Vol 18 No 9, September 2009
1674-1056/2009/18(09)/3940-12

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Localized self-trapping in the two-dimensional discrete molecular lattice with the interaction between Frenkel excitons and phonons*

Xu Quan(徐 权)[a)b)†] and Tian Qiang(田 强)[b)]

[a)] *Department of Physics, Daqing Normal University, Daqing* 163712, *China*

[b)] *Department of Physics, Beijing Normal University, Beijing* 100875, *China*

(Received 19 March 2009; revised manuscript received 22 April 2009)

We investigate the interactions of lattice phonons with Frenkel exciton, which has a small radius in a two-dimensional discrete molecular lattice, by the virtue of the quasi-discreteness approximation and the method of multiple-scale, and obtain that the self-trapping can also appear in the two-dimensional discrete molecular lattice with harmonic and nonlinear potential. The excitons' effect on the molecular lattice does not distort it but only causes it to localize which enables it to react again through phonon coupling to trap the energy and prevent its dispersion.

Keywords: self-trapping, Frenkel exciton, two-dimensional discrete molecular lattice, phonon
PACC: 6320R, 6320P, 0547

1. Introduction

The first application of the localized theory of the nonlinear lattice to the biological field was in 1973 when Davydov proposed a novel mechanism for the localization and transport of vibrational energy in protein.[1] Referring to the simplified structure of an α-helix region of protein, this mechanism can be described classically as follows. Vibrational energy of the stretching oscillators localized on the helix acts to distort the structure of the helix through phonon coupling. The helical distortion reacts again through phonon coupling to trap the energy and prevents its dispersion. This effect is called self-localization or self-trapping.

In the following decades, the biological mechanism was studied in detail by Davydov and his colleagues, and significant progress was made in the studies of biological mechanism. The effects of the phonon coupling on the transport of a single exciton in the helical molecular chain were discussed in the study of the solitons in a molecular system,[2,3] and the conclusion was that the exciton transfers along this structure as a soliton. Shortly thereafter, many results were presented in, for example, biology and quantum mechanics,[4] quantum theory of muscular contraction,[5] solitons and energy transfer along protein molecules,[6] solitons bioenergetics and the mechanism of muscle contraction[7] etc. A more perfect soliton theory of a molecular system was found. This soliton is known as Davydov's soliton.

The polaron suggested by Landau[8] and studied by many scholars[9−12] has been used over the past half-century to denote a wide variety of excitations that are self-localized through interactions with lattice phonons. But Davydov's soliton is a exciton not a polaron, and it only corresponds to a polaron in the following fields. First, all of them are to large to be suitable for the continuum limit approximation. Besides, the self-localization arises from their interactions with the coustic modes of the lattice. Finally, their anharmonic energies are smaller than the phonon bandwidth, hence their couplings are weak.

From the 1970s to 1990s, the Davydov's solitons in a nonlinear molecular lattice were studied to the full. In 1982, Scott modified Davydov's model and added ten dipole–dipole coupling terms to represent the helical symmetry, and reached a conclusion that such solitons should appear under normal physiological conditions through the numerical method and the continuum limit approximation.[13] Lomdahl *et al* pointed out that the random displacements produced

*Project supported by the National Natural Science Foundation of China (Grant No 1057400), and the Natural Science Foundation of Heilongjiang Province of China (Grant No A200506).
†E-mail: xuquan@dqsy.net

by thermal motions are so much larger than the displacements needed to form the coherent soliton pattern that the solitons cannot be formed at the temperature relevant to the biological procedure.[14] Wang *et al* obtained that the exact equations can reduce to the usual nonlinear Schrödinger equation in the continuum limit approximation, when they studied Davydov's theory of energy transferring in a biological molecular system.[15] They also discussed the initial states and the soliton formation process, and time and length scales governing soliton coherence properties were determined. In 1992, Scott systematically studied the behaviour of Davydov's soliton,[16] and formed a more perfect theoretical system of Davydov's soliton. Thereafter the thermal stability of Davydov's soliton was studied to the full.[17−19]

In the 21st century, the Hamiltonian and the wave function in the Davydov theory have simultaneously been improved and extended, based on some physical and biological grounds and results from other models. The equations of motion for the improved Davydov model with a quasicoherent two-quantum state and a new interaction term in the Hamiltonian describe bioenergy transport along the molecular chains in protein molecules by a soliton mechanism. Some elementary properties of the soliton, including the nonlinear coupling energy and greatly increased binding energy of the soliton, are also given. The results obtained suggest that the model could be a candidate for a bioenergy transport mechanism in protein molecules.[20] But most scholars focus their interests on the interactions of a single electron with lattice phonons, when they study the quasi-discrete and discrete molecular systems.[21−25] Thus the studies on the interactions of the excitons with lattice phonons only remain at the condition of the Wannier–Mott exciton which has a large radius. This paper focus its interests on the interactions of lattice phonons with Frenkel excitons which has a small radius, by the virtue of the quasi-discreteness approximation and the method of multiple-scale in Ref.[26]–[28].

2. The self-trapping in a two-dimensional discrete harmonic molecular lattice

We first consider a two-dimensional discrete harmonic molecular lattice. By taking into account the interaction between the lattice and its nearest-neighbours, the Hamiltonian of the system with a harmonic potential is given by

$$\begin{aligned} H = H_{\rm ex} + H_{\rm ph} + H_{\rm ex-ph} &= J_l \sum_{l,m} \left[2\left|\psi_{l,m}\right|^2 - \psi^*_{l,m}\left(\psi_{l+1,m} + \psi_{l-1,m}\right)\right] \\ &+ J_m \sum_{l,m} \left[2\left|\psi_{l,m}\right|^2 - \psi^*_{l,m}\left(\psi_{l,m+1} + \psi_{l,m-1}\right)\right] \\ &+ \sum_{l,m} \left[\frac{p^2_{l,m}}{2M} + \frac{1}{2}k_l\left(u_{l+1,m} + u_{l-1,m} - u_{l,m}\right)^2\right] + \sum_{l,m} \left[\frac{1}{2}k_m\left(u_{l,m+1} + u_{l,m-1} - u_{l,m}\right)^2\right] \\ &+ \chi_l \sum_{l,m} \left[\left|\psi_{l,m}\right|^2\left(u_{l+1,m} - u_{l-1,m}\right)\right] + \chi_m \sum_{l,m} \left[\left|\psi_{l,m}\right|^2\left(u_{l,m+1} - u_{l,m-1}\right)\right], \end{aligned} \tag{1}$$

where $\psi_{l,m}$ is the wave function of the exciton at (l,m)th site, and J_l, J_m are matrixes transferring between neighbour sites; M, $u_{l,m}$, $p_{l,m}$ denote the mass, displacement and momentum, respectively; k_l, k_m are Hook coefficients. The last term in the Hamiltonian denotes the interactions of lattice phonons with the excitons. χ_l, χ_m describe the parameters of the strength of the interaction between the excitons and phonons.

The nonlinear vibration equation of the system is obtained by $\dot{p}_{l,m} = \ddot{u}_{l,m} = -\partial H/\partial u_{l,m}$ and $\mathrm{i}\dot{\psi}_{l,m} = \partial H/\partial\psi^*_{l,m}$

$$\begin{aligned} M\ddot{u}_{l,m} &= k_l\left(u_{l+1,m} + u_{l-1,m} - 2u_{l,m}\right) + k_m\left(u_{l,m+1} + u_{l,m-1} - 2u_{l,m}\right) \\ &+ \chi_l\left(\left|\psi_{l+1,m}\right|^2 - \left|\psi_{l-1,m}\right|^2\right) + \chi_m\left(\left|\psi_{l,m+1}\right|^2 - \left|\psi_{l,m-1}\right|^2\right), \\ \mathrm{i}\dot{\psi}_{l,m} &= -J_l\left(\psi_{l+1,m} + \psi_{l-1,m} - 2\psi_{l,m}\right) - J_m\left(\psi_{l,m+1} + \psi_{l,m-1} - 2\psi_{l,m}\right) \\ &+ \chi_l\left(u_{l+1,m} - u_{l-1,m}\right)\psi_{l,m} + \chi_m\left(u_{l,m+1} - u_{l,m-1}\right)\psi_{l,m}. \end{aligned} \tag{2}$$

According to the method of multiple-scale and the quasi-discreteness approximation, we assume

$$\begin{aligned}
u_{l,m}(t) &= \varepsilon u^{(1)}(\xi_l,\zeta_m,\tau,\phi_{l,m}) + \varepsilon^2 u^{(2)}(\xi_l,\zeta_m,\tau,\phi_{l,m}) + \ldots = \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}u^{(\gamma)}(\xi_l,\zeta_m,\tau,\phi_{l,m}) = \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}u^{(\gamma)}_{lm,lm},\\
\psi_{l,m}(t) &= \varepsilon \psi^{(1)}(\xi_l,\zeta_m,\tau,\phi_{l,m}) + \varepsilon^2 \psi^{(2)}(\xi_l,\zeta_m,\tau,\phi_{l,m}) + \ldots = \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}\psi^{(\gamma)}(\xi_l,\zeta_m,\tau,\phi_{l,m}) = \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}\psi^{(\gamma)}_{lm,lm},
\end{aligned} \tag{3}$$

where ε is a small ordering parameter denoting the relative amplitude of the excitation. Let $u^{(\gamma)}_{lm,lm} = u^{(\gamma)}(\xi_l,\zeta_m,\tau,\phi_{l,m})$, $\psi^{(\gamma)}_{lm,lm} = \psi^{(\gamma)}(\xi_l,\zeta_m,\tau,\phi_{l,m})$, here $\xi_l = \varepsilon(la-\lambda t)$, $\zeta_m = \varepsilon(mb-\eta t)$, $\tau = \varepsilon^2 t$ are multiple-scales slow variables. The fast variable $\phi_{l,m} = nq_l a + mq_m b - \omega t$ represents the phase of the carrier wave. q_l, q_m, ω are the number and frequency of the carrier wave, respectively. Here a and b are lattice constants. λ and η are the parameter to be determined later. We perform the Taylor expansion as follows:

