Effects of dark energy interacting with massive neutrinos and dark matter on universe evolution*

Chen Ju-Hua(陈菊华)[a)b)†] and Wang Yong-Jiu(王永久)[a)]

[a)] College of Physics and Information Science, Hunan Normal University, Changsha 410081, China
[b)] Department of Physics and Astronomy, University of Missouri, Columbia, MO 65211, USA

(Received 2 April 2009; revised manuscript received 18 May 2009)

In this paper we investigate the evolution of the cosmology model with dark energy interacting with massive neutrinos and dark matter. Using the numerical method to investigate the dynamical system, we find that the stronger the interaction between dark energy and dark matter, the lower the ratio of dark matter in the universe is; also, the stronger the interaction between dark energy and massive neutrinos, the lower the ratio of massive neutrinos in the universe is. On the other hand, the interaction between dark energy and dark matter or massive neutrinos has an effect on disturbing the universe's acceleration; we also find that our universe is still accelerating.

Keywords: dark energy, dark matter, massive neutrinos
PACC: 0420, 0490

1. Introduction

The recent astronomical measurements of type-Ia supernovae (SN Ia)[1] and microwave background (CMB) radiation[2] have provided strong evidence for a spatially flat and accelerating universe in the present time. The expansion implies the valuable contribution of matter with negative pressure to the evolution of the universe. The origin of accelerating expansion is regarded as being that the universe is dominated by an exotic component with negative pressure called 'dark energy' which constitutes 70 percent of the energy density of the universe, and about 26 percent of dark matter. The accelerated expansion of the present universe is attributed to the fact that dark energy is an exotic component with negative pressure and the simplest candidate for dark energy is the cosmological constant. However, two difficulties arise from this scenarios, namely the fine-tuning problem and the cosmic coincidence problem.[3] There are currently several proposed candidates for dark energy. The first is the cosmological constant,[4] and the second is the so-called dynamic candidates such as phantoms,[5] quintessence,[6,7] K-essence[8] and quintoms.[9] The difference of these candidates for dark energy is the size of the parameter ω_E, namely the ratio of the pressure and energy density of the dark energy. For quintessence, the state equation is given by the relation between the pressure p_E and the energy density ρ_E, i.e. $p_E = \omega_E \rho_E$ which causes the acceleration. Dark energy has been sought in a wide range of physical phenomena depending on the value of the equation of state parameter w_E as follows: (a) A quintessence field $-1 < w_E < -1/3$, (b) The cosmological constant $w_E = -1$, (c) A phantom field $w_E < -1$. The negative pressure of the dark energy may be the cause of the acceleration of the present universe. However, the nature of the dark energy still remains a complete mystery. No more than five years ago some physicists[10] found that, if we only assume the cosmic fluid to be ideal, i.e. nonviscous, it must bring out the occurrence of a singularity of the universe in the far future, i.e. the big Rip.

Some authors[11,12] found the dark energy scale $\simeq 10.3$ eV is smaller than energy scales in particle physics, but interestingly it is comparable to the neutrino mass. In quintessence-like models, the dark energy $m_{\rm Q} \simeq 10.33$ eV is also connected to the neutrino mass via a see-saw formula $m_{\rm Q} \simeq m_\nu^2/m_{\rm PL}$, where $m_{\rm PL}$ is the Planck mass. They set up cosmological models of neutrino dark energy or interacting dark energy between massive neutrinos. Brookfield *et al.*[13] found that the coupling of neutrinos to dark energy slightly alters the cosmological background, which is

*Project supported by the National Natural Science Foundation of China (Grant No. 10873004), the Scientific Research Fund of Hunan Provincial Education Department (Grant No. 08B051), the Program for Excellent Talents in Hunan Normal University and the State Key Development Program for Basic Research Program of China (Grant No. 2010CB832800).
†Corresponding author. E-mail: jhchen@hunnu.edu.cn
© 2010 Chinese Physical Society and IOP Publishing Ltd

similar to the dark energy/dark matter interaction.

