Quantum entanglement and control in a capacitively coupled charge qubit circuit*

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The macroscopic quantum entanglement in capacitively coupled SQUID (superconducting quantum interference device)-based charge qubits is investigated theoretically. The entanglement characteristic is discussed by employing the quantum Rabi oscillations and the concurrence. An interesting conclusion is obtained, i.e., the magnetic fluxes Φ_{x1} and Φ_{x2} through the superconducting loops can adjust the entanglement degree between the qubits.

Keywords: magnetic flux, Josephson junction, charge qubit, entanglement

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1. Introduction

In order to realize a practical quantum computer, it is necessary to search for suitable quantum systems, which can be easily initialized, controlled, read out, coupled to their neighbours, and decoupled from the environment, to serve as qubits.[1,2] Experimentally, nuclear magnetic resonance in molecules,[3] trapped ions,[4] and quantum optical systems[5] have been investigated for embodying quantum computation. These systems have the advantages of high quantum coherence, but cannot be integrated easily to form large-scale circuits. With the progress of nano-technology and lithography, solid-state units such as quantum dots and Josephson tunnel junctions have become attractive candidates for realizing qubits. Compared with the above schemes, we see that though the solid-state units have the following disadvantages: noise, short decoherence time and manufacturing variablity,[6,7] their advantage of flexibility is encouraging because the designer can determine the layout of the circuit including Josephson junctions or quantum dots, and its parameters can be adjusted continuously over a wide range.[1] In recent years, it has become a hotspot to study mesoscopic circuits including Josephson junctions.[8] Great progress in extensive experimental and theoretical work has been made.[2,9−12] It is believed that with further investigation the advantages can compensate for the disadvantages.

The simplest model of the Josephson junction qubit was proposed earlier,[13,14] and employed an isolated superconducting island to construct qubits. Subsequently, Makhlin *et al.* put forward an improved ideal design that employed a superconducting quantum interference device (SQUID).[2] This design avoided an undesired phase evolution effectively by tuning the energy splitting between logical states during idle periods between operations. The qubits can be manipulated by controlling the external magnetic flux through the superconducting loop. Such qubits are called charge qubits. By coupling two or multiple qubits together, one can perform a logic operation or prepare quantum entangled states.[15,16]

In this work, we couple capacitively two SQUID-based charge qubits (see Fig. 1). We try to investigate whether entangled states can be prepared using this system and whether the entanglement is influenced by externally controlled magnetic fluxes. The structure of this paper is as follows: First, we give the quantization scheme for the SQUID-based charge qubits. Next, we deduce the eigenvalues and eigenstates. Then, we investigate the quantum Rabi oscillations in the system. Finally, we discuss how the entanglement is influenced by the external magnetic flux.

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2. Quantization for the SQUID-based charge qubit circuit

Consider the capacitively coupled SQUID-based charge qubit circuit (see Fig. 1). It consists of three different elementary sections: two SQUID-based charge qubit structures and a coupling capacitor $C_{\rm c}$. For each SQUID-based charge qubit structure, a superconducting island is coupled by two uniform Josephson junctions with capacitance $C_{{\rm j}l}$ and Josephson coupling energy E_l^0 $(l=1,2)$ to a segment of a superconducting loop and through a gate capacitor $C_{{\rm g}l}$ to a gate voltage source $V_{{\rm g}l}$ $(l=1,2)$.

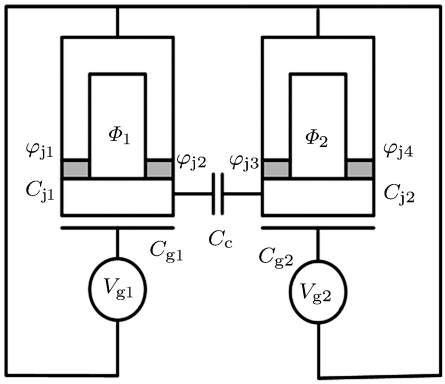


Fig. 1. The capacitively coupled SQUID-based charge qubit circuit.

