# Simulation study of the two-time intensity correlation function of a two-mode laser system with both pump and quantum noises*

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This paper investigates the two-time intensity correlation function of a two-mode ring laser system subjected to both pump and quantum noises by stochastic simulation. It finds that the decay rate of the intensity correlation function of one mode gets faster with decreasing values of relevant parameters, i.e., the coupling constant $\xi$, the cross-correlation coefficient $\lambda$, the difference of the pump parameters $\Delta a$ and the pump parameter $a_1$; however, its variations get complex in the other mode when relevant parameters are changed. The investigating results also show that the effects of the mode competition on intensity correlation function are obvious.

**Keywords:** two-mode laser, correlated noises, correlation function
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## 1. Introduction

The two-mode ring laser system has attracted a great deal of attention over the last decades[1−23] for its application in a laser gyro and mode competition. Many problems from the first-passage-time problem,[1−4] the intensity fluctuations,[5−13] and the backscattering in a laser gyro[14−20] to a full account of saturation effects[21−23] in this field have been investigated.

The two-time intensity correlation function as an important statistical quantity to characterize the dynamic behaviour of stochastic systems in a nonequilibrium steady state is defined by

$$C(t) = \frac{\langle \delta I(t+t')\delta I(t')\rangle_{\rm s}}{\langle (\delta I)^2\rangle_{\rm s}}, \qquad (1)$$

where $\delta I(t') = I(t') - \langle I(t')\rangle_{\rm s}$, and $\langle I(t')\rangle_{\rm s}$ denotes the steady-state mean light intensity. It has been widely used in many stochastic systems to analyse the cases of the fluctuation decay of the dynamical variable.[24−31]

In the present paper, we investigate the two-time intensity correlation function of a two-mode laser subjected to pump noise and quantum noise for different combinations of relevant parameters by stochastic simulation. In Section 2, on the basis of the intensity Langevin equations, the stochastic simulation has been processed and then the two-time intensity correlation function has been investigated. In Section 3, some conclusions are given.

## 2. Investigation of the intensity correlation function

The dimensionless coupled complex electric fields $E_1(t)$ and $E_2(t)$ of a two-mode laser follows Langevin equations[12]

$$\frac{{\rm d}E_1}{{\rm d}t} = (a_1 - |E_1|^2 - \xi|E_2|^2)E_1 + p(t)E_1 + q_1(t), \quad (2)$$

$$\frac{{\rm d}E_2}{{\rm d}t} = (a_2 - |E_2|^2 - \xi|E_1|^2)E_2 + p(t)E_2 + q_2(t), \quad (3)$$

where $a_1$ and $a_2$ are the pump parameters of the two modes, respectively, and $\xi$ is the mode-coupling constant, which depends on the detuning from the atomic line centre,[5,32] $p(t) = p_{\rm R} + {\rm i}\, p_{\rm I}$ is the pump fluctuation, $q_1(t) = q_{1\rm R} + {\rm i}\, q_{1\rm I}$ and $q_2(t) = q_{2\rm R} + {\rm i}\, q_{2\rm I}$ are independent quantum noises, the subscripts I and R represent the real and imaginary parts respectively. These random noise terms are assumed to be zero mean and correlation

$$\langle q_{ki}(t)q_{kj}(t')\rangle = [\delta_{ij} + \lambda(1-\delta_{ij})]P\delta(t-t'), \qquad (4)$$

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and

$$\langle p_i(t)p_j(t')\rangle = P'\delta_{ij}\delta(t-t'), \tag{5}$$

where $i,j = I,R$ and $k = 1,2$, $P$ and $P'$ are the strength of the quantum noise and pump noise, respectively, $\lambda$ is the cross-correlation coefficient which measures the degree of cross-correlation between $q_{k\mathrm{I}}(t)$ and $q_{k\mathrm{R}}(t)$, and $|\lambda| \leq 1$.

