Deterministic and exact entanglement teleportation via the W state*

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Utilizing a three-particle W state, we come up with a protocol for the teleportation of an unknown two-particle entangled state. It is shown that the teleportation can be deterministically and exactly realized. Moreover, two-particle entanglement teleportation is generalized to a system consisting of many particles via a three-particle W state and a multi-particle W state, respectively. All unitary transformations performed by the receiver are given in a concise formula.

Keywords: quantum teleportation, W state, unitary transformations
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1. Introduction

Quantum teleportation,[1] which allows for the disembodied transport of a quantum state between two or more parties via quantum entanglement with the help of local operation and classical communication, is an important ingredient in distributed quantum networks. It also serves as an elementary operation in quantum computers and many quantum communication protocols. Since the seminal work of Bennett *et al.*[1] scientists have experimentally made dramatic progress with quantum teleportation in different quantum systems,[2] inside[3] and outside[4] laboratory conditions. In theory, the teleportation has been generalized to many cases.[5−14]

Multi-particle entangled states, namely the Greenberger–Horne–Zeibinger (GHZ) states,[15] the W states[7,12,16,17] and the cluster states[18,19] have been exploited for carrying out teleportation. Recently, in Ref. [17] a kind of W state $|W\rangle = (|100\rangle + |010\rangle + \sqrt{2}\,|001\rangle)/2$ was shown and utilized to implement the teleportation of a particle state $\alpha\,|0\rangle + \beta\,|1\rangle$. In the protocol,[17] the sender needs to carry out the measurement composed of the following three-qubit W states:

$$\begin{aligned} |\eta^{\pm}\rangle &= (|010\rangle + |001\rangle \pm \sqrt{2}\,|100\rangle)/2, \\ |\xi^{\pm}\rangle &= (|110\rangle + |101\rangle \pm \sqrt{2}\,|000\rangle)/2. \end{aligned} \tag{1}$$

It is known that the W state is much more robust than the GHZ states, as demonstrated in the recent experiment.[20] Especially, due to the importance of the W states introduced by Ref. [17], Zheng[21] utilized the W state $|\eta^{+}\rangle$ in expression (1) to propose a procedure for splitting quantum information and illustrated it in the ion-trap system.

Generally, it is not possible to deterministically and exactly teleport a two-particle state by using a three-qubit W state as the quantum channel.[7,12] However, it is necessary to further research this problem. As especially relevant to this paper, using the Bell state measurement, we propose a deterministic

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and exact two-particle entanglement teleportation via a three-particle W state. It will be shown that the deterministic and exact teleportation of a multi-particle partially entangled state can be completed by using a three-particle W state and a multi-particle W state as quantum channel, respectively. In addition, we summarize all the unitary transformations performed by the receiver in a concise formula.

The rest of the present paper is organized as follows. In Subsection 2.1, we propose a protocol for deterministic and exact teleportation of the two-particle entanglement. Additionally, two-particle entanglement teleportation is generalized to the system consisting of many particles via a three-particle W state and a multi-particle W state in Subsections 2.2 and 2.3, respectively. All unitary transformations performed by the receiver are depicted in a concise formula. A brief discussion and the concluding summary are given in Section 3.

2. Protocol for deterministic and exact teleportation

Although due attention has been paid to the generation of entanglement, quantum entanglement has to be transported among spatially separated spots in some applications.[22] In this section, we propose a protocol for the perfect entanglement teleportation of the two-particle entangled state via a three-particle W state. Multi-particle entanglement teleportation is also investigated via a three-particle W state and a multi-particle W state, separately.

2.1. Teleportation of two-particle entanglement

An unknown two-particle partially entangled state is expressed as

$$|\phi\rangle_{45} = x\,|00\rangle_{45} + y\,|11\rangle_{45}\,, \tag{2}$$

where x and y are non-negative real numbers satisfying $x^2 + y^2 = 1$.

