Generation of multi-atom W states via Raman transition in an optical cavity*

Wu Chun-Wang(吴春旺)[a)†], Han Yang(韩 阳)[a)], Deng Zhi-Jiao(邓志姣)[a)b)], Liang Lin-Mei(梁林梅)[a)], and Li Cheng-Zu(李承祖)[a)]

[a)] *College of Science, National University of Defense Technology, Changsha* 410073, *China*
[b)] *Key Laboratory of Low Dimensional Quantum Structures and Quantum Control (Hunan Normal University), Ministry of Education of China, Changsha* 410081, *China*

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A simple scheme is proposed to generate the W state of N Λ-type neutral atoms trapped in an optical cavity via Raman transition. Conditional on no photon leakage from the cavity, the N-qubit W state can be prepared perfectly by turning on a classical coupling field for an appropriate time. Compared with the previous ones, our scheme requires neither individual laser addressing of the atoms, nor demand for controlling N atoms to go through an optical cavity simultaneously with a constant velocity. We investigate the influence of cavity decay using the quantum jump approach and show that the preparation time decreases and the success probability increases with atom number because of a collective enhancement of the coupling.

Keywords: multi-atom W states, Raman transition, optical cavity

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1. Introduction

Entanglement is one of the most striking features of quantum mechanics. It is not only used to test fundamental quantum-mechanical principles,[1] but also has extensive applications in quantum information processing (QIP) including quantum teleportation,[2] quantum cryptography,[3] quantum dense coding[4] and high-precision frequency measurement.[5] Much effort has been devoted to the controlled generation of entangled states. Up to now, bipartite entangled states have been realized in different kinds of physical systems, such as nonlinear optical systems,[6] cavity quantum electrodynamics (QED),[7] linear ion traps,[8] and so on. Several ingenious proposals aiming to prepare multipartite entanglements have also been demonstrated in experiments.[9−11] Among these physical candidates for generation and preservation of entanglements, cavity QED attracts persistent interest because of its very controllable interaction between the atoms and the cavity.[12]

It is well known that there are two inequivalent classes of tripartite entangled states, i.e., the GHZ class and the W class, which cannot be interconverted with a nonzero probability via local operations and classical communication (LOCC).[13] The N-qubit W state has the general form: $W_N = (1/\sqrt{N})|N-1,\ 1\rangle$, where $|N-1,1\rangle$ denotes all the totally symmetric states including $N-1$ zeros and 1 one. For this state, the entanglement is rather robust in the sense that disposal of any one of the N qubits leads only to a relatively small decrease of the entanglement in the system. Recently, it has been realized that the W state also plays a key role in verifying quantum nonlocality without inequality[14] and optimal cloning protocols.[15] Then the generation of the long-lived N-qubit W state becomes a critical technique in QIP.

In cavity QED, several schemes for W state preparation by Rydberg atoms going through the microwave cavity have been presented. Some of these schemes use the resonant interaction between the atoms and the cavity to shorten the preparation time.[16,17] Others try to suppress the cavity dissipation by employing a large dutuning mechanism.[18,19] However, the above schemes in Refs. [16]–[19] have two unfavorable features in common. On the one hand, it is difficult to control several atoms to go through a cavity simultaneously. In fact, we have not found any experimental report of controlling more than two atoms going through a cavity simultaneously. On the other hand,

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†Corresponding author. E-mail: cwwu@nudt.edu.cn

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although the radiative time of Rydberg states is long enough to complete the preparation procedure, it is relatively short for these atoms to store the prepared entanglement. To generate the long-lived W state, some proposals suggest using Λ-type atoms interacting with an optical cavity.[20,21] Through encoding the qubits in Zeeman sublevels of the electronic ground states, the prepared W state has a much longer coherence time. Reference [20] proposed entanglement of N Λ-type atoms going through an optical cavity by means of adiabatic passages, which still needs the control of several atoms going through the cavity simultaneously. The author in Ref. [21] proposed generation of the W state of N Λ-type atoms trapped in an optical cavity. However, individual laser addressing of the atoms is needed for the latter scheme, so it is also difficult to realize experimentally.

