A simple scheme for implementing four-atom quantum dense coding in cavity QED*

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An experimentally feasible scheme for implementing four-atom quantum dense coding of an atom–cavity system is proposed. The cavity is only virtually excited and no quantum information will be transferred from the atoms to the cavity. Thus the scheme is insensitive to cavity decay and the thermal field. In the scheme, Alice can send faithfully 4 bits of classical information to Bob by sending two qubits. Generalized Bell states can be exactly distinguished by detecting the atomic state, and quantum dense coding can be realized in a simple way.

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Entanglement is one of the distinguishing properties of quantum mechanics, and plays a central role in quantum information processing. Dense coding, originally introduced by Bennett and Wiesner,[1] is one of the important applications of quantum entanglement.[2] In the original protocol,[1] quantum dense coding uses a preshared entangled pair of qubits to transmit two bits of classical information by sending only one qubit. The original dense coding protocol has been generalized by other authors in various directions. Dense coding has been investigated theoretically[3] and experimentally[4] in the context of continuous variables. It has also been considered for more than two entangled bits[5−10] or entangled degrees of freedom.[11,12] An experimental demonstration of the original dense coding[1] was first made using photonic polarization entanglement.[13] The protocol was also simulated in nuclear magnetic resonance using temporal averaging.[14]

Over the past decade, the cavity quantum electrodynamics system, which is one of the possible candidates for engineering quantum entanglement, has received much attention. Recently, a lot of protocols have been proposed for implementing dense coding by using entangled states in cavity QED.[4,6,15−19] Ye and Guo proposed a simple scheme[18] to implement quantum dense coding for two two-level atoms via cavity QED. Their scheme has eliminated the effects of both the thermal field and cavity decay, and its successful probability of dense coding is 1. However, only the two-particle dense coding case was studied. Recently, He *et al.*[19] have proposed a scheme for realizing dense coding by using a class of W-state in cavity quantum electrodynamics (QED) and the probability of implementing quantum dense coding is 1. However, i) this scheme requires the cavity mode to be in the vacuum state. Thus the effect of cavity decay is eliminated and the effect of the thermal field is still present. ii) An auxiliary atom is needed in the scheme. The greater the number of the atoms, the more difficult it is to control them, and the more difficult it is for the atoms to cross the cavity simultaneously. iii) Besides single-qubit rotation operation, two sequential two-qubit operations are needed during the Bell-state measurement. Thus three steps are involved. It is known that the more operation steps there are, the longer the operational time is, because a delay time exists between any two neighbouring operations in actual experiments. Moreover, only the two-particle dense coding case was studied. Here we present an

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experimentally feasible scheme to implement quantum dense coding for realizing four two-level atoms via cavity QED. This scheme is also insensitive to the cavity decay and the thermal field. More importantly, sixteen generalized Bell-state measurements can be exactly discriminated with a success probability of 1. In the scheme, with the assistance of two maximally entangled-qubit pairs, Alice can send faithfully 4 bits of classical information to Bob by sending two qubits.

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity field, and at the same time the atoms are driven by a classical field. Assume the frequency of the driving field ω is equal to the atomic transition frequency ω_0. In the strong driving regime $2\Omega \gg \delta,\ g/2$ (Ω is the Rabi frequency of the classical field, g is the atom–cavity coupling strength and δ is the detuning between the atomic transition frequency and the cavity field frequency), we can neglect the fast oscillating terms. In the large detuning $\delta \gg g/2$, there is no energy exchange between the atomic system and the cavity. Thus the effective Hamiltonian can be described by[20]

$$H_{\rm eff} = 2(\Omega S_x + \lambda S_x^2), \tag{1}$$

where $\lambda = g^2/2\delta$, $S_x = (1/2)\sum_{j=1}^{2}(S_j^+ + S_j^-)$, and $S_j^+ = |e\rangle_j\langle g|$, $S_j^- = |g\rangle_j\langle e|$, with $|e\rangle_j$ and $|g\rangle_j$ being the excited and ground states of the jth atom. We note that the effective Hamiltonian is independent of the cavity field state, allowing it to be in a thermal state. In addition, there is no exchange of energy between the atoms and the cavity mode. So the effects of cavity decay and the thermal field are all eliminated. Then the evolution operator of the system is given by

$$U(t) = {\rm e}^{-{\rm i}H_{\rm eff}t} = {\rm e}^{-{\rm i}\,2(\Omega S_x + \lambda S_x^2)t}. \tag{2}$$