A three-particle W state utilized as a quantum channel is written as

$$|W\rangle_{123} = \frac{1}{2}(|100\rangle + |010\rangle + \sqrt{2}\,|001\rangle)_{123}. \tag{3}$$

Alice is in possession of particles 3, 4, 5, while particle 1 and 2 belong to the receiver Bob. At the beginning of the protocol, the state of the whole system composed of particles 4, 5 and quantum channel is $|\Phi\rangle = |\phi\rangle_{45} \otimes |W\rangle_{123}$. It can be rewritten in the following way by regrouping terms:

$$\begin{aligned}|\Phi\rangle = \frac{1}{2\sqrt{2}}[&|\Phi^{\pm}\rangle_{34}(x|100\rangle_{125} + x|010\rangle_{125} \pm \sqrt{2}y|001\rangle_{125}) \\ &+ |\Psi^{\pm}\rangle_{34}(y|101\rangle_{125} + y|011\rangle_{125} \pm \sqrt{2}x|000\rangle_{125})],\end{aligned} \tag{4}$$

where $|\Phi^{\pm}\rangle = (|00\rangle \pm |11\rangle)/\sqrt{2}$, and $|\Psi^{\pm}\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$.

To begin with, Alice uses the entanglement swapping[23,24] to perform the joint measurement on particles 3 and 4 in the Bell basis $\{|\Phi^+\rangle, |\Phi^-\rangle, |\Psi^+\rangle, |\Psi^-\rangle\}$. For convenience, Alice's measurement result $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, $|\Psi^-\rangle$ is expressed as classical bit strings M_3M_4=00, 10, 01, 11, respectively. The following states can be obtained:

$$\begin{aligned}\langle\Phi^{\pm}|_{34}\Phi\rangle = \frac{1}{4}[&(x|10\rangle + x|01\rangle + (-1)^{M_3}\sqrt{2}y|00\rangle)_{12} \otimes |+\rangle_5 \\ &+ (x|10\rangle + x|01\rangle - (-1)^{M_3}\sqrt{2}y|00\rangle)_{12} \otimes |-\rangle_5],\end{aligned} \tag{5}$$

$$\begin{aligned}\langle\Psi^{\pm}|_{34}\Phi\rangle = \frac{1}{4}[&(y|10\rangle + y|01\rangle + (-1)^{M_3}\sqrt{2}x|00\rangle)_{12} \otimes |+\rangle_5 \\ &+ (-y|10\rangle - y|01\rangle + (-1)^{M_3}\sqrt{2}x|00\rangle)_{12} \otimes |-\rangle_5].\end{aligned} \tag{6}$$

Then Alice measures qubit 5 in the basis $\{|+\rangle, |-\rangle\}$. The measurement result $|+\rangle$, $|-\rangle$ is denoted as classical bit $i_5 = 0$, $i_5 = 1$, respectively.

To make Bob know the collapsed state with certainty, Alice communicates the measurement outcomes to Bob, including measurement outcomes on particles 5 and Bell-state measurement results on particles 3 and 4.

Bob performs the following unitary transformation $\boldsymbol{E}$ on particles 1 and 2:

$$\boldsymbol{E}=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{-\sqrt{2}}{2} & 0 \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{7}$$

The two-particle at the receiving ends collapses into one of the following states: $(x|10\rangle \pm (-1)^{M_3}y\ |00\rangle)_{12}$, $\pm y|10\rangle + (-1)^{M_3}x\ |00\rangle$. Bob performs a CNOT operation with particle 1 as the control qubit and qubit 2 as the target qubit.

Afterwards, according to Alice's different measurement outcomes, Bob uses the following transformation, which has been summarized into a concise formula, on particles 1 and 2 to recover the transmitted state:

$$Z_1^{i_5}Z_1^{M_3}X_1^{M_4}X_2^{M_4}X_1X_2, \tag{8}$$

where $\sigma^0=\boldsymbol{I}$, $\boldsymbol{\sigma}^1=\boldsymbol{\sigma}$, $\boldsymbol{\sigma}$ is the Pauli matrix $\boldsymbol{X}$ or $\boldsymbol{Z}$, and $\boldsymbol{I}$ is an identity matrix.

2.2. Teleportation of multi-particle entanglement via a three-particle W state

In the past few years, many researchers have extensively studied GHZ entangled states due to their important role in quantum information processing and communication. Many theoretical proposals have appeared for the generation of multiqubit GHZ states. Recently, it has been reported that an up to six-photon GHZ state has been created in experiment.[25] In this section, Alice wants to teleport an unknown N-particle entangled GHZ state to the distant receiver Bob.

$$|\phi\rangle=(x|0\rangle^{\otimes N}+y|1\rangle^{\otimes N})_{45\cdots(N+3)}, \tag{9}$$

where x and y are non-negative real numbers satisfying $x^2+y^2=1$, $N\geq 3$.