In this paper, we describe an alternative scheme for preparing the W state of N Λ-type atoms interacting with an optical cavity. With the atoms driven by the cavity mode and an additional strongly detuned laser field, Raman transition will take place between the two lower states under the condition of two-photon resonance. Then the N-qubit W state can be prepared by turning on the coupling laser field for an appropriate time if the cavity is initially prepared in the single-photon state. Compared with the existing schemes, our work has several advantages: (i) Because the qubits are encoded in the metastable states of the atoms, the prepared W state has a much longer coherence time than in Refs. [16]–[19]. (ii) The interaction between atoms and the optical cavity can be interrupted easily by turning off the laser field. In the whole procedure, we do not require controlling the atoms to go through the cavity with a constant velocity like in Ref. [20]. (iii) In contrast to Ref. [21], where individual laser addressing of the atoms is needed, we only need the coupling laser to be focused on the region of the cavity. (iv) In our scheme, the preparation time decreases and the success probability increases with atom number because of a collective enhancement in the coupling.

2. Basic model

To illustrate the basic idea underlying the state preparation scheme discussed in this paper, we give in this section a simplified model without considering the cavity decay. The system we consider is shown schematically in Fig. 1.

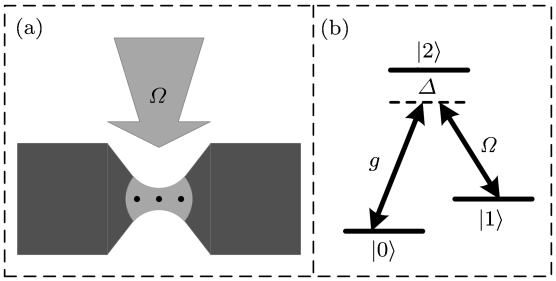


Fig. 1. (a) Schematic illustration of N identical atoms trapped in an optical cavity. They are driven by the cavity mode and an additional classical laser field. For simplicity, we only draw three atoms in the cavity as a sketch. (b) Involved atomic level configuration and the corresponding transitions. The Rabi frequency of the laser field addressing the $|1\rangle - |2\rangle$ transition is denoted by Ω and g is the atom–cavity coupling constant with respect to the $|0\rangle - |2\rangle$ transition. Both transitions have the same detuning Δ.

N identical Λ-type three-level atoms, each with an excited state $|2>$ and two stable ground states $|0\rangle$ and $|1\rangle$, are trapped in an optical cavity. An external classical laser field at frequency $w_{\rm L}$, transmitting along a direction parallel to the cavity mirrors, is focused on the region of the cavity. The $|0\rangle - |2\rangle$ transition is coupled by the cavity mode at frequency $w_{\rm c}$ with the atom–cavity coupling constant g, while the $|1\rangle - |2\rangle$ transition is coupled by the classical field with the Rabi frequency Ω. Both transitions have the same detuning Δ, i.e., under the condition of two-photon resonance. Then the system Hamiltonian (assuming $\hbar = 1$) can be written as

$$H = H_0 + H_1, \tag{1}$$

$$H_0 = w_{\rm c} a^{\dagger} a + \sum_{k=1}^{N} \sum_{i=0,1,2} w_{\rm i} |i\rangle_{kk}\langle i|, \tag{2}$$

$$H_1 = \sum_{k=1}^{N} (g_{\rm k} |2\rangle_{kk}\langle 0| a + \Omega_k |2\rangle_{kk}\langle 1| {\rm e}^{-{\rm i}\, w_{\rm L} t} + {\rm H.c.}), \tag{3}$$

where subscript k represents the kth atom and a ($a^{\dagger}$) denotes the annihilation (creation) operator of the cavity mode. The atomic states $|i\rangle$ ($i = 0,\ 1,\ 2$) have Bohr frequencies w_i. 'H.c.' stands for Hermitian conjugate. In the following, we assume that $\Omega = \Omega_k$ and $g = g_k$ are real for simplicity. Then with respect to the interaction-free Hamiltonian H_0, the interaction Hamiltonian reads

$$H_{\rm I} = \sum_{k=1}^{N} (g |2\rangle_{kk}\langle 0| a {\rm e}^{{\rm i}\,\Delta t} + \Omega |2\rangle_{kk}\langle 1| {\rm e}^{{\rm i}\,\Delta t} + {\rm H.c.}). \tag{4}$$