Next, we analyse this circuit from the point of view of classical Hamilton dynamics. A single Josephson junction is characterized by two Josephson equations (the Josephson current equation and voltage equation)[17]

$$I_{\rm j}=I_{\rm c}\sin\varphi_{\rm j},\quad \dot{\varphi}_{\rm j}=\frac{2eu_{\rm j}}{\hbar}, \tag{1}$$

where the subscript j represents the Josephson junction, $I_{\rm c}=2eE_{\rm j}^0/\hbar$ is the critical current of the junction, $u_{\rm j}$ is the voltage drop across the Josephson junction and $\varphi_{\rm j}$ is the phase difference across the two superconductors of a junction. For the lth $(l=1,2)$ SQUID loop, the following relation can be deduced

$$\varphi_{l2}-\varphi_{l1}=\frac{2e}{\hbar}\oint\mathcal{A}_l\cdot{\rm d}\boldsymbol{s}=\frac{2e}{\hbar}\Phi_l, \tag{2}$$

where φ_{lk} $(k=1,2)$ is the phase difference across the kth junction, and $\mathcal{A}_l$ is the vector potential induced by the magnetic flux Φ_l through the lth $(l=1,2)$ SQUID loop. Consider the mean phase $\varphi_l=(\varphi_{l1}+\varphi_{l2})/2$ as the generalized coordinates; the potential energy of the system is

$$\mathcal{V}=\sum_{l=1}^{2}2E_l^0\left[1-\cos\left(\frac{e}{\hbar}\Phi_l\right)\cos\varphi_l\right], \tag{3}$$

where $E_l^0=I_{cl}\hbar/(2e)$. The kinetic energy related to $\dot{\varphi}_l$ is

$$\mathcal{T}=\sum_{l=1}^{2}\left[\frac{1}{2}C_{\Sigma{\rm j}l}\left(\frac{\hbar}{2e}\right)^2\dot{\varphi}_l^2+N_{{\rm g}l}\hbar\dot{\varphi}_l+\frac{2N_{{\rm g}l}^2e^2}{C_{{\rm g}l}}\right]-C_{\rm c}\left(\frac{\hbar}{2e}\right)^2\dot{\varphi}_1\dot{\varphi}_2, \tag{4}$$

where $C_{\Sigma{\rm j}l}=2C_{{\rm j}l}+C_{{\rm g}l}+C_{\rm c}$ and $N_{gl}=C_{gl}V_{gl}/(2e)$ which is referred to as the gate charge number. Then the Lagrangian of the system is

$$\mathcal{L}=\mathcal{T}-\mathcal{V}. \tag{5}$$

Through each Josephson junction Cooper-pairs can tunnel from or onto the island, and the number n_l of excess Cooper-pairs on the island satisfies the following relations

$$2n_1e=\frac{\hbar}{2e}C_{\Sigma{\rm j}1}\dot{\varphi}_1+2N_{\rm g1}e-\frac{\hbar}{2e}C_{\rm c}\dot{\varphi}_2. \tag{6}$$

$$2n_2e=\frac{\hbar}{2e}C_{\Sigma{\rm j}2}\dot{\varphi}_2+2N_{\rm g2}e-\frac{\hbar}{2e}C_{\rm c}\dot{\varphi}_1. \tag{7}$$

Thus, we obtain the generalized momenta, respectively, conjugate to φ_1 and φ_2

$$p_1\equiv\frac{\partial\mathcal{L}}{\partial\dot{\varphi}_1}=\frac{\hbar}{2e}\left[\frac{\hbar}{2e}C_{\Sigma{\rm j}1}\dot{\varphi}_1+2N_{\rm g1}e-\frac{\hbar}{2e}C_{\rm c}\dot{\varphi}_2\right]=n_1\hbar, \tag{8}$$

$$p_2\equiv\frac{\partial\mathcal{L}}{\partial\dot{\varphi}_2}=\frac{\hbar}{2e}\left[\frac{\hbar}{2e}C_{\Sigma{\rm j}2}\dot{\varphi}_2+2N_{\rm g2}e-\frac{\hbar}{2e}C_{\rm c}\dot{\varphi}_1\right]=n_2\hbar. \tag{9}$$

Note that the generalized momentum p_l is proportional to n_l. From Eqs. (5)–(9) we obtain the classical Hamiltonian of the system

$$\mathcal{H}=\sum_{l=1}^{2}p_l\dot{\varphi}_l-\mathcal{L}=\sum_{l=1}^{2}\mathcal{H}_l+\mathcal{H}_{\rm int}, \tag{10}$$

where $\mathcal{H}_l$ is the equivalent Hamiltonian for the lth $(l=1,2)$ SQUID-based charge qubit structure and $\mathcal{H}_{\rm int}$ is the interacting term, and

$$\mathcal{H}_l=4E_{cl}(n_l-N_{\rm gl})^2+2E_l^0\left[1-\cos\left(\frac{e}{\hbar}\Phi_l\right)\cos\varphi_l\right]-\frac{2N_{{\rm g}l}^2e^2}{C_{{\rm g}l}}, \tag{11}$$