$$\begin{aligned}
u^{(\gamma)}_{l\pm1,m} &= u^{(\gamma)}_{lm,(l\pm1)m} \pm \varepsilon a\frac{\partial u^{(\gamma)}_{lm,(l\pm1)m}}{\partial\xi_l} + \frac{\varepsilon^2a^2}{2!}\frac{\partial^2 u^{(\gamma)}_{lm,(l\pm1)m}}{\partial\xi_l^2} \pm \frac{\varepsilon^3a^3}{3!}\frac{\partial^3 u^{(\gamma)}_{lm,(l\pm1)m}}{\partial\xi_l^3} + \ldots,\\
u_{l\pm1,m} &= \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}\left[\sum_{\mu=0}^{\infty}\frac{1}{\mu!}\left(\pm\varepsilon a\frac{\partial}{\partial\xi_l}\right)^{\mu}u^{(\gamma)}_{lm,(l\pm1)m}\right],\\
u^{(\gamma)}_{l,m\pm1} &= u^{(\gamma)}_{lm,l(m\pm1)} \pm \varepsilon a\frac{\partial u^{(\gamma)}_{lm,l(m\pm1)}}{\partial\zeta_m} + \frac{\varepsilon^2a^2}{2!}\frac{\partial^2 u^{(\gamma)}_{lm,l(m\pm1)}}{\partial\zeta_m^2} \pm \frac{\varepsilon^3a^3}{3!}\frac{\partial^3 u^{(\gamma)}_{lm,l(m\pm1)}}{\partial\zeta_m^3} + \ldots,\\
u_{l,m\pm1} &= \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}\left[\sum_{\mu=0}^{\infty}\frac{1}{\mu!}\left(\pm\varepsilon a\frac{\partial}{\partial\zeta_m}\right)^{\mu}u^{(\gamma)}_{lm,l(m\pm1)}\right],\\
\psi^{(\gamma)}_{l\pm1,m} &= \psi^{(\gamma)}_{lm,(l\pm1)m} \pm \varepsilon a\frac{\partial \psi^{(\gamma)}_{lm,(l\pm1)m}}{\partial\xi_l} + \frac{\varepsilon^2a^2}{2!}\frac{\partial^2 \psi^{(\gamma)}_{lm,(l\pm1)m}}{\partial\xi_l^2} \pm \frac{\varepsilon^3a^3}{3!}\frac{\partial^3 \psi^{(\gamma)}_{lm,(l\pm1)m}}{\partial\xi_l^3} + \ldots,\\
\psi_{l\pm1,m} &= \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}\left[\sum_{\mu=0}^{\infty}\frac{1}{\mu!}\left(\pm\varepsilon a\frac{\partial}{\partial\xi_l}\right)^{\mu}\psi^{(\gamma)}_{lm,(l\pm1)m}\right],\\
\psi^{(\gamma)}_{l,m\pm1} &= \psi^{(\gamma)}_{lm,l(m\pm1)} \pm \varepsilon a\frac{\partial \psi^{(\gamma)}_{lm,l(m\pm1)}}{\partial\zeta_m} + \frac{\varepsilon^2a^2}{2!}\frac{\partial^2 \psi^{(\gamma)}_{lm,l(m\pm1)}}{\partial\zeta_m^2} \pm \frac{\varepsilon^3a^3}{3!}\frac{\partial^3 \psi^{(\gamma)}_{lm,l(m\pm1)}}{\partial\zeta_m^3} + \ldots,\\
\psi_{l,m\pm1} &= \sum_{\gamma=1}^{\infty}\varepsilon^{\gamma}\left[\sum_{\mu=0}^{\infty}\frac{1}{\mu!}\left(\pm\varepsilon a\frac{\partial}{\partial\zeta_m}\right)^{\mu}\psi^{(\gamma)}_{lm,l(m\pm1)}\right].
\end{aligned} \tag{4}$$

According to the differential rule, we make the transformation as follows:

$$\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t} &= -\omega\frac{\partial}{\partial\phi_{l,m}} - \lambda\varepsilon\frac{\partial}{\partial\xi_l} - \eta\varepsilon\frac{\partial}{\partial\zeta_m} + \varepsilon^2\frac{\partial}{\partial\tau},\\
\frac{\mathrm{d}^2}{\mathrm{d}t^2} &= \omega^2\frac{\partial^2}{\partial\phi_{l,m}^2} + 2\lambda\omega\varepsilon\frac{\partial^2}{\partial\phi_{l,m}\partial\xi_l} + 2\eta\omega\varepsilon\frac{\partial^2}{\partial\phi_{l,m}\partial\zeta_m} - 2\omega\varepsilon^2\frac{\partial^2}{\partial\phi_{l.m}\partial\tau} + \lambda^2\varepsilon^2\frac{\partial^2}{\partial\xi_l^2} + \eta^2\varepsilon^2\frac{\partial^2}{\partial\zeta_m^2}\\
&\quad + 2\lambda\eta\varepsilon^3\frac{\partial^2}{\partial\xi_l\partial\zeta_m} - 2\lambda\varepsilon^3\frac{\partial^2}{\partial\xi_l\partial\tau} - 2\eta\varepsilon^3\frac{\partial^2}{\partial\zeta_m\partial\tau} + \varepsilon^4\frac{\partial^2}{\partial\tau^2}.
\end{aligned} \tag{5}$$

Substituting Eqs.(3), (4) and (5) into Eq.(2) and comparing the powers of ε in Eq.(2), we have

$$\varepsilon:\begin{cases}
\omega^2\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\phi_{l,m}^2} - \dfrac{k_l}{M}\left(u^{(1)}_{lm,(l+1)m} + u^{(1)}_{lm,(l-1)m} - 2u^{(1)}_{lm,lm}\right) - \dfrac{k_m}{M}\left(u^{(1)}_{lm,l(m+1)} + u^{(1)}_{lm,l(m-1)} - 2u^{(1)}_{lm,lm}\right) = 0,\\
-\mathrm{i}\,\omega\dfrac{\partial\psi^{(1)}_{lm,lm}}{\partial\phi_{l,m}} + J_l\left(\psi^{(1)}_{lm,(l+1)m} + \psi^{(1)}_{lm,(l-1)m} - 2\psi^{(1)}_{lm,lm}\right) + J_m\left(\psi^{(1)}_{lm,l(m+1)} + \psi^{(1)}_{lm,l(m-1)} - 2\psi^{(1)}_{lm,lm}\right) = 0,
\end{cases} \tag{6}$$

$$
\varepsilon^2:\begin{cases}
\omega^2\dfrac{\partial^2 u^{(2)}_{lm,lm}}{\partial\phi^2_{l,m}}-\dfrac{k_l}{M}\left(u^{(2)}_{lm,(l+1)m}+u^{(2)}_{lm,(l-1)m}-2u^{(2)}_{lm,lm}\right)-\dfrac{k_m}{M}\left(u^{(2)}_{lm,l(m+1)}+u^{(2)}_{lm,l(m-1)}-2u^{(2)}_{lm,lm}\right)\\
=\dfrac{k_l a}{M}\dfrac{\partial}{\partial\xi_l}\left(u^{(1)}_{lm,(l+1)m}-u^{(1)}_{lm,(l-1)m}\right)+\dfrac{k_m b}{M}\dfrac{\partial}{\partial\zeta_m}\left(u^{(1)}_{lm,l(m+1)}-u^{(1)}_{lm,l(m-1)}\right)-2\omega\lambda\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\phi_{l,m}\partial\xi_l}-2\omega\eta\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\phi_{l,m}\partial\zeta_m}\\
+\dfrac{\chi_l}{M}\left(\left|\psi^{(1)}_{lm,(l+1)m}\right|^2-\left|\psi^{(1)}_{lm,(l-1)m}\right|^2\right)+\dfrac{\chi_m}{M}\left(\left|\psi^{(1)}_{lm,l(m+1)}\right|^2-\left|\psi^{(1)}_{lm,l(m-1)}\right|^2\right)\\
-\mathrm{i}\,\omega\dfrac{\partial\psi^{(2)}_{lm,lm}}{\partial\phi_{l,m}}+J_l\left(\psi^{(2)}_{lm,(l+1)m}+\psi^{(2)}_{lm,(l-1)m}-2\psi^{(2)}_{lm,lm}\right)+J_m\left(\psi^{(2)}_{lm,l(m+1)}+\psi^{(2)}_{lm,l(m-1)}-2\psi^{(2)}_{lm,lm}\right)\\
=-J_l a\dfrac{\partial}{\partial\xi_l}\left(\psi^{(1)}_{lm,(l+1)m}-\psi^{(1)}_{lm,(l-1)m}\right)-J_m b\dfrac{\partial}{\partial\zeta_m}\left(\psi^{(1)}_{lm,l(m+1)}-\psi^{(1)}_{lm,l(m-1)}\right)+\mathrm{i}\,\lambda\dfrac{\partial\psi^{(1)}_{lm,lm}}{\partial\xi_l}+\mathrm{i}\,\eta\dfrac{\partial\psi^{(1)}_{lm,lm}}{\partial\zeta_m}\\
+\chi_l\left(u^{(1)}_{lm,(l+1)m}-u^{(1)}_{lm,(l-1)m}\right)\psi^{(1)}_{lm,lm}+\chi_m\left(u^{(1)}_{lm,l(m+1)}-u^{(1)}_{lm,l(m-1)}\right)\psi^{(1)}_{lm,lm},
\end{cases}\tag{7}
$$