In the large detuning regime, i.e., with the choice of $|\Delta| \gg |\Omega|, |g|$, we can neglect the effect of rapidly oscillating terms and use the time-averaging method in Ref. [22]. This allows for an adiabatic elimination of

the excited states and results in the effective interaction Hamiltonian

$$H_{\rm eff} = H_{\rm shift} + H_{\rm ex}, \tag{5}$$

$$H_{\rm shift} = \sum_{k=1}^{N} \Big(- \frac{g^2}{\Delta} |0\rangle_{kk}\langle 0| a^{\dagger} a - \frac{\Omega^2}{\Delta} |1\rangle_{kk}\langle 1| \Big), \tag{6}$$

$$H_{\rm ex} = \sum_{k=1}^{N} \Big(- \frac{g\Omega}{\Delta} |0\rangle_{kk}\langle 1| a^{\dagger} + {\rm H.c.} \Big), \tag{7}$$

where $H_{\rm shift}$ represents the dynamical energy shift of the atomic levels $|0\rangle$ and $|1\rangle$, and $H_{\rm ex}$ represents the dynamical energy exchange between these system states.

Expressing the state of the total system in the form of $|{\rm cavity}\rangle|{\rm atoms}\rangle$, we initially prepare the system in the state $|1\rangle|0\rangle$, which means all atoms are in the ground state $|0\rangle$ and the optical cavity is in the single-photon state. Then the temporal evolution of the system is restricted in an $N+1$ dimensional Hilbert space $\mathcal{H}$ with the basis states $\{|\phi_0\rangle = |1\rangle|0\rangle$, $|\phi_i\rangle = |0\rangle|1_i\rangle$ $(i = 1, 2, \ldots, N)\}$. Here $|\phi_i\rangle$ describes the condition where the ith atom is in the state $|1\rangle$ while the other $N-1$ atoms remain in $|0\rangle$, and the optical cavity is in the vacuum state.

If the coupling strength Ω is chosen to satisfy $\Omega = \sqrt{N} g$, we have the commutation relation

$$[H_{\rm shift}, H_{\rm ex}] = 0. \tag{8}$$

Equation (8) can be verified easily using the fact that $H_{\rm shift} = -\frac{Ng^2}{\Delta} I$, where I is the identity operator acting on the state space $\mathcal{H}$. Then the time-evolution operator of the system can be written as

$$U(t) = {\rm e}^{-{\rm i}\, H_{\rm shift} t} {\rm e}^{-{\rm i}\, H_{\rm ex} t}. \tag{9}$$

Neglecting the global phase factor resulting from $H_{\rm shift}$, the evolution of the system state goes as follows:

$$|\Psi(t)\rangle = \cos\left(\frac{Ng^2}{\Delta} t\right) |\phi_0\rangle + {\rm i} \sin\left(\frac{Ng^2}{\Delta} t\right) \left(\frac{1}{\sqrt{N}} \sum_{i=1}^{N} |\phi_i\rangle\right). \tag{10}$$

With the choice of $Ng^2 t/\Delta = \pi/2$, i.e., $t = \pi\Delta/2Ng^2$, we can get the N-atom W state.

3. Model including cavity decay

In the above basic model we did not take into account the influence of cavity decay. To describe the time evolution of the system in the presence of dissipation we use in the following the quantum jump approach.[23] Let us denote the rate with which a single photon inside the resonator leaks out through the cavity mirrors with κ. Under the condition of no photon emission, the evolution of the system is governed by a conditional Hamiltonian

$$H_{\rm cond} = H_{\rm eff} - {\rm i}\, \frac{\kappa}{2} a^{\dagger} a. \tag{11}$$

Note that the conditional Hamiltonian is non-Hermitian and the norm of a state vector developing with the corresponding Schrödinger equation decreases in general with time. Given the system state $|\Psi(t)\rangle$, the probability that no photon has leaked out of the cavity during the evolution until t equals

$$p(t) = \langle \Psi(t) | \Psi(t) \rangle. \tag{12}$$

Initially prepared in the state $|\phi_0\rangle$ as in the basic model, the system evolves according to $H_{\rm cond}$ conditional on no photon emission. Since the initial atomic state is symmetrical and the atoms play equal roles during the evolution, the relevant Hilbert space is actually two dimensional and contains the basis states $\{|\phi_0\rangle, |\phi_{\rm w}\rangle = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} |\phi_i\rangle\}$. The system state vector $|\Psi(t)\rangle$ can be decomposed as

$$|\Psi(t)\rangle = c_0(t)|\phi_0\rangle + c_1(t)|\phi_{\rm w}\rangle. \tag{13}$$