$$\mathcal{H}_{\rm int} = E_{\rm c,c}(n_1 - N_{\rm g1})(n_2 - N_{g2}), \tag{12}$$

and

$$\begin{aligned} E_{\rm c1} &= \frac{e^2 C_{\Sigma {\rm j}2}}{2(C_{\Sigma {\rm j}1}C_{\Sigma {\rm j}2} - C_{\rm c}^2)}, \\ E_{\rm c2} &= \frac{e^2 C_{\Sigma {\rm j}1}}{2(C_{\Sigma {\rm j}1}C_{\Sigma {\rm j}2} - C_{\rm c}^2)}, \\ E_{\rm c,c} &= \frac{4e^2 C_{\rm c}}{(C_{\Sigma {\rm j}1}C_{\Sigma {\rm j}2} - C_{\rm c}^2)}. \end{aligned} \tag{13}$$

Here, E_{cl} is the single-electron charging energy on the lth ($l = 1, 2$) island. According to the standard canonical quantization principle,[18] the canonical conjugate quantities n_l and φ_l are, respectively, associated with the Hermitian operators $\hat{n}_l$ and $\hat{\varphi}_l$, which are assigned to the following commutation relations

$$[\hat{\varphi}_l, \hat{n}_l] = i. \tag{14}$$

So the Hamilton operator of the system can be written as

$$\hat{\mathcal{H}} = \sum_{l=1}^{2} \hat{\mathcal{H}}_l + \hat{\mathcal{H}}_{\rm int}. \tag{15}$$

Generally, for the lth ($l = 1, 2$) SQUID charge qubit structure, self-inductance needs to be considered and the circulating current in the SQUID loop satisfies the following relation

$$\hat{I}_l = 2I_{\rm cl} \cos\hat{\varphi}_l \sin\left(\frac{e}{\hbar}\Phi_{lx} + \frac{e}{\hbar}L_l\hat{I}_l\right), \tag{16}$$

where Φ_{xl} is the external magnetic flux, and L_l is the self-inductance coefficient for the lth SQUID loop. As $\hat{I}_l$ is a continuous function of $\cos\hat{\varphi}_l$, it can be expanded into Taylor series

$$\hat{I}_l = 2I_{\rm cl} \cos\hat{\varphi}_l \sum_{m=1}^{\infty} a_{lm} \cos^{m-1}\hat{\varphi}_l. \tag{17}$$

Also,

$$\begin{aligned} \hat{I}_l &= 2I_{cl}\cos\hat{\varphi}_l \sin\left(\frac{e}{\hbar}\Phi_{xl} + \frac{e}{\hbar}L_l\hat{I}_l\right) \\ &= 2I_{cl}\cos\hat{\varphi}_l \left\{\sin\left(\frac{e}{\hbar}\Phi_{xl}\right)\left[1 - \frac{1}{2!}\left(\frac{e}{\hbar}L_l\hat{I}_l\right)^2 + \frac{1}{4!}(\frac{e}{\hbar}L_l\hat{I}_l)^4 - \dots \right.\right. \\ &\quad + (-1)^m \frac{1}{(2n)!}\left(\frac{e}{\hbar}L_l\hat{I}_l\right)^{2m}\Big] + \cos\left(\frac{e}{\hbar}\Phi_{xl}\right)\left[\frac{e}{\hbar}L_l\hat{I}_l - \frac{1}{3!}\left(\frac{e}{\hbar}L_l\hat{I}_l\right)^3\right. \\ &\quad \left.\left. + \frac{1}{5!}\left(\frac{e}{\hbar}L_l\hat{I}_l\right)^5 - \dots + (-1)^{m-1}\frac{1}{(2n-1)!}\left(\frac{e}{\hbar}L_l\hat{I}_l\right)^{2m-1}\right]\right\}. \end{aligned} \tag{18}$$

Substituting Eq. (17) into Eq. (18) and comparing the coefficient of $\cos^m\varphi_l$ on both sides lead to

$$\begin{aligned} a_{l1} &= \sin\left(\frac{e}{\hbar}\Phi_{lx}\right), \quad a_{l2} = \frac{1}{2}\left(\frac{2I_{\rm cl}eL_l}{\hbar}\right)\sin\left(\frac{2e}{\hbar}\Phi_{x_l}\right), \\ a_{l3} &= \left(\frac{2I_{\rm cl}eL_l}{\hbar}\right)^2 \sin\left(\frac{e}{\hbar}\Phi_{xl}\right)\left[1 - \frac{3}{2}\sin^2\left(\frac{e}{\hbar}\Phi_{xl}\right)\right], \\ a_{l4} &= \frac{1}{2}\left(\frac{2I_{\rm cl}eL_l}{\hbar}\right)^3 \sin\left(\frac{2e}{\hbar}\Phi_{xl}\right)\left[1 - \frac{8}{3}\sin^2\left(\frac{e}{\hbar}\Phi_{xl}\right)\right]. \\ &\dots . \end{aligned} \tag{19}$$