From Eqs. (2) and (3), the Langevin equations of the laser intensities $I_1$ and $I_2$ for a two-mode laser can be obtained by means of the phase-locking method,[33] and they are

$$\frac{\mathrm{d}I_1}{\mathrm{d}t} = 2(a_1 - I_1 - \xi I_2)I_1 + P(1-|\lambda|) + 2I_1 p_{\mathrm{R}}(t) + 2\sqrt{I_1}\tilde{\varepsilon}_1(t), \tag{6}$$

$$\frac{\mathrm{d}I_2}{\mathrm{d}t} = 2(a_2 - I_2 - \xi I_1)I_2 + P(1-|\lambda|) + 2I_2 p_{\mathrm{R}}(t) + 2\sqrt{I_2}\tilde{\varepsilon}_2(t), \tag{7}$$

where the random noise terms have zero mean and their correlations are

$$\langle p_{\mathrm{R}}(t)p_{\mathrm{R}}(t')\rangle = P'\delta(t-t') \tag{8}$$

and

$$\langle \tilde{\varepsilon}_j(t)\tilde{\varepsilon}_j(t')\rangle = P(1+|\lambda|)\delta(t-t'), \quad j = 1,2. \tag{9}$$

Following Eqs. (6) and (7), one can obtain the corresponding Fokker–Planck equation for the probability function of laser intensity. Generally, there does not exist an analytic solution for a Fokker–Planck equation with arbitrary values of the pump parameters and coupling constant[34] except for the case of equal pump parameters at line centre, i.e., $\xi = 1$ and $a_1 = a_2$.[12]

In order to review the effects of various parameter combinations on the intensity correlation function, stochastic simulation would be a desirable method.[24] The data quoted for the intensity correlation function come from the stochastic simulation of Eqs. (6) and (7) by means of the Euler method, in which the Box–Mueller algorithm is used to generate Gaussian noise from two independent random numbers which are uniformly distributed between 0 and 1 and the time step is 0.001.

The variations of the intensity correlation functions ($C_1(t)$ and $C_2(t)$ respectively stand for the intensity correlation functions for $I_1$ and $I_2$) as the functions of decay time $t$ for combinations of different parameters are presented in Figs. 1–4. The strength of the quantum noise $P = 2$ and the strength of the pump noise $P' = 5$ are held fixed, and the difference of pump parameters $\Delta a \equiv a_1 - a_2$.

The intensity correlation functions are plotted in Fig. 1 as a function of decay time $t$ for several values of the coupling constant $\xi$ with $a_1 = 15$, $\Delta a = 0.8$ and $\lambda = 0$. Figure 1 shows that $C(t)$ exponentially decays as $t$ increases, moreover, the rate of decay gets faster and faster as $\xi$ decreases. When the value of $\xi$ is fixed, $C_1(t)$ decays faster than $C_2(t)$, besides, the rate of decay of $C_2(t)$ is closer and closer to that of $C_1(t)$ as $\xi$ decreases.

