One scheme for remote quantum logical gates with the assistance of a classical field*

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A scheme, based on the two two-level atoms resonantly driven by the classical field separately trapped in two cavities coupled by an optical fibre, for the implementation of remote two-qubit gates is investigated. It is found that the quantum controlled-phase and swap gates can be achieved with the assistance of the classical field when there are detunings of the coupling quantum fields. Moreover, the influence of the dissipation of the cavities and the optical fibre is analysed while the spontaneous emission of the atoms can be effectively suppressed by introducing Λ-type atoms.

Keywords: quantum entanglement, remote phase gate, remote swap gate

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1. Introduction

Distributed quantum computing is an architecture in which a quantum computer is thought of as a network of spatially separated local processors that contain only a few qubits and are connected via quantum transmission lines.[1,2] One of the key problems in the realization of distributed quantum computing is how to implement the two-qubit quantum gates among separate processors since a quantum computer can be built by assembling two-qubit logic gates.[3,4] Therefore, much attention has been paid to the realization of remote two-qubit gates. Many schemes, some of which are based on the sharing of entanglement between the qubits,[5−8] some of which are based on the detection of leaking photons,[9] and some of which are based on the direct linking of the cavities,[10−12] have been proposed.

Recently, several schemes for entangling two distant atoms separately trapped in cavities coupled by optical fibre have been proposed.[13−18] Based on the same system, Serafini *et al.* have investigated the possibility of deterministically realizing remote two-qubit quantum gates and found that reliable swap and controlled-phase gates can be achieved in the presence of spontaneous emission of the atoms and losses of the cavities and the optical fibre when there is no detuning or small detuning of the coupling field.[10] Yin and Li have shown that the quantum swap, entangling, and controlled-Z gates can be deterministically realized between two distant cavities and the operation can be sped up as the number of atoms in the cavities increases if the system fulfils strict requirements.[11] In this scheme, the atoms are resonantly coupled to the cavities and the coupling constant between the cavities and the optical fibre is much larger than that of the atoms and the cavities which would be increased when the number of atoms in the cavity increases. Systems of atoms separately trapped in cavities coupled by optical fibre have been investigated in these schemes. However, they may present interesting features by adding a dressing field when there is detuning between the atomic transition and the frequency of the cavity.[19−21] In this paper, we investigate a system composed of two atoms, resonantly driven by classical fields, separately trapped in two distant cavities coupled by an optical fibre and there are detunings of the coupling quantum fields. It is found that the Hamiltonian of the whole system can be reduced. In the strongly driven regime, it can be equal to that of two atoms coupled to one single-mode cavity for small detunings and a quantum controlled-phase gate can also be achieved. For large detunings, it can be equal to that of the Tavis–Cummings model or double Jaynes–Cummings model when different conditions are fulfilled and the quantum swap gate can also be achieved.

Our paper is organized as follows. In Section 2, the model is introduced. In Section 3, the implemen-

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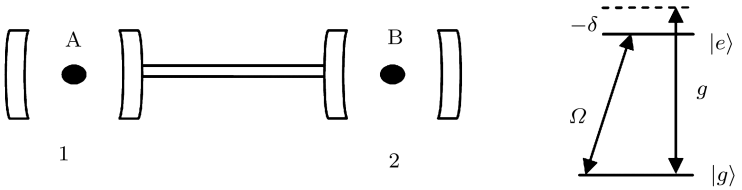
tation of quantum controlled-phase and swap gates is investigated. The influence of the dissipation of the cavities and the optical fibre is analysed while the spontaneous emission of the atoms can be effectively suppressed by introducing Λ-type atoms. The discussion and conclusions are presented in Section 4.