For

$$E_l(\hat{\Phi}_l) = 2E_l^0 \cos\left(\frac{e}{\hbar}\hat{\Phi}_l\right) = 2E_l^0 \cos\left(\frac{e}{\hbar}\Phi_{xl} + \frac{e}{\hbar}L_l I_l\right)$$

can also be expanded in a Taylor series about I_l, and $E_l(\hat{\Phi}_l) = 2E_l^0 \sum\limits_{m=1}^{\infty} b_{lm}\cos^m\varphi_l$, similarly, we can obtain

$$E_l(\hat{\Phi}_l) = 2E_l^0(b_{l0} + b_{l1}\cos\hat{\varphi}_l + b_{l2}\cos^2\hat{\varphi}_l + b_{l3}\cos^3\hat{\varphi}_l + \dots), \tag{20}$$

where

$$b_{l0}=\cos\left(\frac{e}{\hbar}\Phi_{xl}\right),\quad b_{l1}=-\left(\frac{2I_{cl}eL_l}{\hbar}\right)\sin^2\left(\frac{e}{\hbar}\Phi_{xl}\right),$$
$$b_{l2}=-\frac{3}{2}\left(\frac{2I_{cl}eL_l}{\hbar}\right)^2\cos\left(\frac{e}{\hbar}\Phi_{xl}\right)\sin^2\left(\frac{e}{\hbar}\Phi_{xl}\right),$$
$$b_{l3}=-2\left(\frac{2I_{cl}eL_l}{\hbar}\right)^3\sin^2\left(\frac{e}{\hbar}\Phi_{xl}\right)\left[1-\frac{4}{3}\sin^2\left(\frac{e}{\hbar}\Phi_{xl}\right)\right].$$
$$\cdots. \tag{21}$$

Therefore, we can rewrite the Hamiltonian operator $\hat{\mathcal{H}}_l$

$$\hat{\mathcal{H}}_l=4E_{cl}(\hat{n}_l-N_{gl})^2-2E_l^0\sum_{m=0}^{3}(b_{lm}\cos^{m+1}\hat{\varphi}_l-1)-\frac{2N_{gl}^2e^2}{C_{gl}}, \tag{22}$$

where $E_l(\hat{\Phi}_l)=2E_l^0\sum\limits_{m=0}^{3}b_{lm}\cos^m\hat{\varphi}_l$, which is generally a good approximation since typically $2I_{cl}eL_l/\hbar\sim 10^{-3}$.[19]

3. Eigenstates and entanglement of the system

As the system is two coupled SQUID-based charge qubits, it is significant to discuss it by employing the two-state approximation. For this purpose, we choose such a material that the superconducting energy gap Δ is the largest energy in the problem, even larger than the single electron charging energy on the island. In this case quasiparticle tunneling is suppressed at low temperature, and a situation can be reached in which no quasiparticle excitation is found on the island.[9] Moreover, we consider the system in which the single-electron charging energy is much larger than the Josephson coupling energy, i.e., $E_{cl}\gg E_l^0$. In this regime, a convenient basis is formed by charge states, parameterized by the excess number of Cooper-pairs on the island:[9,10,13] $\hat{n}_l\left|n\right\rangle_l=n_l\left|n\right\rangle_l$, where n_l corresponds to the number of excess Cooper-pairs on the lth island. In addition, at the temperature such that $k_BT\ll E_{cl}$,[13] we concentrate on such a voltage (i.e. $N_{gl}=1/2$) near a degeneracy point where only two charge states, say $\left|0\right\rangle_l$ and $\left|1\right\rangle_l$, play an important role, while all other charge states, having much higher energy, can be ignored.[9] Thus, the system can be characterized by the four basis states $\left|+\right\rangle_1\left|+\right\rangle_2$, $\left|+\right\rangle_1\left|-\right\rangle_2$, $\left|-\right\rangle_1\left|+\right\rangle_2$, and $\left|-\right\rangle_1\left|-\right\rangle_2$, where

