The decoherence of the triangular and Coulomb bound potential quantum dot qubit*

Li Hong-Juan(李红娟), Sun Jia-Kui(孙家奎)†, and Xiao Jing-Lin(肖景林)‡

College of Physics and Electromechanics, Inner Mongolia National University, Tongliao 028043, *China*

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We study the eigenenergies and eigenfunctions of the ground and first-excited states of an electron which is strongly coupled to an LO-phonon in a quantum dot with a triangular bound potential and Coulomb bound potential by using the Pekar variational method. This system may be used as a two-level qubit. Phonon spontaneous emission causes the decoherence of the qubit. Numerical calculations are performed on the decoherence rate as a function of the polar angle, the Coulomb binding parameter, the coupling strength, the confinement length of the quantum dot and the dispersion coefficient.

Keywords: quantum dot, triangular bound potential, Coulomb bound potential, decoherence

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1. Introduction

Recently, quantum computing has attracted considerable attention in the information science field and thus considerable experimental[1−3] and theoretical[4−7] works have been devoted to this subject. In those works, the semiconductor quantum dot (QD) has been extensively studied because of its novel physical properties and the potential applications in optoelectronics.[8] The two-level QD system can be employed as a single qubit. For such a qubit, Li *et al.*[9,10] presented a novel parameter phase diagram and defined the usage range of a single QD qubit. Ezaki, Mori and Hamaguchi[11] investigated the electronic structures in a triangular QD. In practice, it is required that a qubit be well isolated away from the external environment. However, quantum systems are very frail and the interaction of a quantum memory with its environment destroys the quantum coherence of the stored information, a process called decoherence. Therefore, quantum decoherence plays a very important role in the formalism of quantum computing. So a great deal of considerable effort[12−19] has been made to investigate the quantum decoherence and to prolong the decoherence rate.

We obtain the eigenenergies of the ground state and the first-excited state, and the eigenfuctions of the ground state and the first-excited state, by using a variational method of Pekar type on the condition of electron–LO-phonon strong coupling in a triangular quantum dot with a hydrogenized impurity in the centre of the quantum dot. This system in a quantum dot may be employed as a two-level quantum system qubit. Phonon spontaneous emission causes the decoherence of the qubit. We discuss the relations between the decoherence rate and the polar angle, the Coulomb binding parameter, the coupling strength, the confinement length of the QD and the dispersion coefficient in this paper.

2. Theoretical model

The confining potential of the electrons in the z direction is much stronger than that in x and y directions, and then the electrons are assumed to be moving in the $x-y$ plane. The confining potential in the QD is taken as the triangular bound potential form[11]

$$V_{\rho,\theta} = m^*\omega_0^2\rho^2\left(1+\frac{2}{7}\cos(3\theta)\right)\Big/2, \qquad (1)$$

where m^* is the band mass of the electron, ρ is the two-dimensional coordinate vector, ω_0 is the confinement strength and θ is the polar angle in the polar coordinate system. The Hamiltonian of an electron–phonon system with a hydrogenized impurity in the

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†E-mail: cfsjk1980@yahoo.cn

‡Corresponding author. E-mail: xiaojlin1939@162.com

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centre of the quantum dot is given by

$$H = p^2/2m^* + m^*\omega_0^2\rho^2\left(1+\frac{2}{7}\cos(3\theta)\right)\Big/2 + \sum_q \hbar\omega_{\rm LO} b_q^+ b_q + \sum_q (V_q e^{{\rm i}\,q\cdot r} b_q + {\rm h.c}) - e^2/\varepsilon_\infty r, \quad (2)$$

where $b_q^+(b_q)$ is the creation (annihilation) operator of the bulk LO-phonon with wave vector $\boldsymbol{q}(q = q_\parallel, q_\perp)$ and frequency $\omega_{\rm LO}$, $\boldsymbol{r} = (\rho, z)$ is the coordinate of the electron. The $-e^2/\varepsilon_\infty r$ denotes the interaction between the electron and the hydrogenized impurity.