Moreover, in recent years some interacting DE models have been brought out to overcome the problem. In these models it is assumed that there exists a nonzero interaction between DE and DM in the universe and gauges DE transfers to DM, which allows us to create an equilibrium balance in the evolution of the universe, so the density of DE remains of the same order as that of DM at late times. Some authors[14] investigated dynamical behaviours of dark energy models, i.e. dark energy with a linear equation of state interacting with dark matter in different cosmology models. They all found that, under some proper parameters of the dynamical system, the universe will enter an era dominated by dark energy and dark matter with interaction between them, and accelerate at late times. However, they all only considered the interaction between dark energy and dark matter under different cosmology models. In this paper we extend the interaction between dark energy, dark matter and massive neutrinos to investigate the effects of dark energy interacting with mass neutrinos and dark matter on the universe's evolution.

The paper is organized as follows: In Section 2, we set up an autonomous dynamical system of dark energy interacting with massive neutrinos and dark matter in the flat FRW universe. In Section 3, we will focus on the evolution of the above dynamical system in phase space with different parameters and effects of dark energy interacting with massive neutrinos and dark matter on our universe's evolution. A brief conclusion is given in the last section.

2. Dynamics of dark energy interacting with massive neutrinos and dark matter in a flat FRW universe

Based on Einstein's general relativity theory, the standard Friedmann equation acts as follows in the flat FRW universe

$$H^2 = \frac{k}{3}(\rho_E + \rho_M + \rho_\nu), \tag{1}$$

where H is the Hubble constant and $k = 8\pi G$. The conservation theory of the energy–momentum tensor $\boldsymbol{T}^{\mu\nu}_{;\nu}$ leads to the evolution equation for the cosmological total energy density,

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2}$$

where the total energy density is $\rho = \rho_E + \rho_M + \rho_\nu$.

Wetterich[15] considered that a pressureless dark matter field interacts with a scalar field, so the evolution equation is given by

$$\dot{\rho}_M + 3H\rho_M = \alpha H^{\frac{3}{2}}\rho_M\dot{\phi}. \tag{3}$$

Based on his proposal, the scalar field stands for the dark energy property and $\alpha H^{3/2}\rho_M\dot{\phi}$ is the coupling term of dark energy to dark matter.

On the other hand we take $\beta H^{3/2}(\rho_\nu - 3p_\nu)\dot{\phi}$ as the interaction term between dark energy and massive neutrinos,[13] so the massive neutrino field evolution equation is

$$\dot{\rho}_\nu + 3H(\rho_\nu + p_\nu) = \beta H^{3/2}(\rho_\nu - 3p_\nu)\dot{\phi}. \tag{4}$$

The coefficient β connects with the mass of the neutrinos. Here β will be considered as a phenomenological coefficient which couples the dark energy field with the neutrinos. Noting that if $p_\nu = \rho_\nu/3$, there is no coupling between the fields of dark energy and neutrinos. So it is important to note that the neutrinos in the past must behave as massless particles, for which the relationship between the pressure and the energy density is $p_\nu = \rho_\nu/3$. Due to the coupling of the neutrinos with the scalar field, they become massive and non-relativistic. For these reasons a barotropic equation of state for the neutrinos is proposed, i.e. the ratio between the pressure and the energy density $p_\nu = \omega_\nu\rho_\nu$, so Eq. (4) becomes

$$\dot{\rho}_\nu + 3H(1 + \omega_\nu)\rho_\nu = \beta H^{3/2}(1 - 3\omega_\nu)\rho_\nu\dot{\phi}. \tag{5}$$

So the evolution equation for dark energy density is obtained from Eqs. (1) and (2), here we take the ratio between the dark energy pressure and the energy density as $p_E = \omega_E\rho_E$,

$$\begin{aligned}&\dot{\rho}_E + 3H(1 + \omega_E)\rho_E\\ &= -[\alpha H^{\frac{3}{2}}\rho_M\dot{\phi} + \beta H^{\frac{3}{2}}(1 - 3\omega_\nu)\rho_\nu\dot{\phi}].\end{aligned} \tag{6}$$