$$\left|+\right\rangle_1=\frac{\left|0\right\rangle_1+\left|1\right\rangle_1}{\sqrt{2}},\quad \left|+\right\rangle_1=\frac{\left|0\right\rangle_1-\left|1\right\rangle_1}{\sqrt{2}},$$
$$\left|+\right\rangle_2=\frac{\left|0\right\rangle_2+\left|1\right\rangle_2}{\sqrt{2}},\quad \left|+\right\rangle_2=\frac{\left|0\right\rangle_2-\left|1\right\rangle_2}{\sqrt{2}}, \tag{23}$$

which are just the eigenstates of $\hat{\mathcal{H}}_l$ of the lth qubits. Using the above four basis states, the Hamiltonian matrix of the system reads

$$\hat{\mathcal{H}}=\begin{pmatrix}\alpha & 0 & 0 & \frac{E_{c,c}}{2}\\ 0 & \beta & \frac{E_{c,c}}{2} & 0\\ 0 & \frac{E_{c,c}}{2} & \gamma & 0\\ \frac{E_{c,c}}{2} & 0 & 0 & \zeta\end{pmatrix}, \tag{24}$$

where

$$\alpha=\sum_{l=1}^{2}\left[E_{cl}+\left(2E_l^0-\frac{e^2}{2C_{gl}}\right)-\sum_{m=0}^{3}E_l^0\frac{b_{lm}}{2^m}\right], \tag{25}$$

$$\beta=\sum_{l=1}^{2}\left\{E_{cl}+\left(2E_l^0-\frac{e^2}{2C_{gl}}\right)-E_l^0\left[(-1)^{l+1}(b_{l0}+\frac{b_{l2}}{4})+\frac{b_{l1}}{2}+\frac{b_{l3}}{8}\right]\right\}, \tag{26}$$

$$\gamma=\sum_{l=1}^{2}\left\{E_{cl}+(2E_l^0-\frac{e^2}{2C_{gl}})-E_l^0\left[(-1)^l\left(b_{l0}+\frac{b_{l2}}{4}\right)+\frac{b_{l1}}{2}+\frac{b_{l3}}{8}\right]\right\}, \tag{27}$$

$$\zeta=\sum_{l=1}^{2}\left[E_{cl}+(2E_l^0-\frac{e^2}{2C_{gl}})-\sum_{m=0}^{3}E_l^0(-1)^{m+1}\frac{b_{lm}}{2^m}\right]. \tag{28}$$

So from Eq. (24) we can obtain the eigenvalues of the system

$$\varepsilon_{1,2} = \frac{1}{2}\left[\beta+\gamma \mp \sqrt{(\beta-\gamma)^2 + E_{\rm c,c}^2}\right], \tag{29}$$

$$\varepsilon_{3,4} = \frac{1}{2}\left[\alpha+\zeta \mp \sqrt{(\alpha-\zeta)^2 + E_{\rm c,c}^2}\right], \tag{30}$$

and the corresponding normalized eigenstates are, respectively,

$$|\Psi_1\rangle = [|-\rangle_1|+\rangle_2 - \tau_1|+\rangle_1|-\rangle_2]\Big/\sqrt{1+\tau_1^2}, \tag{31}$$

$$|\Psi_2\rangle = [|-\rangle_1|+\rangle_2 - \tau_2|+\rangle_1|-\rangle_2]\Big/\sqrt{1+\tau_2^2}, \tag{32}$$

$$|\Psi_3\rangle = [|-\rangle_1|-\rangle_2 - \tau_3|+\rangle_1|+\rangle_2]\Big/\sqrt{1+\tau_3^2}, \tag{33}$$

$$|\Psi_4\rangle = [|-\rangle_1|-\rangle_2 - \tau_4|+\rangle_1|+\rangle_2]\Big/\sqrt{1+\tau_4^2}, \tag{34}$$

where

$$\tau_{1,2} = \frac{-\beta+\gamma \pm \sqrt{(\beta-\gamma)^2 + E_{\rm c,c}^2}}{E_{\rm c,c}}, \tag{35}$$

$$\tau_{3,4} = \frac{-\alpha+\zeta \pm \sqrt{(\alpha-\zeta)^2 + E_{\rm c,c}^2}}{E_{\rm c,c}}. \tag{36}$$

From Eqs. (31)–(34) we can clearly see that the eigenstates of the system are entangled states and when $\tau_m \to 1$ $(m = 1,2,3,4)$ they will degenerate to the maximally entangled states, i.e. Bell states. So by using such a mesoscopic circuit one can prepare entangled states. It is necessary to investigate the entanglement property of these states. Since Rabi oscillations are characteristic for the entanglement realized in the system, their measurement would provide direct evidence of the presence of entangled states. Next, we will employ Rabi oscillations to study these entangled states.