2. Model

First, we introduce the atom–cavity–fibre system. As shown in Fig. 1, two two-level atoms A and B driven by the classical field are trapped in the single-mode cavities 1 and 2, and the two cavities are connected by the optical fibre. Under the rotating wave approximation, the Hamiltonian of the total system can be written as ($\hbar = 1$)

$$\begin{aligned} H = & \sum_{j=1,2} [\omega_j \sigma_j^z + \omega_{\rm f}^j a_j^+ a_j + g_j (a_j \sigma_j^+ + a_j^+ \sigma_j^-) \\ & + \Omega_j (\mathrm{e}^{-\mathrm{i}\omega_{\rm L}^j t} \sigma_j^+ + \mathrm{e}^{\mathrm{i}\omega_{\rm L}^j t} \sigma_j^-)] + \omega_{\rm b} b^+ b \\ & + \nu b (a_1^+ + a_2^+) + \nu b^+ (a_1 + a_2), \end{aligned} \tag{1}$$

where ω_j is the atomic transition frequency of the jth atom, $\omega_{\rm f}^j$ is the cavity frequency of the jth cavity, $\omega_{\rm L}^j$ is the frequency of the jth classical field, $\omega_{\rm b}$ is the frequency of the optical fibre which is resonantly coupled with the cavities, and $\omega_{\rm f}^1 = \omega_{\rm f}^2 = \omega_{\rm b}$. $\sigma_j^z = (|e\rangle_j \langle e| - |g\rangle_j \langle g|)$, $\sigma_j^+ = |e\rangle_j \langle g|$, $\sigma_j^- = |g\rangle_j \langle e|$ are the jth atom phase, raising and lowering operators. a_j and a_j^+ are the annihilation and creation operators for the jth cavity mode. g_j is the jth atom–cavity coupling strength, Ω_j is the jth Rabi frequency, ν is the cavity–fibre coupling constant.



(a) (b)

Fig. 1. (a) Two two-level atoms A and B are trapped in two spatially separated cavities 1 and 2 coupled by optical fibre. (b) Configuration of the two-level atom in a single-mode cavity and driven by the classical field.

Assuming that $\omega_j = \omega_{\rm L}^j$, the interaction Hamiltonian can be written as

$$\begin{aligned} H_{\rm I} = & \sum_{j=1,2} [g_j (a_j \sigma_j^+ \mathrm{e}^{\mathrm{i}\delta_j t} + a_j^+ \sigma_j^- \mathrm{e}^{-\mathrm{i}\delta_j t}) \\ & + \Omega_j (\sigma_j^+ + \sigma_j^-)] + \nu b (a_1^+ + a_2^+) \\ & + \nu b^+ (a_1 + a_2), \end{aligned} \tag{2}$$

where $\delta_j = \omega_j - \omega_{\rm f}^j$. By use of the canonical transformations,[10,11]

$$\begin{aligned} c_0 &= (a_1 - a_2)/\sqrt{2}, \\ c_1 &= (a_1 + a_2 + \sqrt{2} b)/2, \\ c_2 &= (a_1 + a_2 - \sqrt{2} b)/2, \end{aligned} \tag{3}$$

the Hamiltonian can be written as

$$\begin{aligned} H_{\rm I} = & \frac{g_1}{2} [(\mathrm{e}^{-\mathrm{i}\sqrt{2}\nu t} c_1 + \mathrm{e}^{\mathrm{i}\sqrt{2}\nu t} c_2 + \sqrt{2} c_0) \sigma_{r1}^+ \mathrm{e}^{\mathrm{i}\delta_1 t} \\ & + \mathrm{h.c.}] + \frac{g_2}{2} [(\mathrm{e}^{-\mathrm{i}\sqrt{2}\nu t} c_1 + \mathrm{e}^{\mathrm{i}\sqrt{2}\nu t} c_2 \\ & - \sqrt{2} c_0) \sigma_{r2}^+ \mathrm{e}^{\mathrm{i}\delta_2 t} + \mathrm{h.c.}], \end{aligned} \tag{4}$$