Differentiating Eq.(1), then putting Eq.(2) into it, we can get

$$\dot{H} = -\frac{k}{2}(\rho_M + \rho_E + \rho_\nu + p_\nu + p_E). \tag{7}$$

If we introduce the following dimensionless variables

$$x = \frac{\sqrt{k\rho_E}}{\sqrt{3}H},\ y = \frac{\sqrt{k\rho_M}}{\sqrt{3}H},\ z = \frac{\sqrt{k\rho_\nu}}{\sqrt{3}H},\ \frac{\mathrm{d}}{\mathrm{d}N} = \frac{1}{H}\frac{\mathrm{d}}{\mathrm{d}t}, \tag{8}$$

where $N \equiv \ln a$ is the number of e-folding which presents the cosmological time. The energy density and pressure of the dark energy connects the scalar

field by $\dot{\phi} = \sqrt{\rho_E + p_E}$. We can acquire the Hubble equation and the effective total equation of state in the following forms, respectively

$$x^2 + y^2 + z^2 = 1, \tag{9}$$

$$\frac{\dot{H}}{H^2} = -\frac{3}{2}[1 + \omega_E x^2 + \omega_\nu z^2], \tag{10}$$

$$\omega_{\rm total} = \frac{\omega_E \rho_E + \omega_\nu \rho_\nu}{\rho_E + \rho_M + \rho_\nu} = \omega_E x^2 + \omega_\nu z^2. \tag{11}$$

Under these conditions we can construct the following autonomous dynamical system by Eqs. (1), (3), (4), (5), (6) and (7) for the interacting dark energy with massive neutrinos and dark matter in the flat FRW universe,

$$x' = -\frac{\alpha\sqrt{3(1+\omega_E)}}{2}y^2 - \frac{3\omega_E}{2}x + \frac{3\omega_E}{2}x^3 - \frac{\beta(1-3\omega_\nu)\sqrt{3(1+\omega_E)}}{2}z^2 + \frac{3\omega_\nu}{2}xz^2, \tag{12}$$

$$y' = \frac{\alpha\sqrt{3(1+\omega_E)}}{2}xy + \frac{3\omega_E}{2}yx^2 + \frac{3\omega_\nu}{2}yz^2, \tag{13}$$

$$z' = \frac{\beta(1-3\omega_\nu)\sqrt{3(1+\omega_E)}}{2}xz - \frac{3\omega_\nu}{2}z + \frac{3\omega_E}{2}x^2 z + \frac{3\omega_\nu}{2}z^3. \tag{14}$$

In the following section we will mainly focus on the evolution of the above dynamical system in the phase space with different parameters and effects of dark energy interacting with massive neutrinos and dark matter on our universe's evolution.

3. Evolution of the dynamical system in phase space and effects of the interaction term on cosmological acceleration

Equations (12)–(14) describe the evolution of the cosmology model with dark energy interacting with dark matter and massive neutrinos in phase space. In order to investigate the properties of three variables depending on the interacting constants α, β and state parameters ω_E, ω_ν, because of the complexity of the dynamical system, we cannot give detailed algebra expressions for x, y, z. In this section we can only perform numerical solutions for interacting constants and state parameters.

In Fig. 1, we show the evolution of the cosmology model with dark energy interacting with dark matter and massive neutrinos in phase space for different interacting constants α between dark energy and dark matter by fixed β, ω_E and ω_ν. We can see that if the interaction between dark energy and dark matter is getting stronger, the ratio of the dark matter in our universe will decrease. In Fig. 2, we perform an evolution of the cosmology model with dark energy interacting with dark matter and massive neutrinos in phase space for different interacting constants β between dark energy and massive neutrinos by fixed α, ω_E and ω_ν. We can see that the stronger the interaction between dark energy

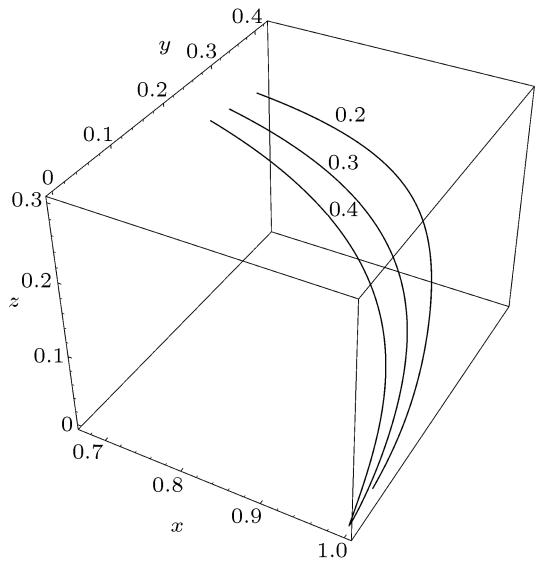


Fig. 1. Evolution of dark energy interacting with the dark matter and massive neutrino cosmology model in phase space with $\alpha = 0.2, 0.3, 0.4$, $\beta = 0.5$ for fixed $\omega_E = -0.25, \omega_\nu = 0.08$.