According to the Schrödinger equation ${\rm i}\, \partial_t |\Psi(t)\rangle \geq H_{\rm cond} |\Psi(t)\rangle$, we deduce that

$$\begin{cases} {\rm i}\, \dot{c_0} = \Big(- \dfrac{Ng^2}{\Delta} - {\rm i}\, \dfrac{\kappa}{2} \Big) c_0 - \dfrac{Ng^2}{\Delta} c_1, \\ {\rm i}\, \dot{c_1} = - \dfrac{Ng^2}{\Delta} c_0 - \dfrac{Ng^2}{\Delta} c_1. \end{cases} \tag{14}$$

Note that we have used the relation $\Omega = \sqrt{N} g$ in Eqs. (14). By employing the transformation $\lambda_{\nu=0,1}(t) = c_{\nu}(t) {\rm e}^{-\frac{{\rm i}\, Ng^2}{\Delta} t}$, Eqs. (14) can be written as

$$\begin{cases} {\rm i}\, \dot{\lambda_0} = - {\rm i}\, \dfrac{\kappa}{2} \lambda_0 - \dfrac{Ng^2}{\Delta} \lambda_1, \\ {\rm i}\, \dot{\lambda_1} = - \dfrac{Ng^2}{\Delta} \lambda_0. \end{cases} \tag{15}$$

Solving Eqs. (15) and neglecting the global phase ${\rm e}^{\frac{{\rm i}\, Ng^2}{\Delta} t}$, we can obtain the system state at time t

$$|\Psi(t)\rangle = \exp\left(-\frac{\kappa}{4}t\right)\left\{\left[\cos\left(\frac{A}{2}t\right) - \frac{\kappa}{2A}\sin\left(\frac{A}{2}t\right)\right]|\phi_0\rangle + \frac{\mathrm{i}\,2B}{A}\sin\left(\frac{A}{2}t\right)|\phi_{\mathrm{w}}\rangle\right\}, \quad (16)$$

where

$$A = \sqrt{4\left(\frac{Ng^2}{\Delta}\right)^2 - \left(\frac{\kappa}{2}\right)^2}, \quad B = \frac{Ng^2}{\Delta}.$$

If the interaction time τ is chosen to satisfy

$$\tan\left(\frac{A\tau}{2}\right) = \frac{2A}{\kappa},$$

we have

$$|\Psi(\tau)\rangle = \mathrm{i}\exp\left(-\frac{\kappa\tau}{4}\right)\frac{2B}{A}\sin\left(\frac{A\tau}{2}\right)|\phi_{\mathrm{w}}\rangle. \quad (17)$$

The corresponding success probability of getting the N-atom W state is

$$p_{\mathrm{suc}} = \langle\Psi(\tau)|\Psi(\tau)\rangle = \frac{4B^2}{A^2}\exp\left(-\frac{\kappa\tau}{2}\right)\sin^2\left(\frac{A\tau}{2}\right). \quad (18)$$

To carry out our scheme, we first prepare the system in the state $|\phi_0\rangle$, i.e., all atoms in the ground state $|0\rangle$ and the optical cavity in the single-photon state. Then we turn on the external laser field, with the laser strength satisfying $\Omega = \sqrt{N}g$. After an appropriate time τ, the laser field is turned off to interrupt the interaction between atoms and the optical cavity. Conditional on no photon leakage from the cavity, the N-atom W state can be prepared perfectly. But there are also possibilities that photon emission occurs during the state preparation. In the case of an emission, which can be observed by photon detectors outside the cavity, the state preparation fails and the experiment has to be repeated.

4. Experimental feasibility

In this section, we analyse the experimental feasibility of our scheme. First, it should be pointed out that our model only works in the large detuning regime, i.e., with the population in the excited state $|2\rangle$ being neglected. It has been shown in Ref. [24] that, with the choice of $\Delta \geq 10\Omega$, $10g$, the evolution of the system can be described by H_{eff} to a very good approximation. In the following discussion, we assume that $\Delta = 10\Omega = 10\sqrt{N}g$.

Cavity decay plays a significant role in the preparation procedure since Raman transition is not adequately faster than the leakage of a photon under the condition of large detuning. Its influences can be estimated by some rough calculations based on the present cavity QED techniques. For a practical situation, we can choose the key cavity QED parameters describing the cavity–atom interaction system as $g/2\pi = 16$ MHz, $\kappa/2\pi = 1.4$ MHz given by Rempe's group.[25] With the laser strength and the transition detuning satisfying $\Omega = \sqrt{N}g$ and $\Delta = 10\Omega$, we compute the preparation time τ and the success probability p_{suc} for atom number N ranging from 2 to 16.

