Evolutionary prisoner's dilemma on Newman–Watts social networks with an asymmetric payoff distribution mechanism*

Du Wen-Bo(杜文博)[a)†], Cao Xian-Bin(曹先彬)[a)‡],
Yang Han-Xin(杨涵新)[b)], and Hu Mao-Bin(胡茂彬)[c)§]

[a)] *School of Computer Science and Technology, University of Science and Technology of China, Hefei* 230026, *China*
[b)] *Department of Modern Physics, University of Science and Technology of China, Hefei* 230026, *China*
[c)] *School of Engineering Science, University of Science and Technology of China, Hefei* 230026, *China*

(Received 18 May 2009; revised manuscript received 22 June 2009)

In this paper, we introduce an asymmetric payoff distribution mechanism into the evolutionary prisoner's dilemma game (PDG) on Newman–Watts social networks, and study its effects on the evolution of cooperation. The asymmetric payoff distribution mechanism can be adjusted by the parameter α: if $\alpha > 0$, the rich will exploit the poor to get richer; if $\alpha < 0$, the rich are forced to offer part of their income to the poor. Numerical results show that the cooperator frequency monotonously increases with α and is remarkably promoted when $\alpha > 0$. The effects of updating order and self-interaction are also investigated. The co-action of random updating and self-interaction can induce the highest cooperation level. Moreover, we employ the Gini coefficient to investigate the effect of asymmetric payoff distribution on the the system's wealth distribution. This work may be helpful for understanding cooperative behaviour and wealth inequality in society.

Keywords: prisoner's dilemma game, cooperation, complex social networks

PACC: 0250, 0175, 0270

1. Introduction

In cellular interactions and human behaviour, the ubiquitous altruism actions apparently contradict the selfish Darwinian evolution theory. Understanding the condition and mechanism for the emergence and persistence of cooperation becomes an interesting research area. The evolutionary game theory, together with its appropriate extensions, provides a suitable tool for this problem.

Since the pioneering work of Axelrod,[1] much attention has been given to the prisoner's dilemma game (PDG).[1−4] In the standard PDG, two agents can synchronously choose to cooperate or to defect, without knowing the opponent's strategy. They get payoffs based on a 2×2 payoff matrix. The agents both receive reward R while they have mutual cooperation and punishment P for mutual defection. If one cooperator meets a defector, the cooperator (C) gets the lowest payoff S (sucker's payoff), while the defector (D) gains the highest payoff T (the temptation to defect). The elements of the payoff matrix must satisfy the conditions: $T > R > P > S$ and $2R > (T + S)$. The original PDG has restrictions in understanding the emergence of cooperation in that the game rules will yield an unresolvable dilemma for 'rational' agents who want to maximize their own income. In a single round of the PDG, it is best to defect no matter what strategy the opponent chooses. Consequently, the evolutionary PDG is proposed as an alternative to the standard PDG,[1] where the agents' wealth come from iterated PDG played with several other agents. In each round of PDG, the agents can choose a 'better' strategy with the knowledge of the co-players' decision history. However, the unstable cooperative behaviour obtained by the evolutionary PDG is still opposite to the observations in real world. This disagreement stimulates a variety of extensions of PDG to explain the emergence and persistence of cooperation.

Network theory provides a natural framework to

*Project supported by the Major State Basic Research Development Program of China (Grant No. 2004CB318109), Program for New Century Excellent Talents in University of China (Grant No. NCET-07-0787) and the National Natural Science Foundation of China (Grant No. 70601026).
†E-mail: wenbodu@mail.ustc.edu.cn
‡Corresponding author. E-mail: xbcao@ustc.edu.cn
§E-mail: humaobin@ustc.edu.cn
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represent social structure: individuals can be denoted by agents occupying the vertexes while contacts between agents can be denoted by edges.[5,6] In the last decades, much attention has been given to the PDG on several population structures.[7] Recently, Nowak and May showed that PDG on a simple spatial structure can produce stable cooperation and amazing spatial chaos.[8] Inspired by the spatial game, much attention has been given to the the evolutionary game on different social networks,[9−20] especially after the complex network theory was proposed at the end of the last century. Real social systems are not regular or complete networks, but are often associated with complex networks, such as the well-known small-world network.[21−24] Much research thus focuses on the interplay between cooperative behaviour and the underlying population topology.[25−38] Some realistic phenomena are also introduced into evolutionary games, such as reward and punishment,[39] variation in strategy transfer capability,[40] and memory effects.[41]

However, previous works usually consider that agents are equal in payoff distribution: they get the same payoff or with a payoff noise[42] while they have mutual cooperation. This rule obviously ignores the effect of individual heterogeneity and contradicts the ubiquitous unfair negotiations in social and economical systems. For example, in commercial activities, some powerful corporations often use their power to force their weak co-players to make an unfair deal. Meanwhile, the rich are often compelled to share part of their income to support the poor (such as the high tax-rate for rich people). In this paper, we introduce an asymmetric payoff distribution mechanism into evolutionary games to represent these phenomena, and investigate its effect on cooperative behaviour.