where

$$\begin{aligned} \sigma_{rj}^+ &= \mathrm{e}^{\mathrm{i}\Omega_j (\sigma_j^+ + \sigma_j^-) t} \sigma_j^+ \mathrm{e}^{-\mathrm{i}\Omega_j (\sigma_j^+ + \sigma_j^-) t} \\ &= (\tilde{\sigma}_j^z - \mathrm{e}^{-2\mathrm{i}\Omega_j t} \tilde{\sigma}_j^+ + \mathrm{e}^{2\mathrm{i}\Omega_j t} \tilde{\sigma}_j^-)/2, \\ \tilde{\sigma}_j^z &= |+\rangle_j \langle +| - |-\rangle_j \langle -|, \tilde{\sigma}_j^+ = |+\rangle_j \langle -|, \\ \tilde{\sigma}_j^- &= |-\rangle_j \langle +|, \end{aligned}$$

and

$$\begin{aligned} |+\rangle &= (|e\rangle + |g\rangle)/\sqrt{2}, \\ |-\rangle &= (|e\rangle - |g\rangle)/\sqrt{2}. \end{aligned}$$

3. Implementation of two remote two-qubit gates

In this section, we present the implementation of quantum remote two-qubit controlled-phase and swap gates based on this model. The influence caused by the dissipation of the cavities and the fibre is analysed.

Case one. Remote two-qubit controlled-phase gate

When the conditions $\nu, \Omega_j \gg g_j, \delta_j$ and $\left|\sqrt{2}\nu - 2\Omega\right| \gg g_j, \delta_j$ are fulfilled, we can realize a rotating-wave approximation and eliminate terms that oscillate with high frequencies.[20] Then the Hamiltonian of the whole system can be reduced to

$$H_{\text{eff}} = \frac{\sqrt{2}}{4} g_1 (c_0 \mathrm{e}^{\mathrm{i}\delta_1 t} + c_0^+ \mathrm{e}^{-\mathrm{i}\delta_1 t}) \tilde{\sigma}_1^z - \frac{\sqrt{2}}{4} g_2 (c_0 \mathrm{e}^{\mathrm{i}\delta_2 t} + c_0^+ \mathrm{e}^{-\mathrm{i}\delta_2 t}) \tilde{\sigma}_2^z. \quad (5)$$

Assuming that $g_1 = g_2 = g$, and $\delta_1 t + \pi = \delta_2 t$, the Hamiltonian can be written as

$$H_{\text{eff}} = \frac{\sqrt{2}}{4} g (c_0 \mathrm{e}^{\mathrm{i}\delta_1 t} + c_0^+ \mathrm{e}^{-\mathrm{i}\delta_1 t})(\tilde{\sigma}_1^z + \tilde{\sigma}_2^z). \quad (6)$$

The evolution operator for the Hamiltonian H_{eff} can be written[19] in the form of

$$U'(t) = \mathrm{e}^{-\mathrm{i}A(t)\tilde{\sigma}_z^2} \mathrm{e}^{-\mathrm{i}B(t)c_0\tilde{\sigma}_z} \mathrm{e}^{-\mathrm{i}C(t)c_0^+\tilde{\sigma}_z}, \quad (7)$$

where

$$\tilde{\sigma}_z = \tilde{\sigma}_1^z + \tilde{\sigma}_2^z,$$
$$A(t) = \frac{g^2}{8\delta_1}\left[t + \frac{1}{\mathrm{i}\delta_1}(\mathrm{e}^{-\mathrm{i}\delta_1 t} - 1)\right],$$
$$B(t) = \frac{\sqrt{2}g}{4\mathrm{i}\delta_1}(\mathrm{e}^{\mathrm{i}\delta_1 t} - 1),$$
$$C(t) = -\frac{\sqrt{2}g}{4\mathrm{i}\delta_1}(\mathrm{e}^{-\mathrm{i}\delta_1 t} - 1).$$