4. Rabi oscillations

The system evolves according to the following unitary transformation

$$\hat{U}(t) = \exp\left(-\mathrm{i}\,\hat{\mathcal{H}}t/\hbar\right). \tag{37}$$

If initially the system is prepared in the state $|-\rangle_1|-\rangle_2$, its time evolution is

$$\begin{aligned}|\Phi(t)\rangle &= \hat{U}(t)|-\rangle_1|-\rangle_2 \\ &= \frac{1}{\sqrt{1+\tau_3^2}}\exp\left(-\mathrm{i}\,\varepsilon_3 t/\hbar\right)|\Psi_3\rangle \\ &\quad + \frac{1}{\sqrt{1+\tau_4^2}}\exp\left(-\mathrm{i}\,\varepsilon_4 t/\hbar\right)|\Psi_4\rangle. \end{aligned} \tag{38}$$

Equations (38) implies that with the time evolution the state $|\Phi(t)\rangle$ oscillates coherently between $|-\rangle_1|-\rangle_2$ and $|+\rangle_1|+\rangle_2$, which is regarded as quantum Rabi oscillations. After a certain time t, the probability distribution $P_1(t)$ for finding the first qubit in the state $|-\rangle_1$ is

$$\begin{aligned}P_1(t) &= |{}_1\langle -|{}_2\langle -|\,\Phi_2(\tau)\rangle|^2 \\ &= \left(\frac{1}{1+\tau_3^2}\right)^2 + \left(\frac{1}{1+\tau_4^2}\right)^2 \\ &\quad + \frac{2}{(1+\tau_3^2)(1+\tau_4^2)}\cos\left[\frac{(\varepsilon_3-\varepsilon_4)t}{\hbar}\right]. \end{aligned} \tag{39}$$

In order to evaluate $P_1(t)$ numerically, we choose the following mesoscopic parameters: $C_{\rm j1} = C_{\rm j2} = 6\times10^{-16}$ F, $C_{\rm g1} = C_{\rm g2} = 6\times10^{-17}$ F, $C_{\rm c1} = C_{\rm c2} = 6\times10^{-17}$ F, $C_{\rm c} = 3\times10^{-15}$ F, $L_1 = L_2 = 0$ nH and $E_1^0 = E_2^0 = 0.1$ K, which can be easily fabricated routinely by present-day technologies. Using the above parameters we plot $P_1(t)$ as a function of time, for different values of Φ_{x1} and Φ_{x2} (see Figs. 2(a)–2(c)).

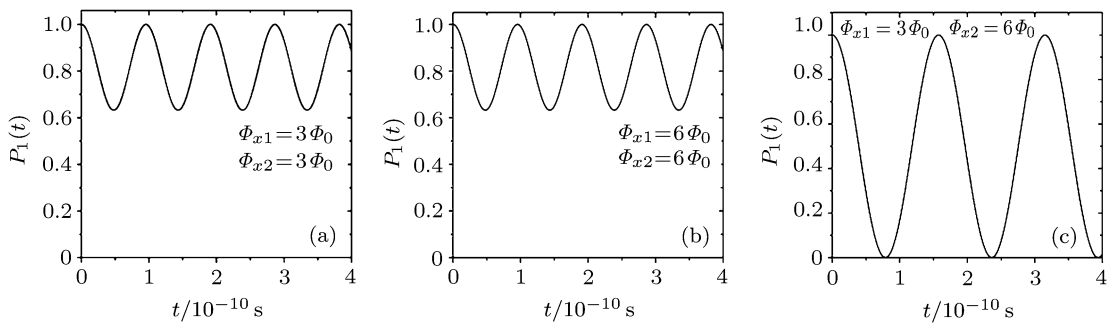


Fig. 2. $P_1(t)$ as a function of time t for different Φ_{x1} and Φ_{x2}.