The quantum channel is still in the W state in expression (3).

Alice possesses particles 3, 4, 5, ..., $(N+3)$, while particles 1 and 2 belong to Bob. The state of the whole system is $|\Phi\rangle=|\phi\rangle\otimes|W\rangle_{123}$.

Firstly, Alice performs a joint measurement on particles 3 and 4 in the Bell basis. Here, the measurement result $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, $|\Psi^-\rangle$ is still expressed as $M_3M_4=00$, 10, 01, 11. As a result, we can obtain the following states at random:

$$\begin{aligned}\langle\Phi^{\pm}|_{34}\Phi\rangle=&[x(|10\rangle+|01\rangle)_{12}\otimes|0\rangle^{\otimes(N-1)}\\&+(-1)^{M_3}\sqrt{2}y|00\rangle_{12}\otimes|1\rangle^{\otimes(N-1)}]/2\sqrt{2},\end{aligned} \tag{10}$$

$$\begin{aligned}\langle\Psi^{\pm}|_{34}\Phi\rangle=&[y(|10\rangle+|01\rangle)_{12}\otimes|1\rangle^{\otimes(N-1)}\\&+(-1)^{M_3}\sqrt{2}x|00\rangle_{12}\otimes|0\rangle^{\otimes(N-1)}]/2\sqrt{2}.\end{aligned} \tag{11}$$

Then, Alice measures qubits 5, 6, ..., $(N+3)$ in the basis $\{\ |+\rangle,|-\rangle\ \}$, separately. For the j-th qubit, we denote the measurement result $|+\rangle$, $|-\rangle$ as classical bit $i_j=0$, $i_j=1$. Alice tells Bob the measurement results including the measurement results on particles 5, 6, ..., $(N+3)$ and Bell-state measurement results on particles 3 and 4.

Bob performs the unitary transformation E in expression (7) on particles 1 and 2. According to the number of the classical bits sent by Alice, Bob introduces $N-2$ additional qubits $|0\rangle_{(N+4)}$, $|0\rangle_{(N+5)}$, ..., $|0\rangle_{(2N+1)}$. Then, he performs a set of CNOT operations with particle 1 as the control qubit, particle 2 and every additional qubit as the target qubit. Afterwards, we can obtain

$$\frac{1}{\sqrt{2^{N+1}}}[x(|111\ldots1\rangle+(-1)^{M_3}(-1)^{\sum_{j=5}^{N+3}i_j}y|000\ldots0\rangle]_{12(N+4)(2N+1)}, \tag{12}$$

$$\frac{1}{\sqrt{2^{N+1}}}[y(-1)^{\sum_{j=5}^{N+3}i_j}(|111\ldots1\rangle+(-1)^{M_3}x|000\ldots0\rangle]_{12(N+4)(2N+1)}. \tag{13}$$

Depending on Alice's measurement results, Bob applies the following transformation:

$$Z_1^{\left(\sum_{j=5}^{N+3}i_j\right)\bmod 2}Z_1^{M_3}(X_1X_2X_{N+4}X_{N+5}\ldots X_{2N+1})^{(M_4+1)\bmod 2}. \tag{14}$$

As a result, particles $1,2,(N+4)$, $(N+5)$, ..., $(2N+1)$ successfully collapse into the state $x|0\rangle^{\otimes(N+1)}+y|1\rangle^{\otimes(N+1)}$, which is just a replica of Alice's original state.

2.3. Teleportation of multi-particle entanglement via a multi-particle W state

Of course, the communicators can use the following $(N+1)$-particle W state, which is a generalization of the state in expression (3), as the quantum channel to teleport an unknown N-particle entangled GHZ state. In such a case, the additional qubits are not required to be introduced by the receiver Bob.

$$|W\rangle = \frac{1}{\sqrt{2N}}[(|10\ldots 00\rangle + |01\ldots 00\rangle + \ldots |00\ldots 10\rangle) + \sqrt{N}|00\ldots 01\rangle]_{123\ldots(N+1)}. \tag{15}$$

The N-particle entangled state that will be teleported can be expressed as

$$|\phi\rangle = (x|0\rangle^{\otimes N} + y|1\rangle^{\otimes N})_{(N+2)(N+3)\ldots(2N+1)}. \tag{16}$$

Alice possesses particles $(N+1)$, $(N+2)$, $\ldots$, $(2N+1)$, while particles 1, 2, $\ldots$, N belong to Bob. The state of the whole system is $|\Phi\rangle = |W\rangle_{12\ldots(N+1)} \otimes |\phi\rangle_{(N+2)(N+3)\ldots(2N+1)}$.