Setting $\delta_1 t = 2k_1\pi$ (k_1 is an integral number), then $B(t) = C(t) = 0$, $U'(t) = \mathrm{e}^{-\mathrm{i}\lambda t\sigma_z^2}$ where $\lambda = g^2/(8\delta_1)$. The evolution operator of the atoms is given by

$$U(t) = \mathrm{e}^{-\mathrm{i}\Omega\tilde{\sigma}_z t} U'(t) = \mathrm{e}^{-\mathrm{i}\Omega t\tilde{\sigma}_z - \mathrm{i}\lambda t\tilde{\sigma}_z^2}. \quad (8)$$

Then it can be shown that

$$U(t)|+\rangle|+\rangle = \mathrm{e}^{-\mathrm{i}(\Omega+\lambda)t}|+\rangle|+\rangle,$$
$$U(t)|+\rangle|-\rangle = |+\rangle|-\rangle,$$
$$U(t)|-\rangle|+\rangle = |-\rangle|+\rangle,$$
$$U(t)|-\rangle|-\rangle = \mathrm{e}^{\mathrm{i}(\Omega-\lambda)t}|-\rangle|-\rangle. \quad (9)$$

Choosing the interaction time t and Rabi frequency to satisfy that $\lambda t = \theta/2$ and $\Omega t = 2k_2\pi + \theta/2$ (k_2 is an integral number), then a controlled-θ gate can be obtained. Setting $k_1 = 1$, and $\delta_1 = g/\sqrt{2}$, the quantum controlled-Z gate can be obtained in the same way.

Based on this model, the quantum controlled-phase gate can be obtained by appropriately choosing the interaction time, Rabi frequency and the atom–cavity coupling constants with fixed detuning. From the Hamiltonian described by Eq. (5) we find that only the mode c_0, which does not contain the fibre mode, is involved, and the cavities are not excited throughout the whole procedure. Thus the dissipation of the cavities has few effects on the implementation of this quantum logical gate.

Case two. Remote two-qubit swap gate

Assuming that $\delta_1 = \delta_2 = \delta$, $\Omega_1 = \Omega_2 = \Omega$ and $\nu, \Omega, |\delta| \gg g$. When the conditions $\left|2\sqrt{2}\nu - 2\Omega\right| \gg g$ and $-\delta = 2\Omega - \sqrt{2}\nu$ are fulfilled, the effective Hamiltonian of the whole system can be written as

$$H_{\text{eff}} = \frac{1}{4} g_1 (c_1\tilde{\sigma}_1^+ + c_1^+\tilde{\sigma}_1^-) + \frac{1}{4} g_2 (c_1\tilde{\sigma}_2^+ + c_1^+\tilde{\sigma}_2^-). \quad (10)$$

On the other hand, when the condition $-\delta = 2\Omega + \sqrt{2}\nu$ is fulfilled, the effective Hamiltonian can be written as

$$H_{\text{eff}} = -\frac{1}{4} g_1 (c_2\tilde{\sigma}_1^+ + c_2^+\tilde{\sigma}_1^-) - \frac{1}{4} g_2 (c_2\tilde{\sigma}_2^+ + c_2^+\tilde{\sigma}_2^-). \quad (11)$$

The Hamiltonians described by Eqs. (10) and (11) are equal to that of the Tavis–Cumming model[22] with different coupling constants. Some tasks, such as entangling the two atoms and transferring the quantum information, can be accomplished using such a system.