$$
\varepsilon^3:\begin{cases}
\omega^2\dfrac{\partial^2 u^{(3)}_{lm,lm}}{\partial\phi^2_{l,m}}-\dfrac{k_l}{M}\left(u^{(3)}_{lm,(l+1)m}+u^{(3)}_{lm,(l-1)m}-2u^{(3)}_{lm,lm}\right)-\dfrac{k_m}{M}\left(u^{(3)}_{lm,l(m+1)}+u^{(3)}_{lm,l(m-1)}-2u^{(3)}_{lm,lm}\right)\\
=\dfrac{k_l a^2}{2M}\dfrac{\partial^2}{\partial\xi_l^2}\left(u^{(1)}_{lm,(l+1)m}+u^{(1)}_{lm,(l-1)m}\right)\\
+\dfrac{k_m b^2}{2M}\dfrac{\partial^2}{\partial\zeta_m^2}\left(u^{(1)}_{lm,l(m+1)}+u^{(1)}_{lm,l(m-1)}\right)+\dfrac{k_l a}{M}\dfrac{\partial}{\partial\xi_l}\left(u^{(2)}_{lm,(l+1)m}-u^{(2)}_{lm,(l-1)m}\right)\\
+\dfrac{k_m b}{2M}\dfrac{\partial}{\partial\zeta_m}\left(u^{(2)}_{lm,l(m+1)}-u^{(2)}_{lm,l(m-1)}\right)-2\omega\lambda\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\phi_{l,m}\partial\xi_l}-2\omega\eta\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\phi_{l,m}\partial\zeta_m}\\
+\dfrac{\chi_l a}{M}\left(\dfrac{\partial\left|\psi^{(1)}_{lm,(l+1)m}\right|^2}{\partial\xi_l}+\dfrac{\partial\left|\psi^{(1)}_{lm,(l-1)m}\right|^2}{\partial\xi_l}\right)+\dfrac{\chi_m b}{M}\left(\dfrac{\partial\left|\psi^{(1)}_{lm,l(m+1)}\right|^2}{\partial\zeta_m}+\dfrac{\partial\left|\psi^{(1)}_{lm,l(m-1)}\right|^2}{\partial\zeta_m}\right)\\
+2\omega\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\phi_{l,m}\partial\tau}-\lambda^2\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\xi_l^2}-\eta^2\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\zeta_m^2}+2\lambda\eta\dfrac{\partial^2 u^{(1)}_{lm,lm}}{\partial\xi_l\partial\zeta_m}\\
+\dfrac{\chi_l}{M}\left(\psi^{(1)}_{lm,(l+1)m}\psi^{(2)*}_{lm,(l+1)m}+\psi^{(1)*}_{lm,(l+1)m}\psi^{(2)}_{lm,(l+1)m}-\psi^{(1)}_{lm,(l-1)m}\psi^{(2)*}_{lm,(l-1)m}-\psi^{(1)*}_{lm,(l-1)m}\psi^{(2)}_{lm,(l-1)m}\right)\\
+\dfrac{\chi_m}{M}\left(\psi^{(1)}_{lm,l(m+1)}\psi^{(2)*}_{lm,l(m+1)}+\psi^{(1)*}_{lm,l(m+1)}\psi^{(2)}_{lm,l(m+1)}-\psi^{(1)}_{lm,l(m-1)}\psi^{(2)*}_{lm,l(m-1)}-\psi^{(1)*}_{lm,l(m-1)}\psi^{(2)}_{lm,l(m-1)}\right)\\
-\mathrm{i}\,\omega\dfrac{\partial\psi^{(3)}_{lm,lm}}{\partial\phi_{l,m}}+J_l\left(\psi^{(3)}_{lm,(l+1)m}+\psi^{(3)}_{lm,(l-1)m}-2\psi^{(3)}_{lm,lm}\right)+J_m\left(\psi^{(3)}_{lm,l(m+1)}+\psi^{(3)}_{lm,l(m-1)}-2\psi^{(3)}_{lm,lm}\right)\\
=-\dfrac{1}{2}J_l a^2\dfrac{\partial^2}{\partial\xi_l^2}\left(\psi^{(1)}_{lm,(l+1)m}+\psi^{(1)}_{lm,(l-1)m}\right)\\
-\dfrac{1}{2}J_m b^2\dfrac{\partial^2}{\partial\zeta_m^2}\left(\psi^{(1)}_{lm,l(m+1)}+\psi^{(1)}_{lm,l(m-1)}\right)-J_l a\dfrac{\partial}{\partial\xi_l}\left(\psi^{(2)}_{lm,(l+1)m}-\psi^{(2)}_{lm,(l-1)m}\right)\\
-J_m b\dfrac{\partial}{\partial\zeta_m}\left(\psi^{(2)}_{lm,l(m+1)}-\psi^{(2)}_{lm,l(m-1)}\right)+\mathrm{i}\,\lambda\dfrac{\partial\psi^{(2)}_{lm,lm}}{\partial\xi_l}+\mathrm{i}\,\eta\dfrac{\partial\psi^{(2)}_{lm,lm}}{\partial\zeta_m}\\
-\mathrm{i}\dfrac{\partial\psi^{(1)}_{lm,lm}}{\partial\tau}+a\chi_l\dfrac{\partial}{\partial\xi_l}\left(u^{(1)}_{lm,(l+1)m}+u^{(1)}_{lm,(l-1)m}\right)\psi^{(1)}_{lm,lm}+b\chi_m\dfrac{\partial}{\partial\xi_l}\left(u^{(1)}_{lm,l(m+1)}+u^{(1)}_{lm,l(m-1)}\right)\psi^{(1)}_{lm,lm}\\
+\chi_l\left(u^{(2)}_{lm,(l+1)m}-u^{(2)}_{lm,(l-1)m}\right)\psi^{(1)}_{lm,lm}+\chi_m\left(u^{(2)}_{lm,l(m+1)}-u^{(2)}_{lm,l(m-1)}\right)\psi^{(1)}_{lm,lm}\\
+\chi_l\left(u^{(1)}_{lm,(l+1)m}-u^{(1)}_{lm,(l-1)m}\right)\psi^{(2)}_{lm,lm}+\chi_m\left(u^{(1)}_{lm,l(m+1)}-u^{(1)}_{lm,l(m-1)}\right)\psi^{(2)}_{lm,lm}.
\end{cases}\tag{8}
$$

Equations (6) have the solutions as follows:

$$
\begin{aligned}
u^{(1)}_{lm,lm}&=u_{10}\left(\xi_l,\zeta_m,\tau\right)+u_1\left(\xi_l,\zeta_m,\tau\right)\mathrm{e}^{\mathrm{i}\,\phi_{lm}}+u_1^*\left(\xi_l,\zeta_m,\tau\right)\mathrm{e}^{-\mathrm{i}\,\phi_{lm}},\\
\psi^{(1)}_{lm,lm}&=\psi_1\left(\xi_l,\zeta_m,\tau\right)\mathrm{e}^{\mathrm{i}\,\phi_{lm}}+\psi_1^*\left(\xi_l,\zeta_m,\tau\right)\mathrm{e}^{-\mathrm{i}\,\phi_{lm}}.
\end{aligned}\tag{9}
$$

Substituting Eqs.(9) into Eq.(7) and eliminating the secular terms, we have

$$\omega = 4J_l \sin^2\frac{q_l a}{2} + 4J_m \sin^2\frac{q_m b}{2} = 2\sqrt{\frac{k_l}{M}\sin^2\frac{q_l a}{2} + \frac{k_m}{M}\sin^2\frac{q_m b}{2}},$$
$$\lambda = 2J_l a\sin(q_l a) = \frac{2k_l a}{\omega M}\sin(q_l a), \quad \eta = 2J_m b\sin(q_m b) = \frac{2q_m b}{\omega M}\sin(q_m b). \tag{10}$$

$$\omega^2\frac{\partial^2 u^{(2)}_{lm,lm}}{\partial\phi^2_{l,m}} - \frac{k_l}{M}\left(u^{(2)}_{lm,(l+1)m} + u^{(2)}_{lm,(l-1)m} - 2u^{(2)}_{lm,lm}\right) - \frac{k_m}{M}\left(u^{(2)}_{lm,l(m+1)} + u^{(2)}_{lm,l(m-1)} - 2u^{(2)}_{lm,lm}\right) = 0,$$
$$-\mathrm{i}\,\omega\frac{\partial\psi^{(2)}_{lm,lm}}{\partial\phi_{l,m}} + J_l\left(\psi^{(2)}_{lm,(l+1)m} + \psi^{(2)}_{lm,(l-1)m} - 2\psi^{(2)}_{lm,lm}\right) + J_m\left(\psi^{(2)}_{lm,l(m+1)} + \psi^{(2)}_{lm,l(m-1)} - 2\psi^{(2)}_{lm,lm}\right)$$
$$= 2\,\mathrm{i}\left(\chi_l\sin(q_l a) + \chi_m\sin(q_m b)\right)u_1\psi_1\mathrm{e}^{2\,\mathrm{i}\,\phi_{lm}}. \tag{11}$$