Two atoms initially in the state $|x_1x_2\rangle$ are simultaneously sent into the cavity. At the same time, they are driven by a classical field. Then the state of the two atoms will evolve into

$$\begin{aligned}|x_1x_2\rangle \longrightarrow {\rm e}^{-{\rm i}\,\lambda t}[&\cos(\lambda t)(\cos(\Omega t)|x_1\rangle - {\rm i}\sin(\Omega t)|\bar{x}_1\rangle)(\cos(\Omega t)|x_2\rangle - {\rm i}\sin(\Omega t)|\bar{x}_2\rangle)\\ &- {\rm i}\sin(\lambda t)(\cos(\Omega t)|\bar{x}_1\rangle - {\rm i}\sin(\Omega t)|x_1\rangle)(\cos(\Omega t)|\bar{x}_2\rangle - {\rm i}\sin(\Omega t)|x_2\rangle)],\end{aligned} \tag{3}$$

where $x \in \{g, e\}$ and $\bar{x}$ is the counterpart of the binary number x. Choosing $\lambda t = \pi/4$ and realizing the condition $\Omega t = n\pi (n = 1, 2, \ldots)$ by modulating the driving field appropriately, we obtain the maximally two-atom entangled state

$$|x_1x_2\rangle \longrightarrow \frac{1}{\sqrt{2}}(|x_1x_2\rangle - {\rm i}\,|\bar{x}_1\bar{x}_2\rangle), \tag{4}$$

where a common factor ${\rm e}^{-{\rm i}\,\pi/4}$ is discarded.

Suppose that there are four atoms initially in the state $|g_1g_2g_3g_4\rangle$; atoms 1, 2 belong to Alice and atoms 3, 4 belong to Bob. Atoms 1 and 3 (2 and 4) are simultaneously sent into the single-mode cavity C_1 (C_2) and at the same time the two atoms are driven by a classical field. After an interaction time $\lambda t = \pi/4$ and $\Omega t = n\pi (n = 1, 2, \ldots)$, we can obtain one of generalized Bell states

$$|\phi_1\rangle = \frac{1}{2}(|gggg\rangle_{1234} - {\rm i}\,|gege\rangle_{1234} - {\rm i}\,|egeg\rangle_{1234} - |eeee\rangle_{1234}). \tag{5}$$

Dense coding can be done in the following way. Suppose Alice and Bob share the entangled state $|\phi_1\rangle$. Firstly, Alice performs one of the four unitary transformations $\{I_1, \sigma_1^z, \sigma_1^y, {\rm i}\,\sigma_1^x\}$ on her atom 1 and performs one of the four unitary transformations $\{I_2, \sigma_2^z, \sigma_2^y, {\rm i}\,\sigma_2^x\}$ on her atom 2, respectively. Through simple calculation, it can be shown that there are $4 \times 4 = 16$ different unitary operators

$$\begin{aligned}&U_1 = I_1 \otimes I_2,\ \ U_2 = \sigma_1^z \otimes I_2,\ \ U_3 = I_1 \otimes \sigma_2^z,\ \ U_4 = \sigma_1^z \otimes \sigma_2^z,\\ &U_5 = I_1 \otimes \sigma_2^y,\ \ U_6 = \sigma_1^z \otimes \sigma_2^y,\ \ U_7 = I_1 \otimes {\rm i}\,\sigma_2^x,\ \ U_8 = \sigma_1^z \otimes {\rm i}\,\sigma_2^x,\\ &U_9 = \sigma_1^y \otimes I_2,\ \ U_{10} = {\rm i}\,\sigma_1^x \otimes I_2,\ \ U_{11} = \sigma_1^y \otimes \sigma_2^z,\ \ U_{12} = {\rm i}\,\sigma_1^x \otimes \sigma_2^z,\\ &U_{13} = \sigma_1^y \otimes \sigma_2^y,\ \ U_{14} = {\rm i}\,\sigma_1^x \otimes \sigma_2^y,\ \ U_{15} = \sigma_1^y \otimes {\rm i}\,\sigma_2^x,\ \ U_{16} = {\rm i}\,\sigma_1^x \otimes {\rm i}\,\sigma_2^x,\end{aligned} \tag{6}$$