The paper is organized as follows. In the next section, we describe the models of the evolutionary prisoner's dilemma games and the asymmetric payoff distribution mechanism. The simulation results and discussions are given in Section 3. The paper is concluded in the last section.

2. The model

In this paper, we adopt the Newman–Watts (NW) small-world network[22] to generate the population structure since real social networks consist of abundant local interactions and sufficient shortcuts. In this model, the network is constructed by randomly adding links to the basis of a bounded one-dimensional lattice (ring). As a reform of the well-known Watts–Strogatz (WS) small-world network,[21] the NW network is proved to be suitable to characterize the social structure.

We firstly re-scale the PDG payoff matrix in accordance with common practices: $T = b$, $R = 1$ and $P = S = 0$ (Eq. (1)). The new matrix contains only one free parameter b, which represents the temptation to defect.[8] Generally, b is set to $1 \leq b \leq 2$. If both agents choose to cooperate, they will get the same payoff regardless of their other information. This disagrees with the real situation. Therefore, we introduce an asymmetric payoff distribution mechanism to the PDG model (Eq. (2)).

$$\begin{pmatrix} (1,1) & (0,b) \\ (b,0) & (0,0) \end{pmatrix}, \tag{1}$$

$$\begin{pmatrix} (R_i', R_j') & (0,b) \\ (b,0) & (0,0) \end{pmatrix} \tag{2}$$

with

$$(R_i', R_j') = \begin{cases} (b, 2-b), \text{ if } U_i \geq b, \\ (U_i, U_j), \text{ if } U_i < b \text{ and } U_j < b, \\ (2-b, b), \text{ if } U_j \geq b, \end{cases} \tag{3}$$

and

$$\begin{cases} U_i = 2 * \dfrac{k_i^\alpha}{k_i^\alpha + k_j^\alpha}, \\ U_j = 2 * \dfrac{k_j^\alpha}{k_i^\alpha + k_j^\alpha}. \end{cases} \tag{4}$$

In Eq. (4), k_i and k_j (assume $k_i > k_j$) are the degrees of agent i (denoted as the strong agent) and j (denoted as the weak agent), respectively. One can see that the mechanism is adjusted by only one parameter α. The total payoff for mutual cooperation is still 2, but it is not always equally distributed. If $\alpha = 0$, the model degenerates to the standard PDG. If $\alpha > 0$, the rich exploit the poor to get richer. If $\alpha < 0$, the rich provide part of their income to the poor. Most importantly, the payoff rank $T > R > P > S$ is strictly satisfied for all values of α.

In each step, all pairs of directly connected agents i and j engage in a single round of game. The total payoff of agent i is stored as P_i and the accumulative payoff (wealth) of player i since the beginning of the simulation is stored as W_i. The strategy updating can be carried out synchronously or randomly. When agent i is updated, it randomly selects a neighbour j

from all its k_i neighbours, and then adopts agent j's strategy with a probability:[10]

$$H_{i\to j} = \frac{1}{1+\exp[(P_i - P_j)/\kappa]}. \quad (5)$$

Here κ characterizes the environmental noise, representing bounded rationality, individual trials, errors in decision, etc. The effect of noise κ on the cooperator frequency has been well studied in Ref. [43]. For simplicity, we set $\kappa = 0.1$ as in other studies.

3. Simulation results and discussions

The simulations are carried out on Newman–Watts networks with net size $N = 1000$, ring degree $m = 4$ and adding-link number $E = 500$. Initially, cooperators and defectors are equally distributed among the population. The equilibrium frequency of cooperators is obtained by averaging the last 4000 generations after a transient time of 10^4 generations. Each datum is obtained by averaging 50 individual runs.