Equations (11) have the following solutions

$$u^{(2)}_{lm,lm} = u_{20}(\xi_l,\zeta_m,\tau) + u_2(\xi_l,\zeta_m,\tau)\,\mathrm{e}^{\mathrm{i}\,\phi_{lm}} + u_2^*(\xi_l,\zeta_m,\tau)\,\mathrm{e}^{-\mathrm{i}\,\phi_{lm}},$$
$$\psi^{(2)}_{lm,lm} = \psi_2(\xi_l,\zeta_m,\tau)\,\mathrm{e}^{\mathrm{i}\,\phi_{lm}} + \psi_2^*(\xi_l,\zeta_m,\tau)\,\mathrm{e}^{-\mathrm{i}\,\phi_{lm}}$$
$$+\ \mathrm{i}\left[\frac{\chi_l\sin(q_l a)}{\omega + J_l\left(\cos(2q_l a) - 1\right)} + \frac{\chi_m\sin(q_m b)}{\omega + J_m\left(\cos(2q_m b) - 1\right)}\right]u_1\psi_1\mathrm{e}^{2\,\mathrm{i}\,\phi_{lm}}. \tag{12}$$

In fact we can let u_{20}=u_2=ψ_2= 0, because they can be transformed into the lowest-order solutions (9). Substituting them and $u^{(1)}_{lm,lm}$, $\psi^{(1)}_{lm,lm}$ into Eqs.(8) and eliminating the secular terms, we have

$$\frac{2\chi_l a}{M}\frac{\partial|\psi_1|^2}{\partial\xi_l} + \frac{2\chi_m b}{M}\frac{\partial|\psi_1|^2}{\partial\zeta_m} + \left(\frac{k_l a^2}{2M} - \lambda^2\right)\frac{\partial^2 u_{10}}{\partial\xi_l^2} - 2\lambda\eta\frac{\partial^2 u_{10}}{\partial\xi_l\partial\zeta_m} + \left(\frac{k_m b^2}{2M} - \eta^2\right)\frac{\partial^2 u_{10}}{\partial\zeta_m^2} = 0,$$
$$\mathrm{i}\frac{\partial u_1}{\partial\tau} + \frac{k_l a^2\cos(q_l a) - M\lambda^2}{2M\omega}\frac{\partial^2 u_1}{\partial\xi_l^2} - \frac{\lambda\eta}{\omega}\frac{\partial^2 u_1}{\partial\xi_l\partial\zeta_m} + \frac{k_m b^2\cos(q_m b) - M\eta^2}{2M\omega}\frac{\partial^2 u_1}{\partial\zeta_m^2}$$
$$-\left\{\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega + J_l\left(\cos(2q_l a) - 1\right)\right]} + \frac{\chi_m^2\sin^2(q_m b)}{M\omega\left[\omega + J_m\left(\cos(2q_m b) - 1\right)\right]}\right\}u_1|\psi_1|^2 = 0,$$
$$\mathrm{i}\frac{\partial\psi_1}{\partial\tau} + J_l a^2\cos(q_l a)\frac{\partial^2\psi_1}{\partial\xi_l^2} + J_m b^2\cos(k_m b)\frac{\partial^2\psi_1}{\partial\zeta_m^2} - a\chi_l\frac{\partial u_{10}}{\partial\xi_l}\psi_1 - b\chi_m\frac{\partial u_{10}}{\partial\zeta_m}\psi_1$$
$$+\left[\frac{2\chi_l^2\sin^2(q_l a)}{\omega + J_l\left(\cos(2q_l a) - 1\right)} + \frac{2\chi_m^2\sin^2(q_m b)}{\omega + J_m\left(\cos(2q_m b) - 1\right)}\right]\psi_1|u_1|^2 = 0. \tag{13}$$

When $\eta = 2J_m b\sin(q_m b) = 0$, Eqs.(13) can be written as follows:

$$\frac{2\chi_l a}{M}\frac{\partial|\psi_1|^2}{\partial\xi_l} + \frac{2\chi_m b}{M}\frac{\partial|\psi_1|^2}{\partial\zeta_m} + \left(\frac{k_l a^2}{2M} - \lambda^2\right)\frac{\partial^2 u_{10}}{\partial\xi_l^2} + \frac{k_m b^2}{2M}\frac{\partial^2 u_{10}}{\partial\zeta_m^2} = 0,$$
$$\mathrm{i}\frac{\partial u_1}{\partial\tau} + \frac{k_l a^2\cos(q_l a) - M\lambda^2}{2M\omega}\frac{\partial^2 u_1}{\partial\xi_l^2} + \frac{k_m b^2}{2M\omega}\frac{\partial^2 u_1}{\partial\zeta_m^2} - \frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega + J_l\left(\cos(2q_l a) - 1\right)\right]}u_1|\psi_1|^2 = 0,$$
$$\mathrm{i}\frac{\partial\psi_1}{\partial\tau} + J_l a^2\cos(q_l a)\frac{\partial^2\psi_1}{\partial\xi_l^2} + J_m b^2\frac{\partial^2\psi_1}{\partial\zeta_m^2} - a\chi_l\frac{\partial u_{10}}{\partial\xi_l}\psi_1 - b\chi_m\frac{\partial u_{10}}{\partial\zeta_m}\psi_1 + \frac{2\chi_l^2\sin^2(q_l a)}{\omega + J_l\left(\cos(2q_l a) - 1\right)}\psi_1|u_1|^2 = 0. \tag{14}$$

Letting $u_1 = \gamma\psi_1$ and taking $\dfrac{2M\lambda^2 - k_l a^2}{4\chi_l a}\dfrac{\partial u_{10}}{\partial\xi_l} = -\dfrac{k_m a}{4\chi_m}\dfrac{\partial u_{10}}{\partial\zeta_m} = |\psi_1|^2$ into account, we have

$$\mathrm{i}\frac{\partial u_1}{\partial\tau} + \frac{k_l a^2\cos(q_l a) - M\lambda^2}{2M\omega}\frac{\partial^2 u_1}{\partial\xi_l^2} + \frac{k_m b^2}{2M\omega}\frac{\partial^2 u_1}{\partial\zeta_m^2} - \frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega + J_l\left(\cos(2q_l a) - 1\right)\right]\gamma^2}|u_1|^2 u_1 = 0,$$
$$\mathrm{i}\frac{\partial\psi_1}{\partial\tau} + J_l a^2\cos(q_l a)\frac{\partial^2\psi_1}{\partial\xi_l^2} + J_m b^2\frac{\partial^2\psi_1}{\partial\zeta_m^2} + \left[\frac{4\chi_m^2}{k_m} - \frac{4\chi_l^2 a^2}{2M\lambda_2 - k_l a^2} + \frac{2\chi_l^2\gamma^2\sin^2(q_l a)}{\omega + J_l\left(\cos(2q_l a) - 1\right)}\right]\psi_1|\psi_1|^2 = 0. \tag{15}$$

Let $\Lambda_1=\dfrac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}$, $\Delta_1=\dfrac{k_m b^2}{2M\omega}$, $\Gamma_1=-\dfrac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]\gamma^2}$, $\Lambda_2=J_l a^2\cos(q_l a)$, $\Delta_2=J_m b^2$, $\Gamma_2=\dfrac{4\chi_m^2}{k_m}-\dfrac{4\chi_l^2 a^2}{2M\lambda_2-k_l a^2}+\dfrac{2\chi_l^2\gamma^2\sin^2(q_l a)}{\omega+J_l\left(\cos(2q_l a)-1\right)}$, Eq.(15) can transform into

$$\mathrm{i}\frac{\partial u_1}{\partial\tau}+\Lambda_1\frac{\partial^2 u_1}{\partial\xi_l^2}+\Delta_1\frac{\partial^2 u_1}{\partial\zeta_m^2}+\Gamma_1\left|u_1\right|^2 u_1=0,\quad \mathrm{i}\frac{\partial\psi_1}{\partial\tau}+\Lambda_2\frac{\partial^2\psi_1}{\partial\xi_l^2}+\Delta_2\frac{\partial^2\psi_1}{\partial\zeta_m^2}+\Gamma_2\left|\psi_1\right|^2\psi_1=0. \tag{16}$$

When $\Lambda_1\Delta_1>0$, $\Lambda_1\Gamma_1>0$ and $\Lambda_2\Delta_2>0$, $\Lambda_2\Gamma_2>0$, Eqs.(16) have the solutions as follows:

$$\begin{aligned}&u_{10}=\pm A_0/B_0\tanh\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right],\quad u_1=\pm A_0\operatorname{sech}\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\mathrm{e}^{\mathrm{i}\,\Omega t},\\&\psi_1=\pm C_0\operatorname{sech}\left[B_1\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\mathrm{e}^{\mathrm{i}\,\Omega t}.\end{aligned} \tag{17}$$

Thus, Eq.(2) has the following solutions

$$\begin{aligned}u_n&=\pm A_0/B_0\tanh\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\\&\quad\pm A_0\operatorname{sech}\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\cos\left[q_l la+q_m mb-(\Omega+\omega)t\right],\\\psi_1&=\pm C_0\operatorname{sech}\left[B_1\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\cos\left[q_l la+q_m mb-(\Omega+\omega)t\right],\end{aligned} \tag{18}$$

where $A_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/\right)}{2\left(\Lambda_1+\Delta_1\right)\Gamma_1}}$, $B_0=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_1+\Delta_1\right)}$, $C_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_2+\Delta_2\right)\Gamma_2}}$, $B_1=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_2+\Delta_2\right)}$. The solutions (18) can be shown in Fig.1.