where I_j $(j=1,2)$ is the identity operator, and σ_j^x, σ_j^y, σ_j^z $(j=1,2)$ are three Pauli operators. The operation U_i $(i=1,2,3\ldots,16)$ will transform the state $|\phi_1\rangle$ of four atoms into one of the following sixteen generalized Bell states, respectively

$$|\phi_1\rangle = \frac{1}{2}(|gggg\rangle_{1234} - \mathrm{i}\,|gege\rangle_{1234} - \mathrm{i}\,|egeg\rangle_{1234} - |eeee\rangle_{1234}), \tag{7}$$

$$|\phi_2\rangle = -\frac{1}{2}(|gggg\rangle_{1234} - \mathrm{i}\,|gege\rangle_{1234} + \mathrm{i}\,|egeg\rangle_{1234} + |eeee\rangle_{1234}), \tag{8}$$

$$|\phi_3\rangle = -\frac{1}{2}(|gggg\rangle_{1234} + \mathrm{i}\,|gege\rangle_{1234} - \mathrm{i}\,|egeg\rangle_{1234} + |eeee\rangle_{1234}), \tag{9}$$

$$|\phi_4\rangle = \frac{1}{2}(|gggg\rangle_{1234} + \mathrm{i}\,|gege\rangle_{1234} + \mathrm{i}\,|egeg\rangle_{1234} - |eeee\rangle_{1234}), \tag{10}$$

$$|\phi_5\rangle = \frac{1}{2}(|ggge\rangle_{1234} - \mathrm{i}\,|gegg\rangle_{1234} - \mathrm{i}\,|egee\rangle_{1234} - |eeeg\rangle_{1234}), \tag{11}$$

$$|\phi_6\rangle = -\frac{1}{2}(|ggge\rangle_{1234} - \mathrm{i}\,|gegg\rangle_{1234} + \mathrm{i}\,|egee\rangle_{1234} + |eeeg\rangle_{1234}), \tag{12}$$

$$|\phi_7\rangle = \frac{1}{2}(|ggge\rangle_{1234} + \mathrm{i}\,|gegg\rangle_{1234} - \mathrm{i}\,|egee\rangle_{1234} + |eeeg\rangle_{1234}), \tag{13}$$

$$|\phi_8\rangle = -\frac{1}{2}(|ggge\rangle_{1234} + \mathrm{i}\,|gegg\rangle_{1234} + \mathrm{i}\,|egee\rangle_{1234} - |eeeg\rangle_{1234}), \tag{14}$$

$$|\phi_9\rangle = \frac{1}{2}(|ggeg\rangle_{1234} - \mathrm{i}\,|geee\rangle_{1234} - \mathrm{i}\,|eggg\rangle_{1234} - |eege\rangle_{1234}), \tag{15}$$

$$|\phi_{10}\rangle = \frac{1}{2}(|ggeg\rangle_{1234} - \mathrm{i}\,|geee\rangle_{1234} + \mathrm{i}\,|eggg\rangle_{1234} + |eege\rangle_{1234}), \tag{16}$$

$$|\phi_{11}\rangle = -\frac{1}{2}(|ggeg\rangle_{1234} + \mathrm{i}\,|geee\rangle_{1234} - \mathrm{i}\,|eggg\rangle_{1234} + |eege\rangle_{1234}), \tag{17}$$

$$|\phi_{12}\rangle = -\frac{1}{2}(|ggeg\rangle_{1234} + \mathrm{i}\,|geee\rangle_{1234} + \mathrm{i}\,|eggg\rangle_{1234} - |eege\rangle_{1234}), \tag{18}$$