The key index to characterize the cooperative behaviour of the system is the density of cooperators defined as $N_C/(N_C + N_D)$ where N_C (N_D) is the number of cooperators (defectors). We firstly investigate cooperator frequency as a function of the tunable parameter α for different payoff parameter b under synchronous updating mechanism. As shown in Fig. 1,

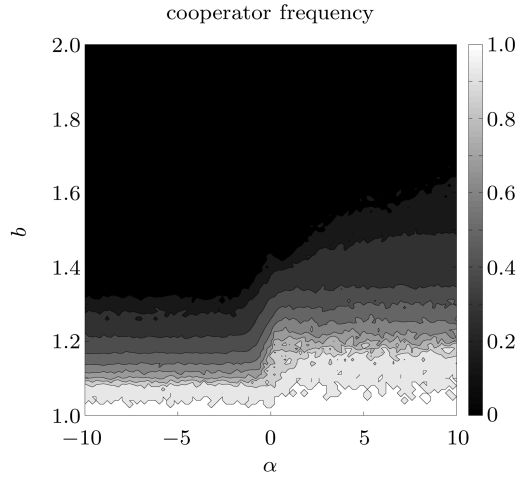


Fig. 1. Cooperator frequency in the $b-\alpha$ space with synchronous updating.

the cooperator frequency has a strong dependence on α. Compared with the standard PDG model ($\alpha = 0$), the cooperation level is remarkably promoted when $\alpha > 0$ whereas it is slightly restricted when $\alpha < 0$. This result exhibits a surprising and interesting phenomenon: the asymmetric payoff distribution mechanism of $\alpha > 0$ harms the interest of the majority but it promotes the cooperative behaviour at the same time. In contrast, the situation of $\alpha < 0$, which should be advocated by the numerous weak agents, yet restrains the cooperation.

Here we give a heuristic explanation for the phenomenon. Though the NW network is quite homogeneous in node degree, there are still some agents with higher degree than their neighbours. When $\alpha > 0$, a strong cooperator can obtain more wealth by exploiting its weak neighbours, so it is more likely to keep cooperation in the next step. Simultaneously, its exploited neighbours become poorer. According to the strategy updating rule (Eq. (5)), the weak agents have a larger probability of being infected by the strong C agent. Thus the cooperator frequency is promoted. That is, the weak agents just follow the strong agent's strategy like sheep regardless of whether the strategy really benefits them or not. When $\alpha < 0$, the strong agents are forced to give part of their payoffs to the weak neighbours for mutual cooperation. They are more in favour of defection and the strong defectors will easily invade their neighbours. Consequently, the cooperation is greatly restricted. We also examine the effect of the random updating mechanism on cooperation. Here, the agents are given a random order and their strategies are updated one by one according to this order. As shown in Fig. 2, compared with synchronous updating, the cooperation is greatly promoted under the random updating rule.

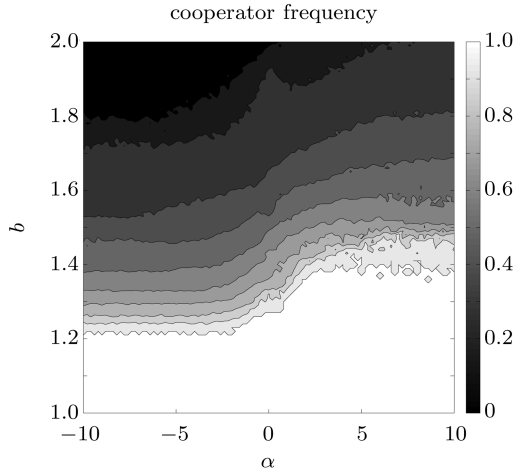


Fig. 2. Cooperator frequency in the $b-\alpha$ space with random updating.

Besides, the agents can play a game with themselves (we call it the self-interaction mechanism).

That is, it can derive income against its own strategy as an opponent. Figure 3 shows the dependence of cooperator frequency on b and α under the self-interaction mechanism. Compared with Fig. 1, the cooperator frequency is remarkably promoted with the self-interaction mechanism. We further figure out the cooperator frequency with both random updating and self-interaction (Fig. 4). One can easily sort out the cooperation level: (random updating + self-interaction) > (synchronous updating + self-interaction) > (random updating + non-self-interaction) > (synchronous updating + non-self-interaction).

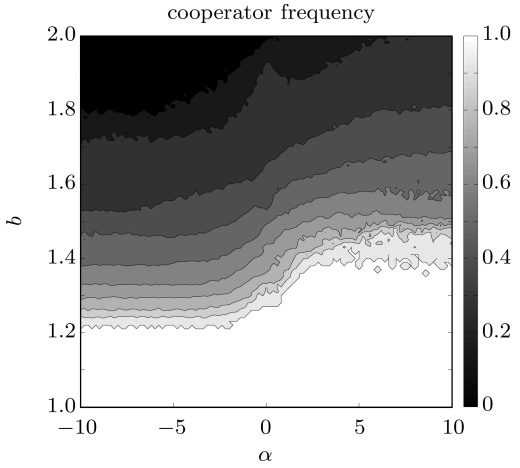


Fig. 3. Cooperator frequency in the $b-\alpha$ space with synchronous updating and self-interaction.