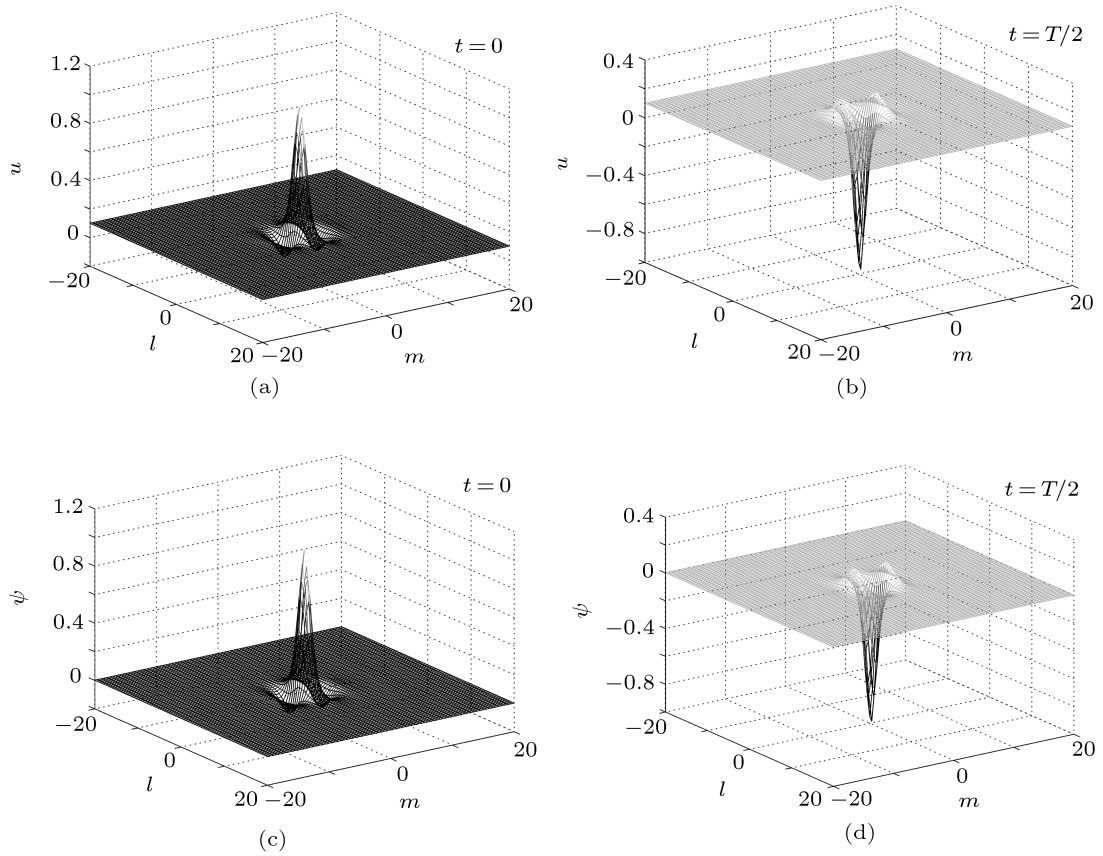


Fig.1. The localized modes in the two-dimensional discrete molecular lattice, (a) motional discrete breathers in the molecular lattice $t=0$, (b) motional discrete breathers in the molecular lattice $t=T/2$, (c) motional discrete exciton-breathers $t=0$, (c) motional discrete exciton-breathers $t=T/2$.

From Fig.1 we know that motional discrete breathers can exist in the two-dimensional discrete molecular lattice with the interactions of excitons with phonons at quasi-discreteness approximation, while Davydov's solitons can be transformed into discrete exciton-breathers and move in the molecular lattice. The difference from the conclusion at continuum limit approximation[29] is that the discreteness of the lattice makes the periodicity ofthe localized mode dominance, which is a discrete breather not a soliton. The difference from the conclusion in the one-dimensional discrete molecular lattice is that the excitons' effect on the molecular lattice does not distort the molecular lattice.

3. The self-trapping in a two-dimensional discrete molecular lattice with cubic nonlinear potential

The nonlinear vibration equation of the system is

$$\begin{aligned}
M\ddot{u}_{l,m} &= k_l\left(u_{l+1,m}+u_{l-1,m}-2u_{l,m}\right)+k_m\left(u_{l,m+1}+u_{l,m-1}-2u_{l,m}\right)\\
&\quad+\alpha_l\left[\left(u_{l+1,m}-u_{l,m}\right)^2-\left(u_{l-1,m}-u_{l,m}\right)^2\right]+\alpha_m\left[\left(u_{l,m+1}-u_{l,m}\right)^2-\left(u_{l,m-1}-u_{l,m}\right)^2\right]\\
&\quad+\chi_l\left(\left|\psi_{l+1,m}\right|^2-\left|\psi_{l-1,m}\right|^2\right)+\chi_m\left(\left|\psi_{l,m+1}\right|^2-\left|\psi_{l,m-1}\right|^2\right),\\
\mathrm{i}\dot{\psi}_{l,m} &= -J_l\left(\psi_{l+1,m}+\psi_{l-1,m}-2\psi_{l,m}\right)-J_m\left(\psi_{l,m+1}+\psi_{l,m-1}-2\psi_{l,m}\right)\\
&\quad+\chi_l\left(u_{l+1,m}-u_{l-1,m}\right)\psi_{l,m}+\chi_m\left(u_{l,m+1}-u_{l,m-1}\right)\psi_{l,m}.
\end{aligned}\tag{19}$$

By the same procedure as above, we obtain

$$\begin{aligned}
&\frac{2\chi_l a}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\xi_l}+\frac{2\chi_m b}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\zeta_m}+\left(\frac{k_l a^2}{2M}-\lambda^2\right)\frac{\partial^2 u_{10}}{\partial\xi_l^2}-2\lambda\eta\frac{\partial^2 u_{10}}{\partial\xi_l\partial\zeta_m}+\left(\frac{k_m b^2}{2M}-\eta^2\right)\frac{\partial^2 u_{10}}{\partial\zeta_m^2}\\
&+4\alpha_l a\left[\cos(q_l a)\left(\cos(q_l a)-1\right)+\sin(q_l a)\right]\frac{\partial\left|u_1\right|^2}{\partial\xi_l}+4\alpha_m b\left[\cos(q_m b)\left(\cos(q_m b)-1\right)+\sin(q_m b)\right]\frac{\partial\left|u_1\right|^2}{\partial\zeta_m}=0,\\
&\mathrm{i}\frac{\partial u_1}{\partial\tau}+\frac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}\frac{\partial^2 u_1}{\partial\xi_l^2}-\frac{\lambda\eta}{\omega}\frac{\partial^2 u_1}{\partial\xi_l\partial\zeta_m}+\frac{k_m b^2\cos(q_m b)-M\eta^2}{2M\omega}\frac{\partial^2 u_1}{\partial\zeta_m^2}+\frac{2\alpha_l a}{\omega}\left(\cos(q_l a)-1\right)u_1\frac{\partial u_{10}}{\partial\xi_l}\\
&+\frac{2\alpha_m b}{\omega}\left(\cos(q_m b)-1\right)u_1\frac{\partial u_{10}}{\partial\zeta_m}-\left\{\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]}+\frac{\chi_m^2\sin^2(q_m b)}{M\omega\left[\omega+J_m\left(\cos(2q_m b)-1\right)\right]}\right\}u_1\left|\psi_1\right|^2\\
&-\left[\frac{2\alpha_l M}{k_l\omega}\cot\frac{q_l a}{2}\sin(2q_l a)\left(\cos(q_l a)-1\right)+\frac{2\alpha_m M}{k_m\omega}\cot\frac{q_m b}{2}\sin(2q_m b)\left(\cos(q_m b)-1\right)\right]\left|u_1\right|^2 u_1\\
&-\left[\frac{4\alpha_l M}{k_l\omega}\cot\frac{q_l a}{2}\sin^3(q_l a)+\frac{4\alpha_m M}{k_m\omega}\cot\frac{q_m b}{2}\sin^3(q_m b)\right]\left|u_1\right|^2 u_1=0,\\
&\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+J_l a^2\cos(q_l a)\frac{\partial^2\psi_1}{\partial\xi_l^2}+J_m b^2\cos(k_m b)\frac{\partial^2\psi_1}{\partial\zeta_m^2}-a\chi_l\frac{\partial u_{10}}{\partial\xi_l}\psi_1-b\chi_m\frac{\partial u_{10}}{\partial\zeta_m}\psi_1\\
&+\left[\frac{2\chi_l^2\sin^2(q_l a)}{\omega+J_l\left(\cos(2q_l a)-1\right)}+\frac{2\chi_m^2\sin^2(q_m b)}{\omega+J_m\left(\cos(2q_m b)-1\right)}\right]\psi_1\left|u_1\right|^2\\
&+\left[\frac{2\alpha_l M\chi_l}{k_l}\cot\frac{q_l a}{2}\sin(2q_l a)+\frac{2\alpha_m M\chi_m}{k_m}\cot\frac{q_m b}{2}\sin(2q_m b)\right]u_1^2\psi_1^*=0.
\end{aligned}\tag{20}$$

When $\eta=2J_m b\sin(q_m b)=0$, Eqs.(20) can be transformed into

$$\begin{aligned}
&\frac{2\chi_l a}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\xi_l}+\frac{2\chi_m b}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\zeta_m}+\left(\frac{k_l a^2}{2M}-\lambda^2\right)\frac{\partial^2 u_{10}}{\partial\xi_l^2}+\frac{k_m b^2}{2M}\frac{\partial^2 u_{10}}{\partial\zeta_m^2}\\
&+4\alpha_l a\left[\cos(q_l a)\left(\cos(q_l a)-1\right)+\sin(q_l a)\right]\frac{\partial\left|u_1\right|^2}{\partial\xi_l}+8\alpha_m b\frac{\partial\left|u_1\right|^2}{\partial\zeta_m}=0,\\
&\mathrm{i}\frac{\partial u_1}{\partial\tau}+\frac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}\frac{\partial^2 u_1}{\partial\xi_l^2}-\frac{k_m b^2}{2M\omega}\frac{\partial^2 u_1}{\partial\zeta_m^2}+\frac{2\alpha_l a}{\omega}\left(\cos(q_l a)-1\right)u_1\frac{\partial u_{10}}{\partial\xi_l}\\
&-\frac{4\alpha_m b}{\omega}u_1\frac{\partial u_{10}}{\partial\zeta_m}-\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]}u_1\left|\psi_1\right|^2+\frac{8\alpha_l M}{k_l\omega}\cot\frac{q_l a}{2}\sin^2\frac{q_l a}{2}\sin(q_l a)\left|u_1\right|^2 u_1=0,
\end{aligned}$$

$$\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+J_la^2\cos(q_la)\frac{\partial^2\psi_1}{\partial\xi_l^2}+J_mb^2\cos(k_mb)\frac{\partial^2\psi_1}{\partial\zeta_m^2}-a\chi_l\frac{\partial u_{10}}{\partial\xi_l}\psi_1-b\chi_m\frac{\partial u_{10}}{\partial\zeta_m}\psi_1$$
$$+\frac{2\chi_l^2\sin^2(q_la)}{\omega+J_l\left(\cos(2q_la)-1\right)}\psi_1\left|u_1\right|^2+\frac{2\alpha_lM\chi_l}{k_l}\cot\frac{q_la}{2}\sin(2q_la)u_1^2\psi_1^*=0. \tag{21}$$