$$|\phi_{13}\rangle = \frac{1}{2}(|ggee\rangle_{1234} - \mathrm{i}\,|geeg\rangle_{1234} - \mathrm{i}\,|egge\rangle_{1234} - |eegg\rangle_{1234}), \tag{19}$$

$$|\phi_{14}\rangle = \frac{1}{2}(|ggee\rangle_{1234} - \mathrm{i}\,|geeg\rangle_{1234} + \mathrm{i}\,|egge\rangle_{1234} + |eegg\rangle_{1234}), \tag{20}$$

$$|\phi_{15}\rangle = \frac{1}{2}(|ggee\rangle_{1234} + \mathrm{i}\,|geeg\rangle_{1234} - \mathrm{i}\,|egge\rangle_{1234} + |eegg\rangle_{1234}), \tag{21}$$

$$|\phi_{16}\rangle = \frac{1}{2}(|ggee\rangle_{1234} + \mathrm{i}\,|geeg\rangle_{1234} + \mathrm{i}\,|egge\rangle_{1234} - |eegg\rangle_{1234}). \tag{22}$$

Then Alice sends her particle back to Bob. Bob takes only one measurement in the basis $\{|\phi_1\rangle, |\phi_2\rangle, \ldots, |\phi_{16}\rangle\}$, and he will know what operation Alice has performed, that is, what messages Alice has encoded in the quantum state. Because the sixteen manipulations result in sixteen orthogonal generalized Bell states, sixteen distinguishable messages, i.e., 4 bits of information,[21] can then be obtained by Bob via collective measurement on the four particles. But how does Bob discriminate the sixteen states?

Next, we demonstrate how to implement the generalized Bell-state measurement, i.e., deterministically distinguish between the sixteen states. To discriminate the sixteen states, Bob lets atom 1 and atom 3 simultaneously interact with a single-mode cavity and be driven by a classical field, and lets atom 2 and atom 4 simultaneously interact with another single-mode cavity and be driven by another classical field. The system evolution is described by Eq. (2). After an interaction time $\lambda t = \pi/4$ and $\Omega t = n\pi (n = 1, 2, \ldots)$, the above sixteen states in Eqs. (7)–(22) will be

$$|\phi_1\rangle \longrightarrow -|eeee\rangle_{1234}), \tag{23}$$

$$|\phi_2\rangle \longrightarrow \mathrm{i}\,|gege\rangle_{1234}), \tag{24}$$

$$|\phi_3\rangle \longrightarrow \mathrm{i}\,|egeg\rangle_{1234}), \tag{25}$$

$$|\phi_4\rangle \longrightarrow |gggg\rangle_{1234}), \tag{26}$$

$$|\phi_5\rangle \longrightarrow -|eeeg\rangle_{1234}), \tag{27}$$

$$|\phi_6\rangle \longrightarrow \mathrm{i}\,|gegg\rangle_{1234}), \tag{28}$$

$$|\phi_7\rangle \longrightarrow -\,\mathrm{i}\,|egee\rangle_{1234}), \tag{29}$$

$$|\phi_8\rangle \longrightarrow -|ggge\rangle_{1234}), \tag{30}$$
$$|\phi_9\rangle \longrightarrow -|eege\rangle_{1234}), \tag{31}$$
$$|\phi_{10}\rangle \longrightarrow -\mathrm{i}\,|geee\rangle_{1234}), \tag{32}$$
$$|\phi_{11}\rangle \longrightarrow \mathrm{i}\,|eggg\rangle_{1234}), \tag{33}$$
$$|\phi_{12}\rangle \longrightarrow -|ggeg\rangle_{1234}), \tag{34}$$
$$|\phi_{13}\rangle \longrightarrow -|eegg\rangle_{1234}), \tag{35}$$
$$|\phi_{14}\rangle \longrightarrow -\mathrm{i}\,|geeg\rangle_{1234}), \tag{36}$$
$$|\phi_{15}\rangle \longrightarrow -\mathrm{i}\,|egge\rangle_{1234}), \tag{37}$$
$$|\phi_{16}\rangle \longrightarrow |ggee\rangle_{1234}). \tag{38}$$

Equations (23)–(38) indicate that the above states are transformed into the product states of atoms 1, 2, 3, and 4, respectively. So a deterministic generalized Bell-state measurement can be achieved by individual detection of the related qubits. With the outcome of the measurement on Bob's four atoms, Bob can distinguish Alice's operation on atoms 1 and 2.

