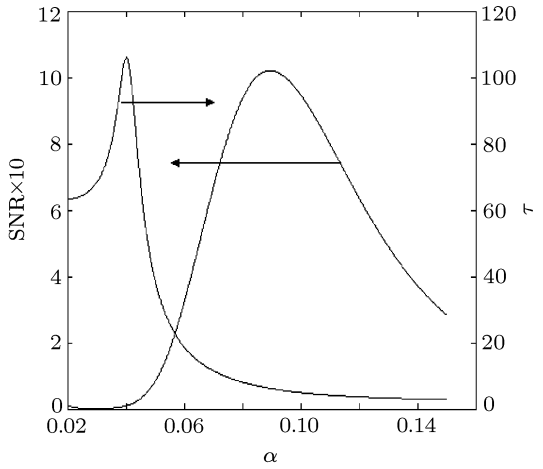


Fig. 7. The curves of SNR $\times$ 10 and τ as a function of α ($\Omega_{\rm in} = 0.1$, $D = 0.1$, $A = 0.15$, $\lambda = 0.7$, $\tau_1 = 0.01$).

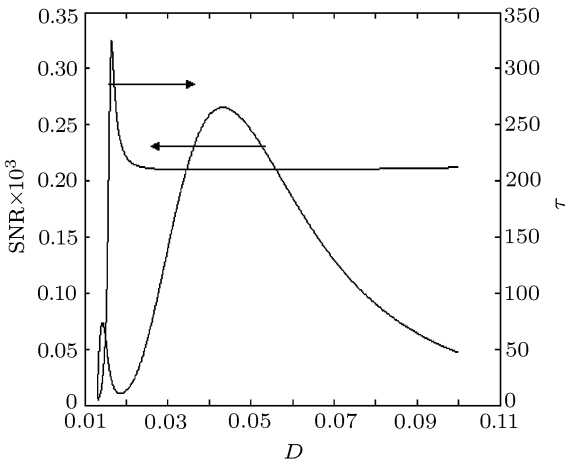


Fig. 8. The curves of SNR $\times$ 10^3 and τ as a function of D ($\Omega_{\rm in} = 0.03$, $\alpha = 0.01$, $A = 0.25$, $\lambda = 0.2$, $\tau_1 = 0.01$).

In Fig. 7, we plot the curves of SNR $\times$ 10 $-$ α and $\tau - \alpha$; the curve in the left is $\tau - \alpha$, and the right one is SNR $\times$ 10 $-$ α. From the figure we can see that the maximum of τ and the output SNR appear at different noise levels with the increasing of α, moreover, as the increase of α, τ arrives at its maximum firstly, then

the output SNR arrives at its maximum. In Fig. 8, we plot the curves of SNR $\times 10^3 - D$ and $\tau - D$, the curve with one peak is $\tau - D$, and the curve with two peaks is SNR $\times 10^3 - D$. From the figure we can see that the maximum of τ and output SNR appear at different time. Moreover, with the increase of D, the output SNR arrives at its maximum first, then τ arrives at its maximum, and the output SNR arrives at its maximum again.

5. Conclusion

SR and the average phase-synchronization time are important concepts in the study of dynamical systems. In this paper, we want to reveal the correlation between SR and the average phase-synchronization time of a bistable system driven by colour-correlated noises. The results show that SR may be a reflection of the phase-synchronization between the input and the output signals; this result can help us to understand SR better. On the other hand, SR and the maximum of the average phase-synchronization time appear at different time. With the increase of additive noise intensity α, the average phase-synchronization time τ arrives at its maximum first, then the output SNR reaches its maximum; while with the increase of multiplicative noise intensity D, the output SNR arrives at its maximum first, then the average phase-synchronization time τ arrives at its maximum, and the output SNR arrives at its maximum again. These results may provide assistance for understanding the principle of SR.

References

[1] Park K, Lai Y C and Krishnamoorthy S 2007 *Chaos* **17** 043111
[2] Park K, Lai Y C, Liu Z H and Nachman A 2004 *Phys. Lett.* A **326** 391
[3] Park K, Lai Y C and Krishnamoorthy S 2007 *Phys. Rev.* E **75** 046205
[4] Casado-Pascual J, Gomez-Ordonez J, Morillo M and Hanggi P 2003 *Phys. Rev. Lett.* **91** 210601
[5] Jia Y, Yu S N and Li J R 2000 *Phys. Rev.* E **62** 1869
[6] Zhang X Y and Xu W 2007 *Chin. Phys.* **16** 928
[7] Novikov E A and Eksp Z 1965 *Sov. Phys. JETP* **20** 1290
[8] Fox R F 1986 *Phys. Rev.* A **34** 4525
[9] Jung P and Hanggi P 1987 *Phys. Rev.* A **35** 4464
[10] Jin Y F and Xu W 2005 *Chaos, Solitons & Fractals* **23** 275
[11] Masoliver J, West B J and Lindenberg K 1987 *Phys. Rev.* A **35** 3086
[12] Jin Y F, Xu W, Ma S J and Li W 2005 *Acta Phys. Sin.* **54** 3480 (in Chinese)
[13] Li J L 2007 *Chin. Phys.* **16** 340
[14] Xu W, Jin Y F, Li W and Ma S J 2005 *Chin. Phys.* **14** 1077