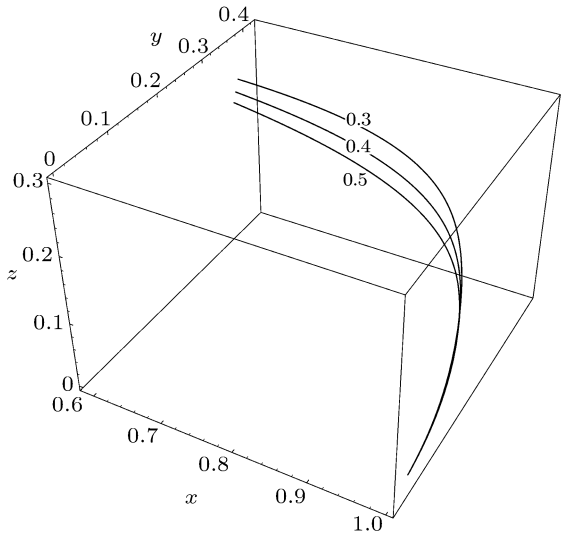


Fig. 2. Evolution of dark energy interacting with the dark matter and massive neutrino cosmology model in phase space with $\beta = 0.3, 0.4, 0.5$ for fixed $\alpha = 0.4, \omega_E = -0.25, \omega_\nu = 0.08$.

and massive neutrinos, the lower the ratio of mass neutrinos in our universe is.

Then in order to investigate the evolution of the universe on a cosmologic time scale, we plot the density evolution of dark energy interacting with the dark matter and massive neutrino cosmology model for fixed parameters $\alpha = 0.4, \beta = 0.5, \omega_E = -0.25, \omega_\nu = 0.08$ in Fig. 3. From this figure, we can find that: 1) During the evolution of our universe, dark energy increases while dark matter and massive neutrinos decrease; 2) The relative ratio of dark energy, dark matter and massive neutrinos will be stable, which agrees with current astronomical observations;[1,2] 3) Based on the dark matter curve, we can see that there is a peak during the evolution of our universe.

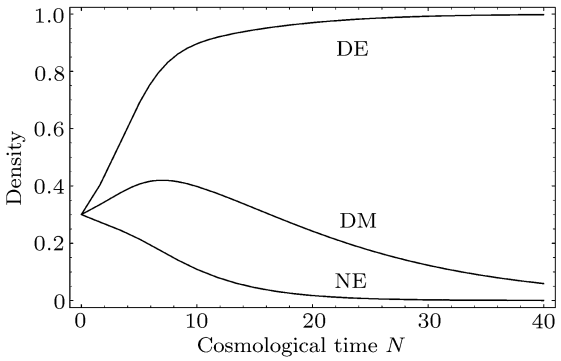


Fig. 3. Density evolution of dark energy interacting with the dark matter and massive neutrino cosmology model for fixed parameters $\alpha = 0.4, \beta = 0.5, \omega_E = -0.25$ and $\omega_\nu = 0.08$.

Finally we studied the effects of the interacting terms on the universe's acceleration. Figs. 4 and 5 show the universe's acceleration depending on the interacting constants between dark energy and dark matter or massive neutrinos, respectively. We can see that: 1) The interaction of dark energy, dark matter and massive neutrinos will disturb the universe's decreasing acceleration during the evolution of our universe; 2) The acceleration of our universe is slowing down.[16] However, obviously, our universe is still accelerating as shown by Figs. 4 and 5. This result is not in contradiction with the current observations; 3) At late times the universe's acceleration will be stable for different conditions.