Alice performs the joint measurement on particles $(N+1)$ and $(N+2)$ in the Bell basis. The measurement result $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Psi^+\rangle$, $|\Psi^-\rangle$ is still expressed as $M_{N+1}M_{N+2}$=00, 10, 01, 11, respectively. Then, Alice measures every qubit $(N+3)(N+4)\ldots(2N+1)$ in the basis $\{|+\rangle, |-\rangle\}$, separately. Alice communicates all the measurement results to Bob. Bob performs the unitary transformation U on particles $1, 2, 3, \ldots, N$. He obtains the following state at random:

$$\begin{aligned}
&\frac{1}{\sqrt{2^{N+1}N}}\left[x(|10\ldots 0\rangle + |01\ldots 0\rangle + \ldots + |00\ldots 1\rangle) + (-1)^{M_{N+1}}(-1)^{\sum_{j=N+3}^{2N+1} i_j}\sqrt{N}y|000\ldots 0\rangle\right]_{12\ldots N} \\
&\xrightarrow{U} \frac{1}{\sqrt{2^{N+1}}}\left[x|10\ldots 0\rangle + (-1)^{M_{N+1}}(-1)^{\sum_{j=N+3}^{2N+1} i_j} y|000\ldots 0\rangle\right]_{12\ldots N} \qquad (17) \\
&\frac{1}{\sqrt{2^{N+1}}}\left[y(-1)^{\sum_{j=N+3}^{2N+1} i_j}(|10\ldots 0\rangle + |01\ldots 0\rangle + \ldots + |00\ldots 1\rangle) + (-1)^{M_{N+1}}\sqrt{N}x|000\ldots 0\rangle\right]_{12\ldots N} \\
&\xrightarrow{U} \frac{1}{\sqrt{2^{N+1}}}\left[y(-1)^{\sum_{j=N+3}^{2N+1} i_j}|10\ldots 0\rangle + (-1)^{M_{N+1}}x|000\ldots 0\rangle\right]_{12\ldots N}. \qquad (18)
\end{aligned}$$

Generally, the unitary transformation U on particles $1, 2, 3, \ldots, N$ is an operation about $2^N \times 2^N$. For example, when N=3, the transformation about $2^3 \times 2^3$ is

$$\boldsymbol{E} = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1/\sqrt{3} & -1/\sqrt{3} & 1/\sqrt{3} & 0 & 0 & 0 & 0 \\
0 & 1/\sqrt{3} & 0 & -1/\sqrt{3} & -1/\sqrt{3} & 0 & 0 & 0 \\
0 & 0 & 1/\sqrt{3} & 1/\sqrt{3} & -1/\sqrt{3} & 0 & 0 & 0 \\
0 & 1/\sqrt{3} & 1/\sqrt{3} & 0 & 1/\sqrt{3} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}. \tag{19}$$

Then, Bob performs a set of CNOT operations with particle 1 as the control qubit and particles $2, 3, \ldots, N$ as the target qubit. Finally, Bob uses the following transformation to recover the transmitted state:

$$Z_1^{\left(\sum_{j=N+3}^{2N+1} i_j\right) \bmod 2} Z_1^{M_{N+1}}(X_1X_2\ldots X_N)^{(M_{N+2}+1) \bmod 2}. \tag{20}$$

3. Conclusions

Using the W state as a quantum channel, one can usually obtain the approximate teleportation deterministically (with imperfect fidelity but unit success probability)[16] or exact teleportation probabilistically (with unit fidelity but a certain probability of failure).[7,12] One of the features in our protocol is that an unknown two-particle entangled state can be deterministically and exactly teleported via a three-particle W state.