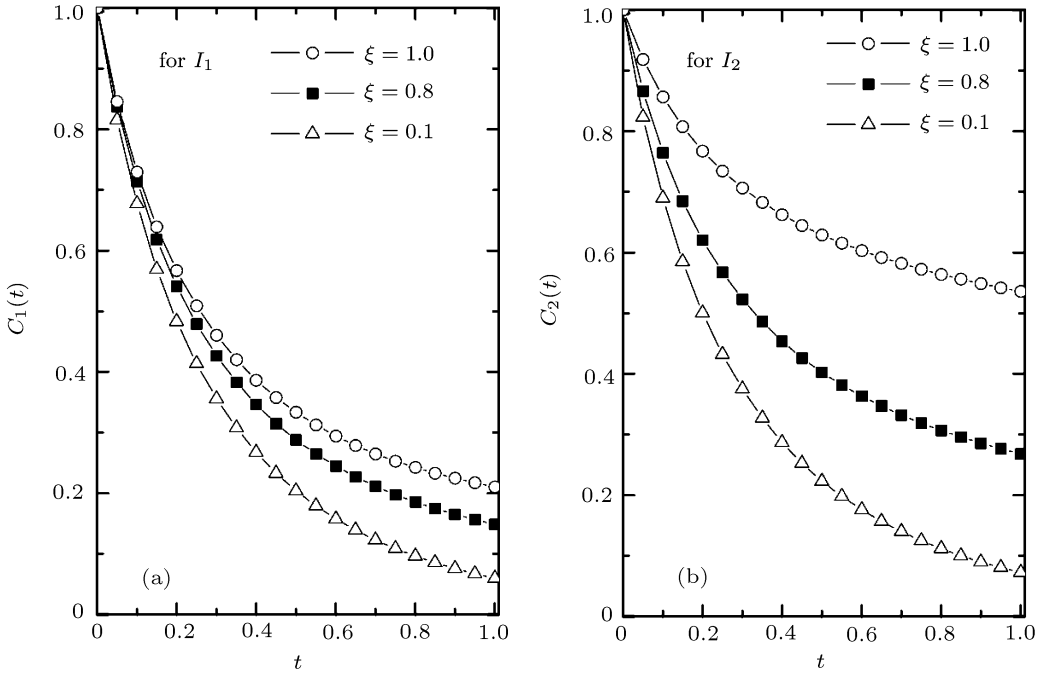



**Fig. 1.** The intensity correlation functions $C_1(t)$ (a) and $C_2(t)$ (b) as functions of decay time $t$ for several values of the coupling constant $\xi$ with $a_1 = 15$, $\Delta a = 0.8$, $\lambda = 0$, $P = 2$ and $P' = 5$.

The intensity correlation functions are plotted in Fig. 2 as a function of decay time $t$ for several values of the cross-correlation coefficient $\lambda$ with $a_1 = 15$, $\Delta a = 0.8$ and $\xi = 1$. Figure 2 shows that the intensity correlation functions decay faster and faster as $\lambda$ decreases. The effects of $\lambda$ on the decay of $C_1(t)$ are more obvious in comparison with the case of $C_2(t)$. When the value of $\xi$ is fixed, $C_1(t)$ decays faster than $C_2(t)$.

The intensity correlation functions are plotted in Fig. 3 as a function of decay time $t$ for several values of the difference of pump parameters $\Delta a$ with $a_1 = 15$, $\xi = 1$ and $\lambda = 0$. Figure 3 presents that $C_1(t)$ decays faster and faster as $\Delta a$ increases. In contrast, $C_2(t)$ decays slower and slower as $\Delta a$ increases. When $\Delta a = 0$ the intensity correlation functions for $I_1$ and $I_2$ decay at the same rate.