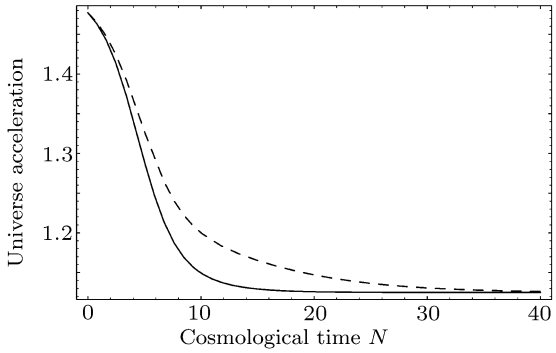


Fig. 4. The effect of interacting constants $\alpha = 0.3$, 0.4 (dash) between dark energy and dark matter on the universe's acceleration for fixed $\beta = 0.5$, $\omega_E = -0.25$, and $\omega_\nu = 0.08$.

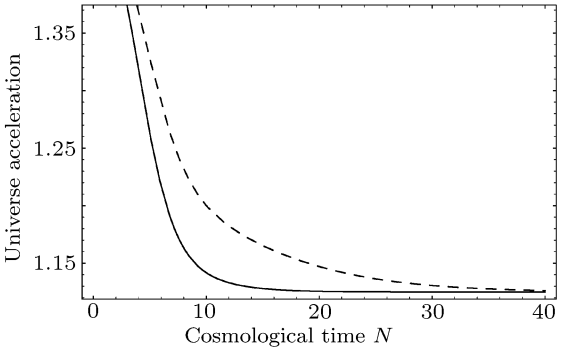


Fig. 5. The effect of interacting constants $\beta = 0.3$, 0.5 (dash) between dark energy and massive neutrinos on the universe's acceleration for fixed $\alpha = 0.4$, $\omega_E = -0.25$, and $\omega_\nu = 0.08$.

4. Conclusions

In this paper we have investigated the evolution of a cosmology model in which dark energy interacts with massive neutrinos and dark matter. Using the numerical method to investigate the dynamical system, we found that the stronger the interaction between dark energy and dark matter, the lower the ratio of dark matter in our universe is; also, the stronger the interaction between dark energy and massive neutrinos, the lower the ratio of mass neutrinos in our universe is. On the other hand, the interacting term between dark energy and dark matter or massive neutrinos has an effect on the universe's acceleration by decreasing it. Our universe is still accelerating.

References

[1] Bennett C L S, Halpern M, Hinshaw G and Jarosik N 2003 *Astrophys. J. Suppl.* **148** 1

Perlmutter S, Aldering G, Goldhaber G and Knop R A 1999 *Astrophys. J.* **517** 565

Riess A G, Peter N E and Ronald G L 2001 *Astrophys. J.*

560 49
Turner M S and Riess A G 2002 *Astrophys. J.* **569** 18
Tonry J, Schmidt P B and Barris B 2003 *Astrophys. J.* **594** 1

[2] Perlmutter S, Gabi S and Goldhaber G 1997 *Astrophys. J.* **483** 565
Peiris H V, Komatsu E and Verde L 2003 *Astrophys. J. Suppl.* **148** 213
Netterfield C B, Ade P A R and Bock J J 2002 *Astrophys. J.* **571** 604
Halverson N, Leitch E M and Pryke C 2002 *Astrophys. J.* **568** 38
Spergel D N, Verde L and Peiris H V 2003 *Astrophys. J. Suppl.* **148** 175

[3] Steinhardt P In: Fitch V L and Marlow D R (editors) 1997 *Critical Problems in Physics* (Princeton: Princeton University Press)
Dicke R H and Peebles P J E In: Hawking S W and Israel W 1979 *General Relativity: an Einstein Centenary Survey* (Cambridge: Cambridge University Press)
Copeland E J, Sami M and Tsujikawa S 2006 *Int. J. Mod. Phys.* D **15** 1753

[4] Padmanabham T 2003 *Phys. Rep.* **380** 235
Alcaniz J S 2004 *Phys. Rev.* D **69** 083501

[5] Caldwelland R R and Rahul D 2002 *Phys. Lett.* B **545** 23
Chimento L P and Lazkoz R 2003 *Phys. Rev. Lett.* **91** 211301
Caldwell R R, Dave R and Steinhardt P J 1998 *Phys. Rev. Lett.* **80** 1582
Sahniv L M and Wang Y 2000 *Phys. Rev.* D **62** 103517