On the other hand, our paper also has the following features by comparison with that given in Ref. [17]. (i) Entanglement teleportation transfers not only the amount of entanglement but also the entanglement structure. There is a higher capability of transmission in our protocol, in which the two-particle state $|\phi\rangle = x|00\rangle + y|11\rangle$ can be perfectly teleported via the same quantum channel. (ii) In our protocol, the sender performs a Bell measurement on two photons, which is easier to realize than the three-qubit measurement in expression (1) used in Ref. [17]. Moreover, based on some techniques[26−28] the Bell measurement is experimentally realized. The disadvantage is that the receiver needs to additionally perform a unitary transformation in expression (7) and a CNOT operation. However, the quantum circuits of the unitary transformation in expression (7) and the quantum channel W state can be constructed by combining a set of one-bit quantum gates with two-bit CNOT gates.[29] Nowadays, a single-qubit gate has been easily implemented in experiment. The quantum CNOT gate has been demonstrated successfully.[30] Hence, the advantages of our paper outweigh the disadvantage in essence. (iii) In many existing protocols,[7,31] the unitary transformations performed by the receiver are complex tables. One of the unique features in this paper is that all unitary transformations performed by the receiver are summarized into a concise formula. (iv) We generalize two-particle entanglement teleportation to a system composed of multi-particle entanglement via a three-particle W state or a multi-particle W state, separately.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 *Nature* **390** 575

[3] Marcikic I, de Riedmatten H, Tittel W, Zbinden H and Gisin N 2003 *Nature* **421** 509

[4] Ursin R, Jennewein T, Aspelmeyer M, Kaltenbaek R, Lindenthal M, Walther P and Zeilinger A 2004 *Nature* **430** 849

[5] Stenholm S and Bardroff P J 1998 *Phys. Rev.* A **58** 4373

[6] Braunstein S L and Kimble H J 1998 *Phys. Rev. Lett.* **80** 869

[7] Cao Z L and Song W 2005 *Physica* A **347** 177

[8] Chen X B, Du J Z, Wen Q Y and Zhu F C 2008 *Chin. Phys.* B **17** 771

[9] Chen X B, Wen Q Y and Zhu F C 2006 *Chin. Phys.* **15** 2240

[10] Chen X B, Wen Q Y and Zhu F C 2007 *Int. J. Quant. Inform.* **5** 717

[11] Bae J, Jin J, Kim J, Yoon C and Kwon Y 2005 *Chaos, Solitons and Fractals* **24** 1047

[12] Chen X B, Zhang N, Lin S, Wen Q Y and Zhu F C 2008 *Opt. Commun.* **281** 2331

[13] Vidal G, Jonathan D and Nielsen M A 2000 *Phys. Rev.* A **62** 012304

[14] Chen X B, Wen Q Y, Xu G, Yang Y X and Zhu F C 2009 *Phys. Rev.* A **79** 036301

[15] Karlsson A and Bourennane M 1998 *Phys. Rev.* A **58** 4394

[16] Joo J, Park Y J, Oh S and Kim J 2003 *New J. Phys.* **5** 136

[17] Agrawal P and Pati A 2006 *Phys. Rev.* A **74** 062320

[18] Raussendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188

[19] Briegel H J and Raussendorf R 2001 *Phys. Rev. Lett.* **86** 910

[20] Roos C F, Riebe M, Haffner H, Hansel W, Benhelm J, Lancaster G P T, Becher C, Schmidt-Kaler F and Blatt R 2004 *Science* **304** 1478

[21] Zheng S B 2006 *Phys. Rev.* A **74** 054303

[22] Gottesman D and Chuang I L 1999 *Nature* **402** 390

[23] Ukowski M, Zeilinger A, Horne M A and Ekert A K 1993 *Phys. Rev. Lett.* **71** 4287

[24] Wang X B, Shi B S, Tomita A and Matsumoto K 2004 *Phys. Rev.* A **69** 014303

[25] Lu C Y, Zhou X Q, Guhne O, Gao W B, Zhang J, Yuan Z S, Goebel A, Yang T and Pan J W 2007 *Nature Physics* **3** 91

[26] Barrett S D, Kok P, Nemoto K, Beausoleil R G, Munro W J and Spiller T P 2005 *Phys. Rev.* A **71** 060302

[27] Walborn S P, Padua S and Monken C H 2003 *Phys. Rev.* A **68** 042313

[28] Kim Y H 2003 *Phys. Rev.* A **67** 040301

[29] Chen X B, Wen Q Y, Guo F Z, Sun Y, Xu G and Zhu F C 2008 *Int. J. Quant. Inform.* **6** 899

[30] O'Brien J L, Pryde G J, White A G, Ralph T C and Branning D 2003 *Nature* **426** 264

[31] Fang J X, Lin Y S, Zhu S Q and Chen X F 2003 *Phys. Rev.* A **67** 014305