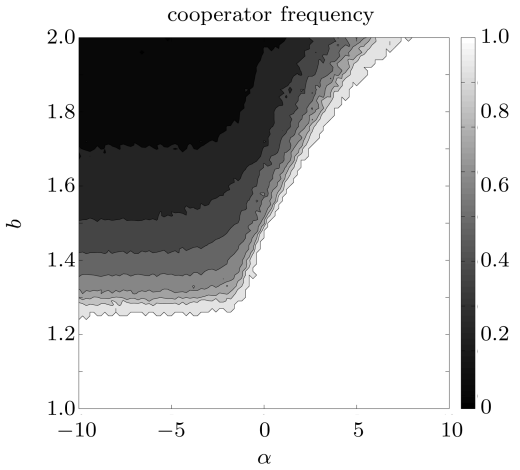


Fig. 4. Cooperator frequency in the $b-\alpha$ space with random updating and self-interaction.

Besides the cooperative behaviour, the game dynamics on networks is also suitable for characterizing the payoff and wealth accumulation behaviour in society.[44] Obviously, our asymmetric payoff distribution mechanism will affect the population's wealth distribution. Hence, we employ the Gini coefficient to study the effect of the asymmetric payoff distribution mechanism on wealth distribution.

The Gini coefficient is a measure of inequality of social wealth distribution. It is defined as a ratio with a value between 0 and 1: the numerator is the area between the Lorenz curve of the wealth distribution and the uniform distribution line; the denominator is the area under the uniform distribution line. Thus, a low Gini coefficient indicates more equal income or wealth distribution, while a high Gini coefficient indicates more unequal distribution. 0 corresponds to perfect equality (everyone having exactly the same income) and 1 corresponds to perfect inequality (where one person has all the wealth, while everyone else has zero income).

Figure 5 shows the dependence of Gini coefficient on α and b under random updating and non-self-interaction mechanisms. One can easily find that the Gini coefficient is greatly affected by α and b. When $\alpha < 0$, the strong agents have many neighbours to collect wealth but get little from each neighbour; the weak agents have few neighbours to collect wealth but get more from each neighbour. Consequently, the Gini coefficient is quite small when $\alpha < 0$ and the wealth distribution is more homogeneous. When $\alpha > 0$ and b is small, the cooperation level is high and every agent can collect wealth (little or a greater amount) from its C-neighbours. Since the NW network is quite homogeneous, the gap between the incomes of the rich and the poor is still not large. However, for large values of b, cooperators and defectors co-exist (see Fig. 2), which may induce more inequality of wealth distribution. We also examine the results under other three updating situations and the simulation results are similar.

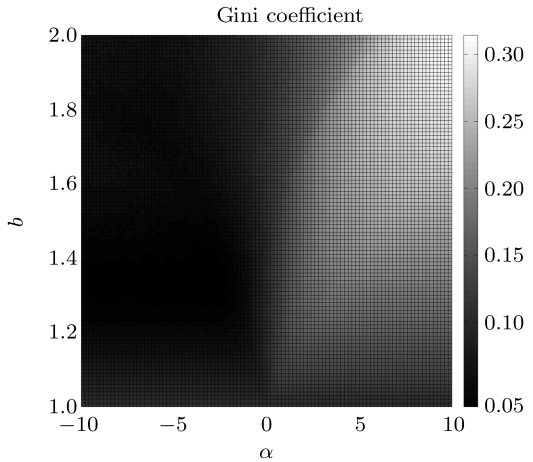


Fig. 5. The Gini coefficient vs b and α. The results are obtained with random updating and without self-interaction.

4. Conclusion

In summary, we introduce an asymmetric payoff distribution mechanism into the evolutionary prisoner's dilemma game on Newman–Watts social networks, and then investigate its influence on cooperative behaviour and wealth distribution in the population.

In our model, the asymmetric payoff distribution can be tuned by only one parameter α. If $\alpha > 0$, the rich get richer and the poor get poorer. If $\alpha < 0$, it is beneficial to the majority of poor agents. Our main result is very interesting: the cooperator frequency monotonously increases as α grows. This phenomenon may be due to people's blind-following behaviour, which is common in real society. Besides, we also investigated the effect of synchronous updating, random updating, self-interaction and non-self-interaction. It is found that a combination of random-updating and self-interaction can induce the highest cooperation level. Moreover, the asymmetric payoff distribution can also affect the system's Gini coefficient. Our work may be helpful in understanding cooperative behaviour and wealth inequality in society.

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