Assume the detunings of the two coupling fields are different. When the conditions that $\nu, \Omega_j, |\delta_j| \gg g_j, \left|2\sqrt{2}\nu - 2\Omega_1\right| \gg g_1$, $-\delta_1 = 2\Omega_1 - \sqrt{2}\nu$ and $-\delta_2 = 2\Omega_2 + \sqrt{2}\nu$ are fulfilled, the effective Hamiltonian can be written as

$$H_{\text{eff}} = \frac{1}{4} g_1 (c_1\tilde{\sigma}_1^+ + c_1^+\tilde{\sigma}_1^-) - \frac{1}{4} g_2 (c_2\tilde{\sigma}_2^+ + c_2^+\tilde{\sigma}_2^-). \quad (12)$$

On the other hand, when the conditions $\nu, \Omega_j, |\delta_j| \gg g_j, \left|2\sqrt{2}\nu - 2\Omega_2\right| \gg g_2$, $-\delta_1 = 2\Omega_1 + \sqrt{2}\nu$ and $-\delta_2 = 2\Omega_2 - \sqrt{2}\nu$ are fulfilled, the effective Hamiltonian can be written as

$$H_{\text{eff}} = -\frac{1}{4} g_1 (c_2\tilde{\sigma}_1^+ + c_2^+\tilde{\sigma}_1^-) + \frac{1}{4} g_2 (c_1\tilde{\sigma}_2^+ + c_1^+\tilde{\sigma}_2^-). \quad (13)$$

The Hamiltonians described by Eqs. (12) and (13) are equal to that of the double Jaynes–Cummings model[23] but with different coupling constants. In the following, we present the realization of the remote quantum swap gate.

Assume that $g_1 = g_2 = g$. First, the evolution of the system is governed by the Hamiltonian described by Eq. (12) until $gt_1 = 2\pi$. At this moment, adjust the classical field to ensure that the evolution of the system is governed by the Hamiltonian described by Eq. (13) until $gt_2 = 6\pi$, and choose the right Rabi frequency to satisfy $\Omega t_1 = 2k\pi$ (k is an integer). Then the following can be obtained:

$$
\begin{aligned}
&|+\rangle|+\rangle|0\rangle|0\rangle \xrightarrow{gt_1=2\pi} |-\rangle|-\rangle|1\rangle|1\rangle \xrightarrow{gt_1=6\pi} |+\rangle|+\rangle|0\rangle|0\rangle\,;\\
&|+\rangle|-\rangle|0\rangle|0\rangle \xrightarrow{gt_1=2\pi} -\mathrm{i}|-\rangle|-\rangle|1\rangle|0\rangle \xrightarrow{gt_1=6\pi} |-\rangle|+\rangle|0\rangle|0\rangle\,;\\
&|-\rangle|+\rangle|0\rangle|0\rangle \xrightarrow{gt_1=2\pi} \mathrm{i}|-\rangle|-\rangle|0\rangle|1\rangle \xrightarrow{gt_1=6\pi} |+\rangle|-\rangle|0\rangle|0\rangle\,;\\
&|-\rangle|-\rangle|0\rangle|0\rangle \xrightarrow{gt_1=2\pi} |-\rangle|-\rangle|0\rangle|0\rangle \xrightarrow{gt_1=6\pi} |-\rangle|-\rangle|0\rangle|0\rangle\,.
\end{aligned}
\tag{14}
$$

In this way we get a distant two-qubit swap gate in the ideal case.

However, the system is inevitably influenced by quantum decoherence caused by the environments during the evolution. Spontaneous emission can be suppressed effectively by introducing the Λ-type atom to encode in the hyperfine ground levels (with negligible 'intrinsic' spontaneous emission rates). With the assistance of the classical field, the Λ-type atom, trapped in the cavity where the allowed transitions $|g_0\rangle \leftrightarrow |e\rangle$ and $|g_1\rangle \leftrightarrow |e\rangle$ are excited off-resonantly by laser fields and a cavity mode which is shown in Fig. 2(a), can be effectively equal to one two-level atom with only two ground levels.[18,24,25] Here we only consider the influence caused by the cavities and the optical fibre losses. In Fig. 2(b), the fidelity F of the quantum swap gate is plotted for the cases with $\Gamma_1 = \Gamma_2 = \Gamma_\mathrm{b} = \Gamma$, where Γ_1, Γ_2, Γ_b are the damping constants of cavities 1 and 2 and the optical fibre. For $\Gamma = 10^{-2}g$, the maximum fidelity $F = 0.941$ can be reached, and for $\Gamma = 10^{-3}g$, the maximum fidelity $F = 0.991$ can be reached.