To simplify our calculations, we choose the units $\hbar = 2m^* = \omega_{\rm LO} = 1$ and $\beta = e^2/\varepsilon_\infty$. Then the Hamiltonian of the system can be rewritten as

$$H = -\nabla_\rho^2 + \omega_0^2\rho^2\left(1+\frac{2}{7}\cos(3\theta)\right)\Big/4 + \sum_q \omega_{\rm LO} b_q^+ b_q + \sum_q (V_q e^{{\rm i}\,q\cdot r} b_q + {\rm h.c}) - \beta/r, \quad (3)$$

where

$$V_q = {\rm i}(8\pi\alpha/V)^{1/2}/q, \quad (4)$$

$$\alpha = (e^4/16)^{1/2}(1/\varepsilon_\infty - 1/\varepsilon_0), \quad (5)$$

where α is the electron–LO coupling constant, β is the Coulomb binding parameter.

Using the Fourier expansion to Eq. (3),

$$-\beta/r = -\sum_q 4\pi\beta/Vq^2\exp({\rm i}\,\boldsymbol{q}\cdot\boldsymbol{r}). \quad (6)$$

By introducing the LLP transformation to Eq. (3)

$$U = \exp\left[\sum_q (f_q b_q^+ - f_q^* b_q)\right], \quad (7)$$

where f_q will be treated as a variational function, we have

$$H' = U^{-1}HU. \quad (8)$$

Supposing that the Gaussian function approximation is still valid in the ground state and the first-excited state wave functions of the electron–phonon system are chosen as follows by a variational method of Pekar type

$$|\varphi_0\rangle = |0\rangle|0_{\rm ph}\rangle = \lambda_0\exp\left(-\lambda_0^2\rho^2/2\right)|0_{\rm ph}\rangle/\sqrt{\pi}, \quad (9)$$

$$|\varphi_1\rangle = |1\rangle|0_{\rm ph}\rangle = \lambda_1^2\rho\exp\left(-\lambda_1^2\rho^2/2\right)\exp^{\pm{\rm i}\phi}|0_{\rm ph}\rangle/\sqrt{\pi}, \quad (10)$$

where λ_0, λ_1 are the variational parameters, and $|0_{\rm ph}\rangle$ is the unperturbed zero phonon state which satisfies $b_q|0_q\rangle = 0$. The $|0\rangle$ and $|1\rangle$ are the ground and the first-excited state wave functions, respectively. They satisfy the following relations

$$\langle 0|0\rangle = 1, \quad \langle 1|1\rangle = 1, \quad \langle 1|0\rangle = 0. \quad (11)$$

We then obtain the ground and the first-excited state energies in the following form

$$E_0 = \langle\phi_0|H'|\phi_0\rangle, \quad E_1 = \langle\varphi_1|H'|\varphi_1\rangle. \quad (12)$$

We can obtain λ_0 and λ_1 using the variational method and thus get the eigenenergies and the eigenfuctions. It is easy to find that the energy difference between the ground state and the first-excited state in various directions is

$$\Delta E(\theta) = E_1(\theta) - E_0(\theta) = \lambda_0^2 - 2\lambda_1^2 + \left(1+\frac{2}{7}\cos(3\theta)\right)\Big/\lambda_0^2 l_0^4 - \left(2+\frac{4}{7}\cos(3\theta)\right)\Big/\lambda_1^2 l_0^4 + \sqrt{2\pi\omega_{\rm LO}}(11\lambda_1/32 - \lambda_0/2) + \beta\sqrt{\pi}\lambda_0 - \beta\sqrt{\pi}\lambda_1/2, \quad (13)$$

where $l_0 = \sqrt{2/\omega_0}$.

The superposition state of the electron can be expressed as

$$|\psi_{01}\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, \quad (14)$$

where

$$|0\rangle = \lambda_0\exp\left(-\lambda_0^2\rho^2/2\right)/\sqrt{\pi}, \quad (15)$$

$$|1\rangle = \lambda_1^2\rho\exp\left(-\lambda_1^2\rho^2/2\right)\exp^{\pm{\rm i}\phi}/\sqrt{\pi}. \quad (16)$$

On the condition of low temperature, we introduce the phonon dispersion $\omega = \omega_{\rm LO} - \eta c q^2$. Under the dipole approximation based on the Fermi Golden Rule, the spontaneous emission rate can be written in the following form

$$T^{-1} = (e^2\Delta E/2\pi c\varepsilon_0\hbar^2\eta)\,|\langle 0\,|\rho|\,1\rangle|^2, \quad (17)$$

where c is the speed of light in vacuum, ε_0 is the material (vacuum) dielectric constant, η is the dispersion coefficient, T^{-1} is the decoherence rate and ΔE is the energy level spacing between the states $|0\rangle$ and $|1\rangle$.