Here we will discuss the experimental feasibility of the proposed scheme. For Rydberg atoms, the radiative time is about $T_{\mathrm{r}} = 3 \times 10^{-2}$ s and the coupling constant is $g = 2\pi \times 24$ kHz.[22] We choose $\delta = 10g$, the required atom–cavity–field interaction time is $T \approx 10^{-4}$ s, much shorter than the spontaneous radiative time. Thus the losses due to spontaneous emission can be neglected. Meanwhile from Eq. (1) it is noted that the atomic state evolution is independent of the cavity field state, so the effects of cavity decay and thermal field have been eliminated. So our scheme is realizable by using cavity QED techniques.

One of the difficulties for the present scheme is that the atoms 1 and 3 are required to be simultaneously sent through a cavity, and the atoms 2 and 4 are required to be simultaneously sent through another cavity, otherwise there will be an error. Firstly, we assume that during the generation of the state $|\phi_1\rangle$, the whole time for the atom pair (1, 3) passing through the cavity C_1 is $t_1 = t$, and the whole time for the atom pair (2, 4) passing through the cavity C_2 is $t_2 = t$. Atom 1 enters the cavity C_1 earlier than atom 3 by the difference Δt and atom 2 enters the cavity C_2 earlier than atom 4 by the difference Δt. Another difficulty for the present scheme is that the Rabi frequency Ω has a fluctuation $\Omega + \Delta\Omega$. Assume $\Delta\Omega = 0.01\Omega$. We choose $\Omega t = \pi$, then the four atoms (after interaction) are prepared in the state

$$\begin{aligned}|\phi_1\rangle' \approx{}& \cos^2(\Omega+\Delta\Omega)\Delta t\cos^2(\Omega+\Delta\Omega)(t-\Delta t)\\ &\times[\cos^2(\lambda(t-\Delta t))|gggg\rangle_{1234} - \mathrm{i}\cos(\lambda(t-\Delta t))\sin(\lambda(t-\Delta t))|gege\rangle_{1234}\\ &-\mathrm{i}\sin(\lambda(t-\Delta t))\cos(\lambda(t-\Delta t))|egeg\rangle_{1234} - \sin^2(\lambda(t-\Delta t))|eeee\rangle_{1234}],\end{aligned} \tag{39}$$

where a common factor $\mathrm{e}^{-\mathrm{i}\,2(\lambda t)}$ is discarded. In this case, the difference between the state in Eq. (5) and the state in Eq. (39) can be characterized in terms of the fidelity

$$\begin{aligned}F = {}&\cos^4((\Omega+\Delta\Omega)\Delta t)\cos^4((\Omega+\Delta\Omega)(t-\Delta t))\\ &\times\frac{[\cos^2(\lambda(t-\Delta t)) + 2\cos(\lambda(t-\Delta t))\sin(\lambda(t-\Delta t)) + \sin^2(\lambda(t-\Delta t))]^2}{4}.\end{aligned} \tag{40}$$

We choose $\lambda t = \pi/4$ and $\Delta t = 0.01t$, the fidelity is $F = 0.9979$. Therefore we can conclude that the small fluctuation of the Rabi frequency Ω and the small error of the interaction time only affect the fidelity of the result state slightly.

In summary, we have proposed a simple scheme to realize quantum dense coding using four atoms in thermal cavities. In cavity QED, the photon number-dependent parts in the effective Hamiltonian are cancelled with the assistance of a strong classical driving field. Thus our schemes are insensitive to both cavity decay and the thermal field. More importantly, we can make a complete measurement of generalized Bell states with our scheme. The sixteen states codified by Alice can be completely discriminated only via atom–cavity interaction. The success probability of this scheme is equal to 1.

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