In Fig. 2, we plot τ as a function of N, where the preparation time is of the order of 10^{-7} s and decreases sharply with increasing atom number. In Fig. 3, we plot p_{suc} versus N and show that the success probability monotonically increases as N grows. Here, we give a simple explanation for these results. Each Λ-type atom considered in our model

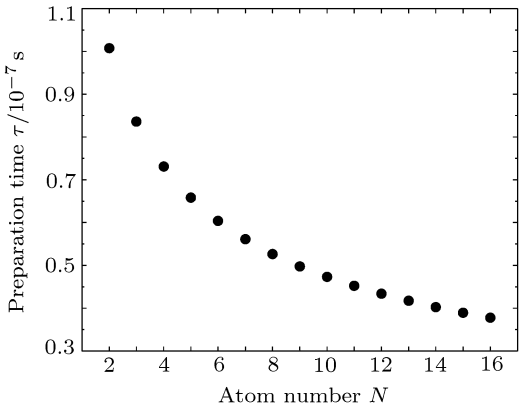


Fig. 2. The preparation time τ as a function of the atom number N, for $g/2\pi = 16$ MHz, $\kappa/2\pi = 1.4$ MHz, $\Omega = \sqrt{N}g$ and $\Delta = 10\Omega$.

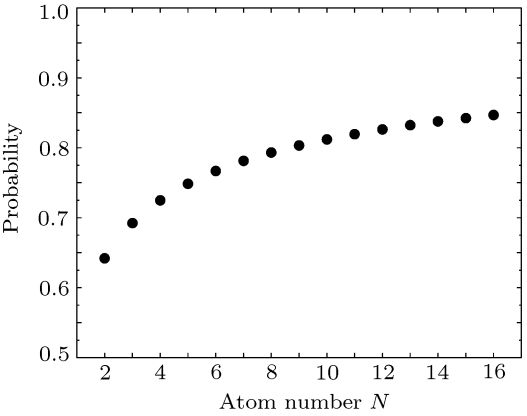


Fig. 3. The success probability p_{suc} as a function of the atom number N, for $g/2\pi = 16$ MHz, $\kappa/2\pi = 1.4$ MHz, $\Omega = \sqrt{N}g$ and $\Delta = 10\Omega$.

behaves like a two-level atom which is resonantly coupled to the cavity with the effective Rabi frequency $g_{\rm eff} = g\Omega/\Delta$. Due to the symmetric interaction of all N atomic dipoles with the cavity mode, the coupling strength is collectively enhanced to $\sqrt{N}g_{\rm eff}$. So the preparation time decreases and the success probability increases with the atom number, which helps our scheme prevail in the case of large-scale entanglement preparation.

In this scheme, we need to trap N atoms in an optical cavity and ensure that all these atoms have the same atom–cavity coupling constant g. One attainable technology that can be used for our experiment may be the one-dimensional (1D) optical lattice.[26] Atoms, trapped in the antinodes of a standing wave, can be transferred in and out of the cavity mode. In the present case, we can use N parallel 1D optical lattices to trap N atoms in the cavity. The optical cavity has a general mode function described by $\chi(z,r) = \sin(kz)\exp(-r^2/w_0^2)$, where w_0 is the waist, k is the wave vector of the Gaussian cavity mode, r is the atom position in the direction parallel to cavity mirrors with $r = 0$ on the cavity axis, and z denotes the direction along the cavity axis. As a function of the atom location, the atom–cavity coupling constant equals $g(z,r) = g_0\chi(z,r)$, where g_0 is the maximum coupling rate assumed only at an antinode of the field mode inside the resonator. If we assume that all the atoms are addressed at different antinodes in the cavity mode, then the atom–cavity coupling constant for each atom is identical.

Our scheme also requires an efficient single photon source and its injection into an optical cavity. Recently, Kimble's group has demonstrated a reliable single photon source in an ingenious experiment.[27] With a trapped cesium atom strongly coupled to the optical cavity, long-lived internal atomic states can be mapped to quantum states of the electromagnetic field by way of 'dark' eigenstates of the atom–cavity system.

In summary, we have presented a simple scheme for generating the W state of N trapped atoms in an optical cavity via Raman transition. Apart from the finally prepared state being stable, the interaction between the atoms and the cavity mode can be controlled easily by an external lser field. We also investigate the influence of cavity decay using the quantum jump approach and show that the preparation time decreases and the success probability increases with atom number, which helps our scheme prevail in the case of large-scale entanglement generation.

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