3. Results and discussion

The numerical results of the decoherence rate versus the polar angle, the electron–LO-phonon coupling constant, the confinement length of the QD and the coefficient of phonon dispersion in a triangular QD are presented in Figs. 1–3.

Figure 1 presents the decoherence rate as a function of the polar angle θ and the Coulomb binding parameter β for the confinement length of the QD $l_0 = 0.2$, the electron–LO-phonon coupling constant $\alpha = 7.0$ and the coefficient of phonon dispersion $\eta = 1.0$. It shows that the decoherence rate varies periodically with the changing of the polar angle, and the effects of the polar angle on the decoherence rate are cardinal when the Coulomb binding parameter increases. This phenomenon is driven to the triangular potential. It can also be seen that the decoherence rate increases with increasing Coulomb binding parameter. This is because the effect of the Coulomb potential in the first-excited state is weaker than in the ground state while the binding parameter is increasing. So the increase in energy spacing between the first-excited and the ground states causes an increase in the decoherence rate. But the influence of the Coulomb potential on the decoherence rate is weaker than that of the bound potential with increasing binding parameter. The bound potential usually exists in real crystals and it is derived from the existence of the hydrogenized impurity. So we should try our best to choose a quantum dot material with fewer impurities, in order to reduce the decoherence rate.

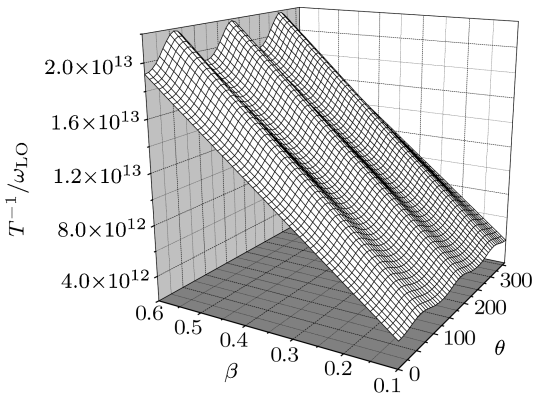


Fig. 1. The decoherence rate T^{-1} changes with the polar angle θ and the Coulomb binding parameter β.

The strengths of the electron–LO-phonon coupling constant and the bound potential are different in different material quantum dots. So the choice of material and bound potential for making a qubit using a quantum dot has to be appropriate.

Figure 2 presents the decoherence rate as a function of the Coulomb binding parameter β and the electron–LO-phonon coupling constant α for the confinement length of the QD $l_0 = 0.2$, the coefficient of phonon dispersion $\eta = 1.0$ and the polar angle $\theta = \pi$. It shows that the decoherence rate increases with increasing Coulomb binding parameter for the same reason as in Fig. 1. It is also shown that the decoherence rate decreases with increasing coupling constant. This is because the coupling strength of the electron–LO-phonon in the first-excited state is weaker than in the ground state while the coupling constant is increasing. So the increase in energy spacing between the first-excited and the ground state causes a decrease in the decoherence rate. It also shows that the influence of the Coulomb potential on the decoherence rate is stronger than that of the coupling constant.

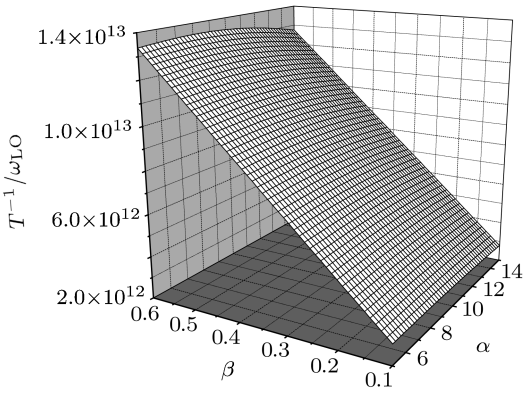


Fig. 2. The decoherence rate T^{-1} changes with the Coulomb binding parameter β and the electron–LO-phonon coupling constant α.