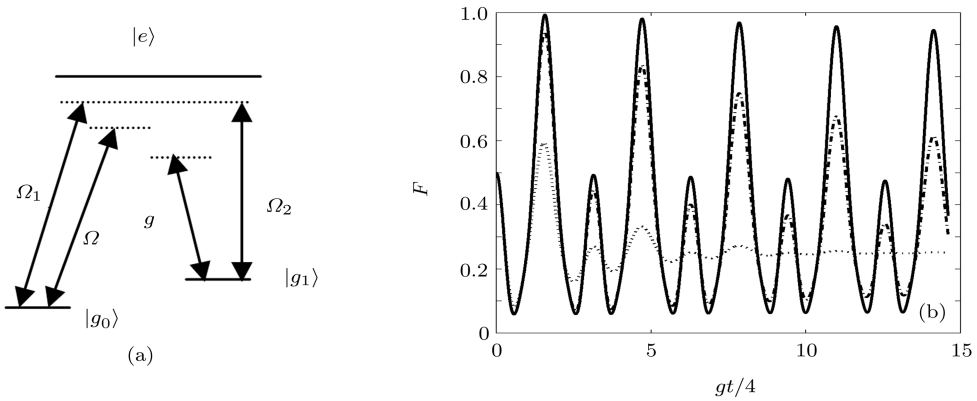


Fig. 2. (a) Λ-type atom driven by classical fields trapped in a single-mode cavity. (b) Fidelity of the swap gate for $\Gamma = 10^{-1}g$ (dotted line), $\Gamma = 10^{-2}g$ (dash doted line), $\Gamma = 10^{-3}g$ (solid line).

4. Summary

In summary, one scheme for implementation of the remote quantum controlled-phase and swap gates is presented. This scheme is based on a system composed of two two-level atoms, driven by the classical field, separately trapped in two single-mode cavities coupled by an optical fibre. In the scheme, the atoms are strongly driven by the classical fields and there are detunings of the coupling quantum fields. When the detuning is small, the quantum controlled-phase gate can be achieved by choosing an appropriate time and Rabi frequency. The dissipation of the cavities

and the fibre has few effects on the implementation of this gate because they are not excited throughout the whole procedure. When there is a large detuning, the effective Hamiltonian of this system can be equal to the Hamiltonian of two atoms trapped in one single-mode cavity or the sum of the Hamiltonians of two Jaynes–Cummings models. The quantum swap gate can be achieved by choosing the right time to adjust the Rabi frequencies. Spontaneous emission can be suppressed effectively by introducing a Λ-type atom to encode in the ground levels. The influence of the dissipation of the cavities and the optical fibre is analysed. For $\Gamma = 10^{-2}g$, the maximum fidelity $F = 0.941$ can be reached, and for $\Gamma = 10^{-3}g$, the maximum fidelity $F = 0.991$ can be reached. In conclusion, the remote quantum controlled-phase and swap gates can be achieved with the assistance of the classical fields when there is detuning between the atomic transition and the frequency of the field.

This scheme is based on the following assumptions: (i) strong coupling between the cavities and the fibre, (ii) the atoms are strongly resonantly driven by the classical fields with tunable Rabi frequency. These conditions can be fulfilled with the current experiments.[26] On the other hand, the three-level atom in Λ configuration placed inside an optical cavity with the assistance of classical fields can be equal to one two-level atom which only involves two ground states.[25,27] Taking advantage of this result, the spontaneous emission of the atom can be effectively suppressed. In recent experiments,[28,29] the condition $\Gamma \leq 10^{-2}g$ is achieved. With the progress of fibre–cavity coupling techniques, the quantum controlled-phase and swap gates can be achieved reliably with the assistance of classical fields with tunable Rabi frequency.

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