[6] Capozziello S, Cardone V F, Piedipalumbo E and Rubano C 2006 *Class. Quantum Grav.* **23** 1205
Zhang Y and Gui Y X 2006 *Class. Quantum Grav.* **23** 6141
Vikman A 2005 *Phys. Rev.* D **71** 023515
Rogerio R and Joshua A F 2007 *Phys. Rev.* D **75** 043513

[7] Chen J H and Wang Y J 2008 *Chin. Phys.* B **17** 1184
Chen J H and Wang Y J 2007 *Chin. Phys.* **16** 3212
Chen J H and Wang Y J 2004 *Chin. Phys.* **13** 583
Chen Guang 2008 *Acta Phys. Sin.* **57** 3299 (in Chinese)

[8] Chiba T, Okabe T and Yamaguchi M 2000 *Phys. Rev.* D **62** 023511
Scherrer R J 2004 *Phys. Rev. Lett.* **93** 011301

[9] Wei H, Cai R G and Zeng D F 2005 *Class. Quantum Grav.* **22** 3189
Zhao G B, Xia J Q, Li M Z, Feng B and Zhang X M 2005 *Phys. Rev.* D **72** 123515.

[10] Caldwell R R, Kamionkowski M and Weinberg N N 2003 *Phys. Rev. Lett.* **91** 071301
McInnes B 2002 *J. High Energy Phys.* **0208** 029
Barrow J D 2004 *Class. Quantum Grav.* **21** L79

[11] Bi X J, Feng B, Li H and Zhang X 2005 *Phys. Rev.* D **72** 123523
Gu P H, Bi X J, Feng B, Young B L and Zhang X M 2008 *Chin. Phys.* C **32** 530
Li H, Feng B, Xia J Q and Zhang X M 2006 *Phys. Rev.* D **73** 103503
Wu S Q and Cai X 2002 *Chin. Phys.* **11** 661

[12] Kremer G M 2007 *Gen. Rel. Grav.* **39** 965
Lü J B, Xu L X, Liu M L and Gui Y X 2009 *Chin. Phys.* B **18** 1711

[13] Brookfield A W, van de Bruck C, Mota D F and Tocchini V D 2006 *Phys. Rev. Lett.* **96** 061301
Brookfield A W, van de Bruck C, Mota D F and Tocchini V D 2006 *Phys. Rev.* D **73** 083515

[14] Guo Z K, Cai R G and Zhang Y Z 2005 *J. Cosmol. Astropart. Phys.* **0505** 002
Guo Z K, Piao Y S, Zhang X M and Zhang Y Z 2005 *Phys. Lett.* B **608** 177
Guo Z K and Zhang Y Z 2005 *Phys. Rev.* D **71** 023501
He J H, Wang B and Jing Y P 2009 *JCAP* **0907** 030
Wang B, Zang J D, Lin E, Abdalla C Y and Micheletti S 2007 *Nucl. Phys.* B **778** 69
Wu P X and Zhang S N 2008 *JCAP* **06** 007
Fu X Y, Yu H W and Wu P X 2008 *Phys. Rev.* D **78** 063001
Chen S B and Jing J L 2009 *Class. Quant. Grav.* **26** 155006
Bi X J, Feng B, Li H and Zhang X M 2005 *Phys. Rev.* D **72** 123523
Chen X M, Gong Y G and Saridakis E N 2009 *JCAP* 04 001
Shu W X, Yu H W and Wu P X 2004 *Chin. Phys.* **13** 2025
Shu W X, Yu H W, Li F, Wu P X and Ren Z Z 2006 *Chin. Phys.* **15** 934
Deng Q and Yan J 2008 *Acta Phys. Sin.* **57** 3978 (in Chinese)
Liu L and Pei S Y 2006 *Acta Phys. Sin.* **55** 4980 (in Chinese)

[15] Wetterich C 1988 *Nucl. Phys.* B **302** 645
Wetterich C 1995 *Astron. Astrophys.* **301** 321

[16] Frieman J, Turner M and Huterer D 2008 *Ann. Rev. Astron. Astrophys.* **46** 385
Linder E V 2003 *Phys. Rev. Lett.* **90** 091301