Let $u_1=\gamma\psi_1$, and $\dfrac{2M\lambda^2-k_la^2}{4\chi_la+8M\alpha_la\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]}\dfrac{\partial u_{10}}{\partial\xi_l}=-\dfrac{k_mb^2}{4\chi_mb+16M\alpha_mb\gamma^2}\dfrac{\partial u_{10}}{\partial\zeta_m}=\left|\psi_1\right|^2$, we have

$$\mathrm{i}\frac{\partial u_1}{\partial\tau}+\frac{k_la^2\cos(q_la)-M\lambda^2}{2M\omega}\frac{\partial^2u_1}{\partial\xi_l^2}-\frac{k_mb^2}{2M\omega}\frac{\partial^2u_1}{\partial\zeta_m^2}$$
$$+\frac{8\alpha_la\left(\cos(q_la)-1\right)\left\{\chi_la+2M\alpha_la\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]\right\}}{\omega\left(2M\lambda^2-k_la^2\right)\gamma^2}\left|u_1\right|^2u_1+\frac{4\alpha_m\left(4\chi_m+16M\alpha_m\gamma^2\right)}{k_m\omega\gamma^2}\left|u_1\right|^2u_1$$
$$-\frac{\chi_l^2\sin^2(q_la)}{M\omega\left[\omega+J_l\left(\cos(2q_la)-1\right)\right]\gamma^2}\left|u_1\right|^2u_1+\frac{8\alpha_lM}{k_l\omega}\cot\frac{q_la}{2}\sin^2\frac{q_la}{2}\sin(q_la)\left|u_1\right|^2u_1=0,$$
$$\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+J_la^2\cos(q_la)\frac{\partial^2\psi_1}{\partial\xi_l^2}+J_mb^2\cos(k_mb)\frac{\partial^2\psi_1}{\partial\zeta_m^2}-\frac{a^2\chi_l\left\{4\chi_l+8M\alpha_l\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]\right\}}{2M\lambda^2-k_la^2}\left|\psi_1\right|^2\psi_1$$
$$-\frac{\chi_m\left(4\chi_m+16M\alpha_m\gamma^2\right)}{k_m}\left|\psi_1\right|^2\psi_1+\frac{2\chi_l^2\gamma^2\sin^2(q_la)}{\omega+J_l\left(\cos(2q_la)-1\right)}\left|\psi_1\right|^2\psi_1+\frac{2\alpha_lM\chi_l\gamma^2}{k_l}\cot\frac{q_la}{2}\sin(2q_la)\left|\psi_1\right|^2\psi_1=0. \tag{22}$$

Let $\Lambda_1=\dfrac{k_la^2\cos(q_la)-M\lambda^2}{2M\omega}$, $\Delta_1=-\dfrac{k_mb^2}{2M\omega}$, $\Lambda_2=J_la^2\cos(q_la)$, $\Delta_2=J_mb^2$,

$$\Gamma_1=-\frac{\chi_l^2\sin^2(q_la)}{M\omega\left[\omega+J_l\left(\cos(2q_la)-1\right)\right]\gamma^2}+\frac{8\alpha_la\left(\cos(q_la)-1\right)\left\{\chi_la+2M\alpha_la\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]\right\}}{\omega\left(2M\lambda^2-k_la^2\right)\gamma^2}$$
$$+\frac{4\alpha_m\left(4\chi_m+16M\alpha_m\gamma^2\right)}{k_m\omega\gamma^2}+\frac{8\alpha_lM}{k_l\omega}\cot\frac{q_la}{2}\sin^2\frac{q_la}{2}\sin(q_la),$$
$$\Gamma_2=\frac{2\chi_l^2\gamma^2\sin^2(q_la)}{\omega+J_l\left(\cos(2q_la)-1\right)}-\frac{a^2\chi_l\left\{4\chi_l+8M\alpha_l\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]\right\}}{2M\lambda^2-k_la^2}$$
$$+\frac{2\alpha_lM\chi_l\gamma^2}{k_l}\cot\frac{q_la}{2}\sin(2q_la)-\frac{\chi_m\left(4\chi_m+16M\alpha_m\gamma^2\right)}{k_m}.$$

Equations (22) can be rewritten as follows:

$$\mathrm{i}\frac{\partial u_1}{\partial\tau}+\Lambda_1\frac{\partial^2u_1}{\partial\xi_l^2}+\Delta_1\frac{\partial^2u_1}{\partial\zeta_m^2}+\Gamma_1\left|u_1\right|^2u_1=0,$$
$$\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+\Lambda_2\frac{\partial^2\psi_1}{\partial\xi_l^2}+\Delta_2\frac{\partial^2\psi_1}{\partial\zeta_m^2}+\Gamma_2\left|\psi_1\right|^2\psi_1=0. \tag{23}$$

When $\Lambda_1\Delta_1>0$, $\Lambda_1\Gamma_1>0$ and $\Lambda_2\Delta_2>0$, $\Lambda_2\Gamma_2>0$, Eqs.(23) have the following solutions

$$u_{10}=\pm A_0/B_0\tanh\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right],$$
$$u_1=\pm A_0\operatorname{sech}\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\mathrm{e}^{\mathrm{i}\,\Omega t},$$
$$\psi_1=\pm C_0\operatorname{sech}\left[B_1\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\mathrm{e}^{\mathrm{i}\,\Omega t}. \tag{24}$$

Thus, Eqs.(19) have the solutions as follows:

$$u_n=\pm A_0/B_0\tanh\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]$$
$$\pm A_0\operatorname{sech}\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\cos\left[q_lla+q_mmb-(\Omega+\omega)t\right],$$
$$\psi_1=\pm C_0\operatorname{sech}\left[B_1\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\cos\left[q_lla+q_mmb-(\Omega+\omega)t\right], \tag{25}$$

where $A_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_1+\Delta_1\right)\Gamma_1}}$, $B_0=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_1+\Delta_1\right)}$, $C_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_2+\Delta_2\right)\Gamma_2}}$, $B_1=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_2+\Delta_2\right)}$.

The solutions (25) can also be shown in Fig.1 and that is that motional discrete breathers exis in the two-dimensional discrete cubic nonlinear molecular lattice with the interactions of excitons with phonons at quasi-discreteness approximation, while Davydov's solitons can also be transformed into discrete exciton-breathers and move in it.

4. The self-trapping in a two-dimensional discrete molecular lattice with quartic nonlinear potential

The nonlinear vibration equation of the system is

$$\begin{aligned}
M\ddot{u}_{l,m}&=k_l\left(u_{l+1,m}+u_{l-1,m}-2u_{l,m}\right)+k_m\left(u_{l,m+1}+u_{l,m-1}-2u_{l,m}\right)+\beta_l\left[\left(u_{l+1,m}-u_{l,m}\right)^3+\left(u_{l-1,m}-u_{l,m}\right)^3\right]\\
&\quad+\beta_m\left[\left(u_{l,m+1}-u_{l,m}\right)^3+\left(u_{l,m-1}-u_{l,m}\right)^3\right]+\chi_l\left(\left|\psi_{l+1,m}\right|^2-\left|\psi_{l-1,m}\right|^2\right)+\chi_m\left(\left|\psi_{l,m+1}\right|^2-\left|\psi_{l,m-1}\right|^2\right),\\
\mathrm{i}\dot{\psi}_{l,m}&=-J_l\left(\psi_{l+1,m}+\psi_{l-1,m}-2\psi_{l,m}\right)-J_m\left(\psi_{l,m+1}+\psi_{l,m-1}-2\psi_{l,m}\right)\\
&\quad+\chi_l\left(u_{l+1,m}-u_{l-1,m}\right)\psi_{l,m}+\chi_m\left(u_{l,m+1}-u_{l,m-1}\right)\psi_{l,m}.
\end{aligned}\tag{26}$$