Because the self-inductance of the two superconducting loops is very small,[20] the magnetic fluxes through the two loops are, respectively, approximately equal to the classical magnetic fluxes Φ_{x1} and Φ_{x2}. In addition, it is well known that the magnetic fluxes through the superconducting loop must be integral times magnetic fluxon Φ_0 ($\Phi_0 = 2.0678 \times 10^{-15}$ Wb).[17] Based on the above two conditions, from Fig. 2 we see some interesting phenomena, i.e., when the magnetic fluxes through both loops are all even or odd times Φ_0, we can clearly see Rabi oscillations with periodicity $T_{\rm Rabi} \approx 96$ ps (see Figs. 2(a) and 2(b)) and the amplitude peak–peak value $A_{\rm P-P} < 0.4$. But when the magnetic fluxes through one loop are odd times Φ_0 and those through the other loop are even times Φ_0, the periodicity ($T_{\rm Rabi} \approx 157$ ps) of Rabi oscillations and the amplitude peak–peak value ($A_{\rm P-P} = 1$) become large (see Fig. 2(c))). Such phenomena imply that the entanglement between $|-\rangle_1 |-\rangle_2$ and $|+\rangle_1 |+\rangle_2$ is much stronger when the magnetic fluxes through one loop are odd times and those through the other loop are even times Φ_0 than when the magnetic fluxes through both loops are all even or all odd times Φ_0. Thus we can speculate that the magnetic fluxes can control the entanglement between the qubits. Next, we validate such a speculation.

One measure of entanglement is the concurrence[21] $\mathcal{C}$ for an arbitrary state of coupled two qubits. The concurrence ranges from 0 for a disentangled to 1 for a maximally entangled state. For a normalized pure state $|\Psi\rangle = a|11\rangle + b|01\rangle + c|10\rangle + d|00\rangle$, the concurrence is given by[22]

$$\mathcal{C}(|\Psi\rangle) = 2|ad - bc|. \tag{40}$$

So from Eqs. (31)–(34) we obtain

$$\mathcal{C}_1(|\Psi_1\rangle) = \frac{2|\tau_1|}{1+\tau_1^2}, \quad \mathcal{C}_2(|\Psi_2\rangle) = \frac{2|\tau_2|}{1+\tau_2^2},$$
$$\mathcal{C}_3(|\Psi_3\rangle) = \frac{2|\tau_3|}{1+\tau_3^2}, \quad \mathcal{C}_4(|\Psi_4\rangle) = \frac{2|\tau_4|}{1+\tau_4^2}. \tag{41}$$

In order to evaluate the concurrence in Eq. (41) numerically we choose the same mesoscopic parameters as the above. Figure 3 shows the concurrences $\mathcal{C}_3$ and $\mathcal{C}_4$ as functions of Φ_{x1} and Φ_{x2}. The bright tint means the value of the concurrence is large and the dark, small. From Fig. 3, one can see clearly that the contours exhibit a regular distribution, i.e., when the magnetic fluxes through one loop are odd times Φ_0 and those through the other loop are even times Φ_0, the concurrence takes the larger value, implying that the state $|-\rangle_1 |-\rangle_2$ is entangled with the state $|+\rangle_1 |+\rangle_2$ perfectly. But the concurrence will take a small value when the magnetic fluxes through both superconducting loops are all even or all odd times Φ_0, indicating that entanglement becomes feeble. Such phenomena just validate the above speculation well.

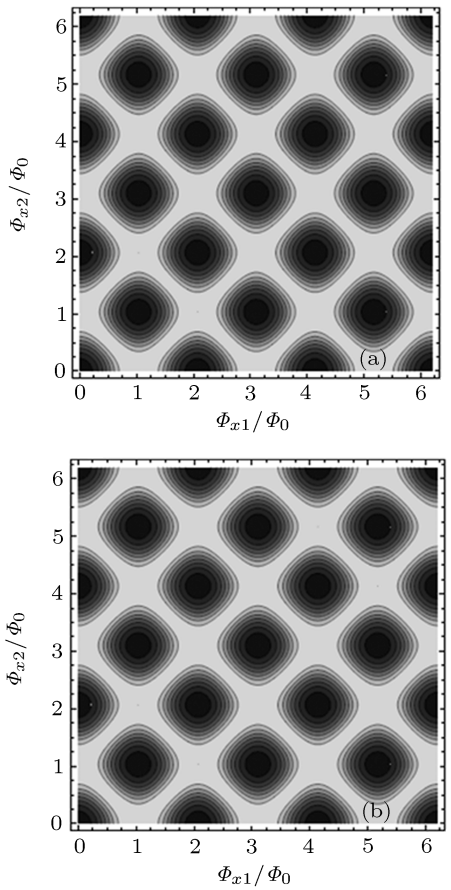


Fig. 3. Contours of $\mathcal{C}_3$ and $\mathcal{C}_4$ as functions of Φ_{x1} and Φ_{x2}: (a) corresponding to $\mathcal{C}_3$; (b) corresponding to $\mathcal{C}_4$.