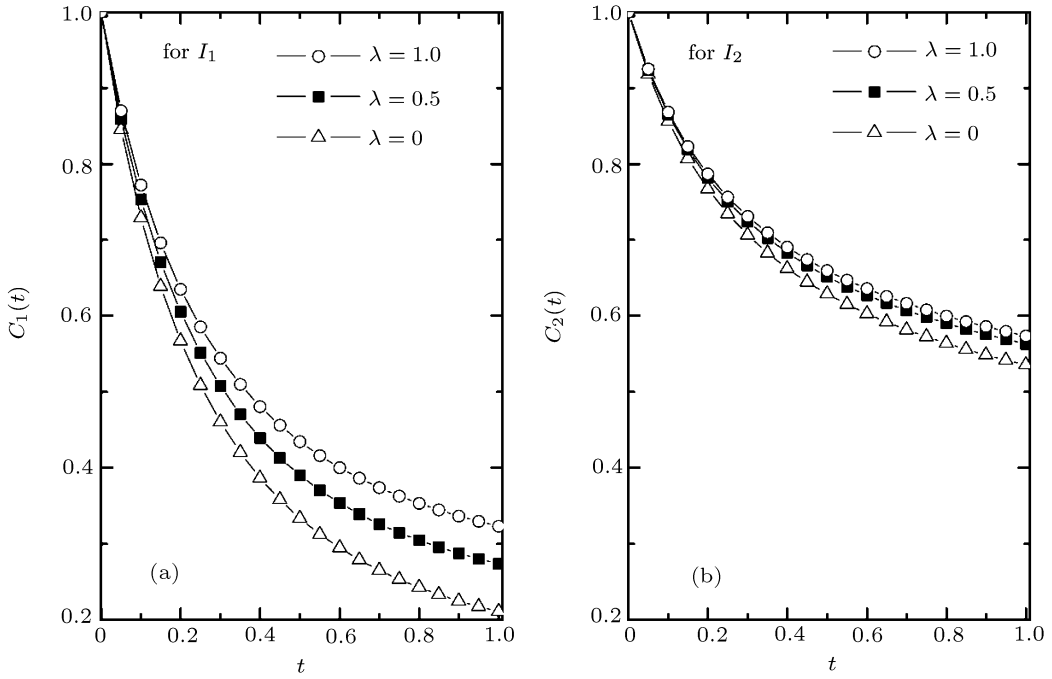



**Fig. 2.** The intensity correlation functions $C_1(t)$ (a) and $C_2(t)$ (b) as functions of decay time $t$ for several values of the cross-correlation coefficient $\lambda$ with $a_1 = 15$ $\Delta a = 0.8$, $\xi = 1$, $P = 2$ and $P' = 5$.

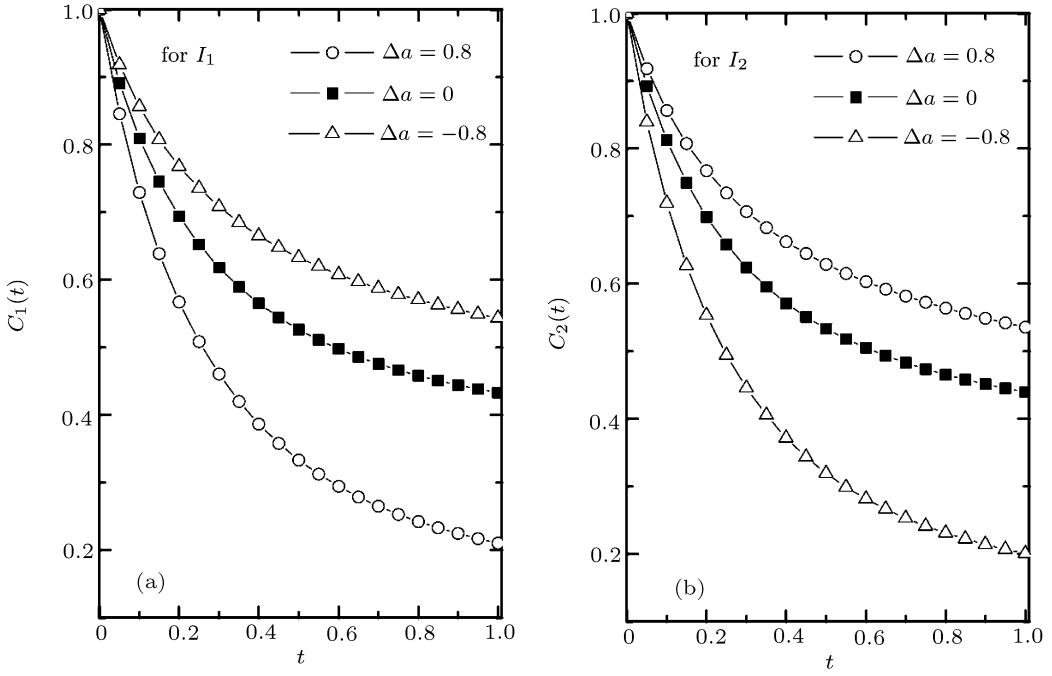



**Fig. 3.** The intensity correlation functions $C_1(t)$ (a) and $C_2(t)$ (b) as functions of decay time $t$ for several values of the difference of pump parameter $\Delta a$ with $a_1 = 15$, $\xi = 1$, $\lambda = 0$, $P = 2$ and $P' = 5$.