Figure 3 presents the decoherence rate as a function of the Coulomb binding parameter β and the confinement length of the QD l_0 for the electron–LO-phonon coupling constant $\alpha = 7$, the coefficient of phonon dispersion $\eta = 1.0$ and the polar angle $\theta = \pi$. It shows that the decoherence rate increases with an increase in the Coulomb binding parameter for the same reason as in Fig. 1. Also, the decoherence rate is a decreasing function of the confinement length. This is because the electron thermal motion energy and the interaction between the electron and the phonon, which takes the phonon as a medium, are induced when the particle motion range becomes smaller with a decrease of the confinement length of the QD. So the decrease of the energy spacing between

the ground and the first-excited states reduces the decoherence rate, which can be attributed to interesting quantum size effects. So we have to choose a material whose confinement length is appropriate for making a qubit using a quantum dot.

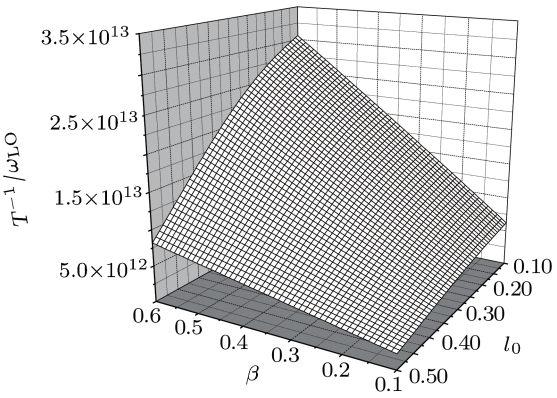


Fig. 3. The decoherence rate T^{-1} as a function of the Coulomb binding parameter β and the the confinement length of the QD l_0.

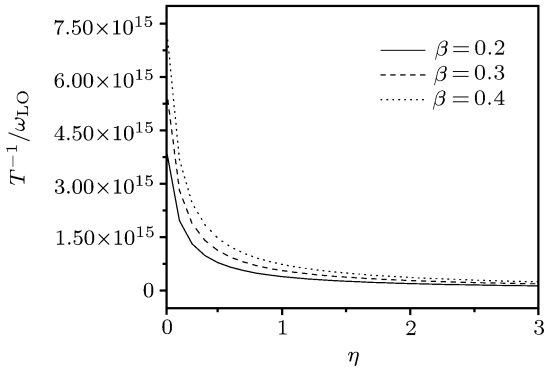


Fig. 4. The decoherence rate T^{-1} as a function of the coefficient of phonon dispersion η.

Figure 4 presents the decoherence rate as a function of the coefficient of phonon dispersion for different Coulomb binding parameters 0.2, 0.3, 0.4, the confinement length of the QD l_0= 0.2, the electron–LO-phonon coupling constant $\alpha = 7$, the coefficient of phonon dispersion $\eta = 1.0$ and the polar angle $\theta = \pi$. It can be seen that the decoherence rate decreases with increasing coefficient of phonon dispersion. The coefficient dispersion is a property of the material. As a result, we should choose a material whose coefficient of phonon dispersion is large for making a qubit using a quantum dot to get a longer decoherence rate.

4. Conclusion

The eigenenergies and eigenfunctions of the ground and first-excited states of an electron that is strongly coupled to an LO-phonon are studied by using the Pekar variational method in a QD with a triangular bound potential and Coulomb bound potential. Our numerical results indicate that the decoherence rate is a decreasing function of the confinement length of the QD and Coulomb binding parameter, while it is an increasing function of the electron–LO-phonon coupling constant and the coefficient of phonon dispersion. Meanwhile, the decoherence rate varies periodically with respect to the polar angle. The influences of the polar angle on the decoherence rate are dominant when the Coulomb binding parameter increases, and the effects of the coupling constant on it are weaker than the Coulomb binding parameter. We should choose a material whose coefficient of phonon dispersion, coupling constant and confinement length are large, and in which there are fewer impurities, for making a qubit using a quantum dot to get a longer decoherence rate.

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