By the same procedure as above, we obtain

$$\begin{aligned}
&\frac{2\chi_l a}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\xi_l}+\frac{2\chi_m b}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\zeta_m}+\left(\frac{k_l a^2}{2M}-\lambda^2\right)\frac{\partial^2u_{10}}{\partial\xi_l^2}-2\lambda\eta\frac{\partial^2u_{10}}{\partial\xi_l\partial\zeta_m}+\left(\frac{q_m b^2}{2M}-\eta^2\right)\frac{\partial^2u_{10}}{\partial\zeta_m^2}=0,\\
&\mathrm{i}\frac{\partial u_1}{\partial\tau}+\frac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}\frac{\partial^2u_1}{\partial\xi_l^2}-\frac{\lambda\eta}{\omega}\frac{\partial^2u_1}{\partial\xi_l\partial\zeta_m}+\frac{k_m b^2\cos(q_m b)-M\eta^2}{2M\omega}\frac{\partial^2u_1}{\partial\zeta_m^2}\\
&-\left\{\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]}+\frac{\chi_m^2\sin^2(q_m b)}{M\omega\left[\omega+J_m\left(\cos(2q_m b)-1\right)\right]}\right\}u_1\left|\psi_1\right|^2\\
&-\left(24\frac{\beta_l}{M\omega}\sin^4\frac{q_l a}{2}+24\frac{\beta_m}{M\omega}\sin^4\frac{q_m b}{2}\right)u_1\left|u_1\right|^2=0,\\
&\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+J_l a^2\cos(q_l a)\frac{\partial^2\psi_1}{\partial\xi_l^2}+J_m b^2\cos(k_m b)\frac{\partial^2\psi_1}{\partial\zeta_m^2}-a\chi_l\frac{\partial u_{10}}{\partial\xi_l}\psi_1-b\chi_m\frac{\partial u_{10}}{\partial\zeta_m}\psi_1\\
&+\left[\frac{2\chi_l^2\sin^2(q_l a)}{\omega+J_l\left(\cos(2q_l a)-1\right)}+\frac{2\chi_m^2\sin^2(q_m b)}{\omega+J_m\left(\cos(2q_m b)-1\right)}\right]\psi_1\left|u_1\right|^2=0.
\end{aligned}\tag{27}$$

When $\eta=2J_m b\sin(q_m b)=0$, Eqs.(27) can be transformed into

$$\begin{aligned}
&\frac{2\chi_l a}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\xi_l}+\frac{2\chi_m b}{M}\frac{\partial\left|\psi_1\right|^2}{\partial\zeta_m}+\left(\frac{k_l a^2}{2M}-\lambda^2\right)\frac{\partial^2u_{10}}{\partial\xi_l^2}+\frac{k_m b^2}{2M}\frac{\partial^2u_{10}}{\partial\zeta_m^2}=0,\\
&\mathrm{i}\frac{\partial u_1}{\partial\tau}+\frac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}\frac{\partial^2u_1}{\partial\xi_l^2}+\frac{k_m b^2}{2M\omega}\frac{\partial^2u_1}{\partial\zeta_m^2}-\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]}u_1\left|\psi_1\right|^2\\
&-\left(24\frac{\beta_l}{M\omega}\sin^4\frac{q_l a}{2}+24\frac{\beta_m}{M\omega}\right)u_1\left|u_1\right|^2=0,\\
&\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+J_l a^2\cos(q_l a)\frac{\partial^2\psi_1}{\partial\xi_l^2}+J_m b^2\frac{\partial^2\psi_1}{\partial\zeta_m^2}-a\chi_l\frac{\partial u_{10}}{\partial\xi_l}\psi_1-b\chi_m\frac{\partial u_{10}}{\partial\zeta_m}\psi_1+\frac{2\chi_l^2\sin^2(q_l a)}{\omega+J_l\left(\cos(2q_l a)-1\right)}\psi_1\left|u_1\right|^2=0.
\end{aligned}\tag{28}$$

Let $u_1=\gamma\psi_1$ and $\dfrac{2M\lambda^2-k_l a^2}{4\chi_l a}\dfrac{\partial u_{10}}{\partial\xi_l}=-\dfrac{k_m a}{4\chi_m}\dfrac{\partial u_{10}}{\partial\zeta_m}=\left|\psi_1\right|^2$, we have

$$\begin{aligned}
&\mathrm{i}\frac{\partial u_1}{\partial \tau}+\frac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}\frac{\partial^2 u_1}{\partial \xi_l^2}+\frac{k_m b^2}{2M\omega}\frac{\partial^2 u_1}{\partial \zeta_m^2}-\left[\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]\gamma^2}\right.\\
&\left.+\left(24\frac{\beta_l}{M\omega}\sin^4\frac{q_l a}{2}+24\frac{\beta_m}{M\omega}\right)\right]|u_1|^2 u_1=0,\\
&\mathrm{i}\frac{\partial \psi_1}{\partial \tau}+J_l a^2\cos(q_l a)\frac{\partial^2 \psi_1}{\partial \xi_l^2}+J_m b^2\frac{\partial^2 \psi_1}{\partial \zeta_m^2}+\left[\frac{4\chi_m^2}{k_m}-\frac{4\chi_l^2 a^2}{2M\lambda_2-k_l a^2}+\frac{2\chi_l^2\gamma^2\sin^2(q_l a)}{\omega+J_l\left(\cos(2q_l a)-1\right)}\right]\psi_1|\psi_1|^2=0.
\end{aligned}\tag{29}$$

Let $\Lambda_1=\dfrac{k_l a^2\cos(q_l a)-M\lambda^2}{2M\omega}$, $\Delta_1=\dfrac{k_m b^2}{2M\omega}$,

$$\Gamma_1=-\left[\frac{\chi_l^2\sin^2(q_l a)}{M\omega\left[\omega+J_l\left(\cos(2q_l a)-1\right)\right]\gamma^2}+\left(24\frac{\beta_l}{M\omega}\sin^4\frac{q_l a}{2}+24\frac{\beta_m}{M\omega}\right)\right],$$

$\Lambda_2=J_l a^2\cos(q_l a)$, $\Delta_2=J_m b^2$, $\Gamma_2=\dfrac{4\chi_m^2}{k_m}-\dfrac{4\chi_l^2 a^2}{2M\lambda_2-k_l a^2}+\dfrac{2\chi_l^2\gamma^2\sin^2(q_l a)}{\omega+J_l\left(\cos(2q_l a)-1\right)}$, Eq.(29) can be rewritten as follows:

$$\begin{aligned}
&\mathrm{i}\frac{\partial u_1}{\partial \tau}+\Lambda_1\frac{\partial^2 u_1}{\partial \xi_l^2}+\Delta_1\frac{\partial^2 u_1}{\partial \zeta_m^2}+\Gamma_1|u_1|^2 u_1=0,\\
&\mathrm{i}\frac{\partial \psi_1}{\partial \tau}+\Lambda_2\frac{\partial^2 \psi_1}{\partial \xi_l^2}+\Delta_2\frac{\partial^2 \psi_1}{\partial \zeta_m^2}+\Gamma_2|\psi_1|^2\psi_1=0.
\end{aligned}\tag{30}$$

When $\Lambda_1\Delta_1>0$, $\Lambda_1\Gamma_1>0$ and $\Lambda_2\Delta_2>0$, $\Lambda_2\Gamma_2>0$, Eqs.(30) have the solutions as follows:

$$\begin{aligned}
u_{10}&=\pm A_0/B_0\tanh\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right],\\
u_1&=\pm A_0\operatorname{sech}\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot \mathrm{e}^{\mathrm{i}\,\Omega t},\\
\psi_1&=\pm C_0\operatorname{sech}\left[B_1\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot \mathrm{e}^{\mathrm{i}\,\Omega t}.
\end{aligned}\tag{31}$$

Thus, Eqs.(26) have the following solutions

$$\begin{aligned}
u_n&=\pm A_0/B_0\tanh\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\\
&\quad\pm A_0\operatorname{sech}\left[B_0\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\cos\left[q_l la+q_m mb-(\Omega+\omega)t\right],\\
\psi_1&=\pm C_0\operatorname{sech}\left[B_1\sqrt{(la-\lambda t)^2+m^2b^2}\right]\cdot\cos\left[q_l la+q_m mb-(\Omega+\omega)t\right],
\end{aligned}\tag{32}$$

where $A_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_1+\Delta_1\right)\Gamma_1}}$, $B_0=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_1+\Delta_1\right)}$, $C_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_2+\Delta_2\right)\Gamma_2}}$, $B_1=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_2+\Delta_2\right)}$.

The solutions (32) can also be shown in Fig.1, that is that motional discrete breathers can exist in the two-dimensional discrete quartic nonlinear molecular lattice with the interactions of excitons with phonons at quasi-discreteness approximation, while Davydov's solitons can also be transformed into discrete exciton-breathers and move in it.

5. The self-trapping in a two-dimensional discrete molecular lattice with cubic & quartic nonlinear potential

The nonlinear vibration equation of the system is

$$
\begin{aligned}
M\ddot{u}_{l,m} &= k_l\left(u_{l+1,m}+u_{l-1,m}-2u_{l,m}\right)+k_m\left(u_{l,m+1}+u_{l,m-1}-2u_{l,m}\right)\\
&\quad+\alpha_l\left[\left(u_{l+1,m}-u_{l,m}\right)^2+\left(u_{l-1,m}-u_{l,m}\right)^2\right]\\
&\quad+\alpha_m\left[\left(u_{l,m+1}-u_{l,m}\right)^2+\left(u_{l,m-1}-u_{l,m}\right)^2\right]+\beta_l\left[\left(u_{l+1,m}-u_{l,m}\right)^3+\left(u_{l-1,m}-u_{l,m}\right)^3\right]\\
&\quad+\beta_m\left[\left(u_{l,m+1}-u_{l,m}\right)^3+\left(u_{l,m-1}-u_{l,m}\right)^3\right]\\
&\quad+\chi_l\left(\left|\psi_{l+1,m}\right|^2-\left|\psi_{l-1,m}\right|^2\right)+\chi_m\left(\left|\psi_{l,m+1}\right|^2-\left|\psi_{l,m-1}\right|^2\right),\\
\mathrm{i}\dot{\psi}_{l,m} &= -J_l\left(\psi_{l+1,m}+\psi_{l-1,m}-2\psi_{l,m}\right)-J_m\left(\psi_{l,m+1}+\psi_{l,m-1}-2\psi_{l,m}\right)\\
&\quad+\chi_l\left(u_{l+1,m}-u_{l-1,m}\right)\psi_{l,m}+\chi_m\left(u_{l,m+1}-u_{l,m-1}\right)\psi_{l,m}.
\end{aligned}\tag{33}
$$