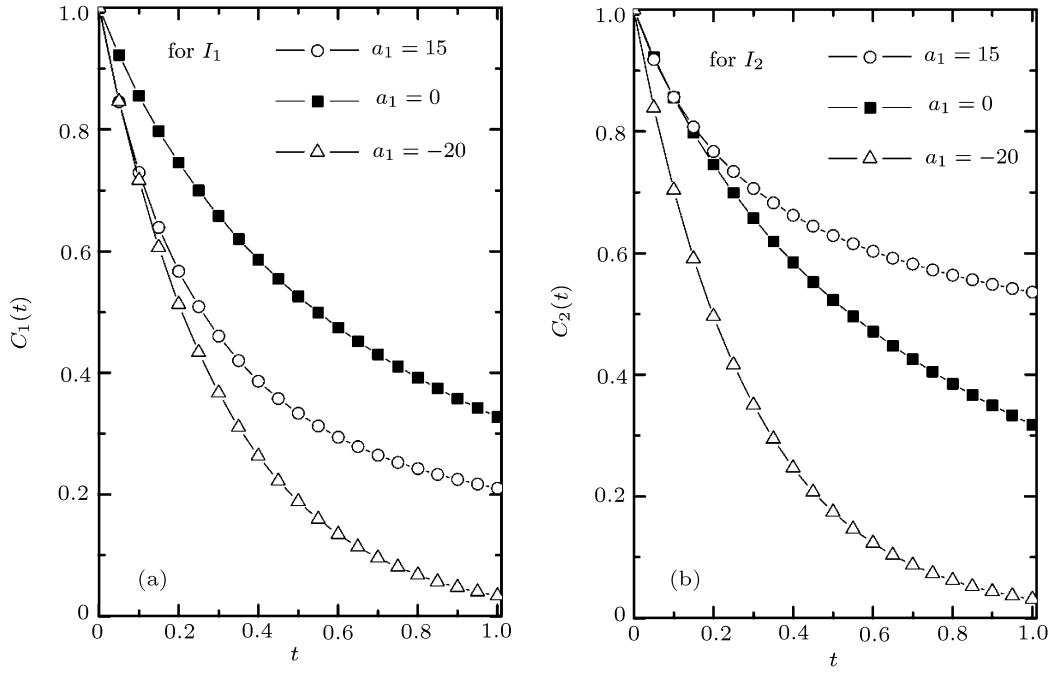



**Fig. 4.** The intensity correlation functions $C_1(t)$ (a) and $C_2(t)$ (b) as functions of decay time $t$ for the different values of the pump parameter $a_1$ with $\Delta a = 0.8$, $\xi = 1$, $\lambda = 0$, $P = 2$ and $P' = 5$.

The intensity correlation functions are plotted in Fig. 4 as a function of decay time $t$ for different values of the pump parameter $a_1$ with $\Delta a = 0.8$, $\xi = 1$ and $\lambda = 0$. Figure 4 shows that $C_2(t)$ decays faster and faster as $a_1$ increases. The decay of $C_1(t)$ is almost the same as that of $C_2(t)$ when $a_1 = 0$ and $a_1 = -20$, however, the decay of $C_1(t)$ is much faster than in the case of $C_2(t)$ if $a_1 = 15$.

## 3. Conclusions

This paper studies the two-time intensity correlation functions in a two-mode laser with different combinations of the coupling constant $\xi$, the cross-correlation coefficient $\lambda$, the difference of the pump parameters $\Delta a$ and the pump parameter $a_1$ studied by stochastic simulation. Some conclusions are given as follows:

(i) When any one of the parameters $\xi$, $\lambda$, $\Delta a$ and $a_1$ gets smaller, the decay rate of the intensity correlation function $C_2(t)$ gets faster.

(ii) The decay rate of the intensity correlation function $C_1(t)$ increases as $\xi$ and $\lambda$ decreases; however, it gets slower as $\Delta a$ decreases. In comparison with the case of $C_2(t)$, we find that the decay rate of $C_1(t)$ has no obvious difference when $a_1$ takes 0 and $-20$; however, it is much faster than that of $C_2(t)$ when $a_1 = 15$.

(iii) The effects of the same parameter on the intensity correlation functions $C_1(t)$ and $C_2(t)$ are quite different, and this difference shows the competition between the two modes of the laser.

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