Then how about the entanglement between $|-\rangle_1 |+\rangle_2$ and $|+\rangle_1 |-\rangle_2$? Similar to the above, we discuss the entanglement between $|-\rangle_1 |+\rangle_2$ and $|+\rangle_1 |-\rangle_2$, and by analysing Figs. 4 and 5 we can draw the following conclusions: Variation of the characteristic of entanglement between $|-\rangle_1 |+\rangle_2$ and $|+\rangle_1 |-\rangle_2$ with Φ_{x1} and Φ_{x2} is quite contrary to that between $|-\rangle_1 |-\rangle_2$ and $|+\rangle_1 |+\rangle_2$ with Φ_{x1} and Φ_{x2}; i.e., when the magnetic fluxes through both superconducting loops are all even or all odd times Φ_0, the state $|-\rangle_1 |+\rangle_2$ is entangled with the state $|+\rangle_1 |-\rangle_2$ perfectly. But when the magnetic fluxes through one loop are odd times Φ_0 and those through the other loop are even times Φ_0, the entanglement becomes feeble. Thus from the above it is found that the magnetic fluxes can be used to adjust the entanglement degree between qubits, which may provide some reference and even basis for the quantum control of multiple qubits.

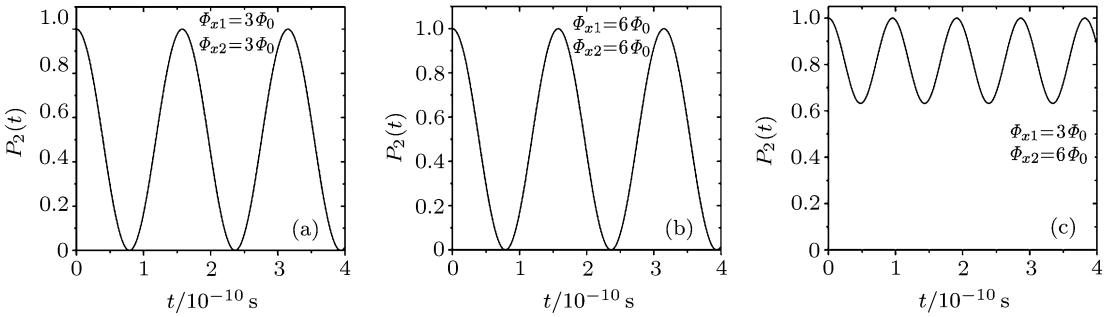


Fig. 4. $P_2(t)$ as a function of time t.

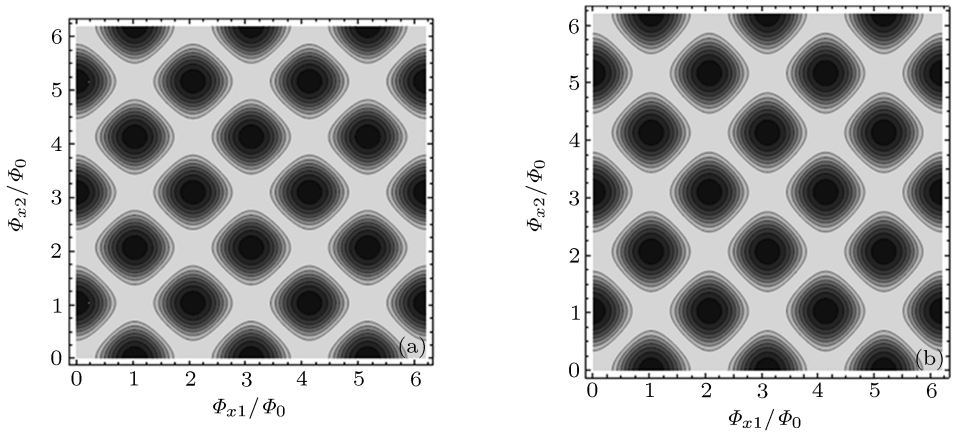


Fig. 5. Contours of $\mathcal{C}_1$ and $\mathcal{C}_2$ as functions of Φ_{x1} and Φ_{x2}. (a) corresponding to $\mathcal{C}_1$, (b) corresponding to $\mathcal{C}_2$.

5. Conclusions

In summary, in the quantization scheme of SQUID-based charge qubit circuits, the self-inductance role was considered. We obtained the eigenvalues and eigenstates which are entangled states. By virtue of numerical calculation, we investigated the characteristic of entanglement between qubits. We have found that the magnetic fluxes Φ_{x1} and Φ_{x2} can control the entanglement between the qubits. We believe that such a conclusion will provide some reference and even basis for the quantum control of multiple qubits.

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