By the same procedure as above, we obtain

$$
\begin{aligned}
&\mathrm{i}\frac{\partial u_1}{\partial\tau}+\Lambda_1\frac{\partial^2 u_1}{\partial\xi_l^2}+\Delta_1\frac{\partial^2 u_1}{\partial\zeta_m^2}+\Gamma_1\left|u_1\right|^2u_1=0,\\
&\mathrm{i}\frac{\partial\psi_1}{\partial\tau}+\Lambda_2\frac{\partial^2\psi_1}{\partial\xi_l^2}+\Delta_2\frac{\partial^2\psi_1}{\partial\zeta_m^2}+\Gamma_2\left|\psi_1\right|^2\psi_1=0,
\end{aligned}\tag{34}
$$

where $\Lambda_1=\dfrac{k_la^2\cos(q_la)-M\lambda^2}{2M\omega}$, $\Delta_1=-\dfrac{k_mb^2}{2M\omega}$, $\Lambda_2=J_la^2\cos(q_la)$, $\Delta_2=J_mb^2$,

$$
\begin{aligned}
\Gamma_1 &= -\frac{\chi_l^2\sin^2(q_la)}{M\omega\left[\omega+J_l\left(\cos(2q_la)-1\right)\right]\gamma^2}+\frac{8\alpha_la\left(\cos(q_la)-1\right)\left\{\chi_la+2M\alpha_la\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]\right\}}{\omega\left(2M\lambda^2-k_la^2\right)\gamma^2}\\
&\quad+\frac{4\alpha_m\left(4\chi_m+16M\alpha_m\gamma^2\right)}{k_m\omega\gamma^2}+\frac{8\alpha_lM}{k_l\omega}\cot\frac{q_la}{2}\sin^2\frac{q_la}{2}\sin(q_la)-\left(24\frac{\beta_l}{M\omega}\sin^4\frac{q_la}{2}+24\frac{\beta_m}{M\omega}\right),\\
\Gamma_2 &= \frac{2\chi_l^2\gamma^2\sin^2(q_la)}{\omega+J_l\left(\cos(2q_la)-1\right)}-\frac{a^2\chi_l\left\{4\chi_l+8M\alpha_l\gamma^2\left[\cos(q_la)\left(\cos(q_la)-1\right)+\sin(q_la)\right]\right\}}{2M\lambda^2-k_la^2}\\
&\quad+\frac{2\alpha_lM\chi_l\gamma^2}{k_l}\cot\frac{q_la}{2}\sin(2q_la)-\frac{\chi_m\left(4\chi_m+16M\alpha_m\gamma^2\right)}{k_m}.
\end{aligned}
$$

When $\Lambda_1\Delta_1>0$, $\Lambda_1\Gamma_1>0$ and $\Lambda_2\Delta_2>0$, $\Lambda_2\Gamma_2>0$, Eqs.(34) have the solutions as follows:

$$
\begin{aligned}
u_{10} &= \pm A_0/B_0\tanh\left[B_0\sqrt{\left(la-\lambda t\right)^2+m^2b^2}\right],\\
u_1 &= \pm A_0\operatorname{sech}\left[B_0\sqrt{\left(la-\lambda t\right)^2+m^2b^2}\right]\cdot\mathrm{e}^{\mathrm{i}\,\Omega t},\\
\psi_1 &= \pm C_0\operatorname{sech}\left[B_1\sqrt{\left(la-\lambda t\right)^2+m^2b^2}\right]\cdot\mathrm{e}^{\mathrm{i}\,\Omega t},
\end{aligned}\tag{35}
$$

Thus, Eqs.(33) have the following solutions

$$
\begin{aligned}
u_n &= \pm A_0/B_0\tanh\left[B_0\sqrt{\left(la-\lambda t\right)^2+m^2b^2}\right]\\
&\quad\pm A_0\operatorname{sech}\left[B_0\sqrt{\left(la-\lambda t\right)^2+m^2b^2}\right]\cdot\cos\left[q_lla+q_mmb-\left(\Omega+\omega\right)t\right],\\
\psi_1 &= \pm C_0\operatorname{sech}\left[B_1\sqrt{\left(la-\lambda t\right)^2+m^2b^2}\right]\cdot\cos\left[q_lla+q_mmb-\left(\Omega+\omega\right)t\right],
\end{aligned}\tag{36}
$$

where $A_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_1+\Delta_1\right)\Gamma_1}}$, $B_0=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_1+\Delta_1\right)}$, $C_0=\sqrt{\dfrac{\Omega\left(\Omega-2\lambda/l\right)}{2\left(\Lambda_2+\Delta_2\right)\Gamma_2}}$, $B_1=\dfrac{\sqrt{\Omega\left(\Omega-2\lambda/l\right)}}{2\left(\Lambda_2+\Delta_2\right)}$.

The solutions (32) can also be shown in Fig.1, that is that motional discrete breathers can exist in the two-dimensional discrete cubic and quartic nonlinear molecular lattice with the interactions of excitons with phonons at quasi-discreteness approximation, while Davydov's solitons can also be transformed into discrete exciton-breathers and move in it.

6. Conclusions

In this paper we investigate the interactions of lattice phonons with the Frenkel exciton (the exciton with a small radius) in a two-dimensional discrete molecular lattice, by virtue of the quasi-discreteness approximation and the method of multiple-scale, and obtain the following conclusions:

(i) When the interactions of lattice phonons with Frenkel excitons are taken into account, the self-trapping (or self-localization) can also appear in the two-dimensional discrete molecular lattice. Motional discrete breathers can exist in the two-dimensional discrete molecular lattice with the interactions of excitons with phonons at quasi-discreteness approximation, while Davydov's solitons can be transformed into discrete exciton-breathers and move in the molecular lattice.

(ii) The difference from the conclusion at continuum limit approximation[29] is that the discreteness of the lattice makes the periodicity of the localized mode dominance. The difference from the conclusion in the one-dimensional discrete molecular lattice is that the excitons' effect on the molecular lattice does not distort the molecular lattice but only cause it to localize.

(iii) When consider the two-dimensional discrete molecular lattice with cubic nonlinear potential, quartic nonlinear potential, and cubic and quartic potential, the conclusions are the same as in the two-dimensional discrete molecular lattice with harmonic potential.

Therefore, we conclude that the self-trapping can also appear in the two-dimensional discrete molecular lattice with harmonic and nonlinear potential. The excitons' effect on the molecular lattice does not distort the molecular lattice but only causes it to localize and the localization can also react again through phonon coupling to trap the energy and prevent its dispersion.

References

[1] Davydov A S 1973 *J. Theor. Biol.* **38** 559
[2] Davydov A S 1985 *Solitons in Molecular System* (Reidel: Dordrecht)
Davydov A S 1982 *Usp. Fiz. Nauk.* **25** 898
[3] Davydov A S 1991 *Solitons in Molecular System* 2nd ed. (Reidel: Dordrecht)
[4] Davydov A S 1982 *Biology and Quantum mechanics* (New York: Pergamon)
[5] Davydov A S 1974 *Biofiz* **19** 670 [Biophys 1974 **19** 684]
[6] Davydov A S 1977 *J. Theor. Biol.* **66** 379
[7] Davydov A S 1979 *Int. J. Quantum Chem.* **16** 5
[8] Landau L D 1933 *Phys. Z. Sowjetunion.* **3** 664
[9] Pekar I 1954 *Untersuchungen Uber Die Electronentheorie der Kristalle* (Berlin: Akadamie Verlag)
[10] Frohlich H 1952 *Proc. R. Soc. London* A **215** 291
[11] Holstein T 1959 *Ann. Phys.* **8** 325
[12] Gerlach S and Lowen H 1991 *Rev. Mod. Phys.* **63** 63
[13] Scott A C 1982 *Phys. Rev.* A **26** 570
[14] Lomdahl P S and Ker W C 1985 *Phys. Rev. Lett.* **55** 1235
[15] Wang X D, Brown D W, Lindenberg K and West B 1988 *Phys. Rev.* A **37** 3557
[16] Scott A 1992 *Phys. Rep.* **217** 1
[17] Kenkre V M, Raghavan S and Cruzeiro-Hansson L 1994 *Phys. Rev.* B **49** 9511
[18] Pang X F 1994 *Phys. Rev.* E **49** 4747
[19] Cruzeiro-Hansson L 1994 *Phys. Rev. Lett.* **73** 2927
[20] Pang X F 2000 *Phys. Rev.* E **62** 6989
[21] Kalosakas G and Aubry S 1998 *Physica* D **113** 228
[22] Kalosakas G, Aubry S and Tsironis G P 1998 *Phys. Rev.* B **58** 3094
[23] Maniadis P, Kalosakas G, Rasmussen K Ø and Bishop A R 2003 *Phys. Rev.* B **68** 174304.
[24] Fuentes M A, Maniadis P, Kalosakas G, Rasmussen K Ø, Bishop A R, Kenkre V M and Gaididei Y B 2004 *Phys. Rev.* E **70** 025601
[25] Cuevas J, Kevrekidis P G., Frantzeskakis D J and Bishop A R 2006 *Phys. Rev.* B **74** 064304
[26] Xu Q, Tang F Y and Tian Q 2008 *Chin. Phys.* B **17** 1881
[27] Xu Q and Tian Q 2005 *Chin. Sci. Bull.* **50** 5
[28] Xu Q and Tian Q 2009 *Chin. Phys.* B **18** 0259
[29] Xu Q and Tian Q 2004 *Acta. Phys. Sin.* **53** 2811 (in Chinese)