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A quasi-discrete Hankel transform for nonlinear beam propagation*

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We propose and implement a quasi-discrete Hankel transform algorithm based on Dini series expansion (DQDHT) in this paper. By making use of the property that the zero-order Bessel function derivative $J_0'(0)$=0, the DQDHT can be used to calculate the values on the symmetry axis directly. In addition, except for the truncated treatment of the input function, no other approximation is made, thus the DQDHT satisfies the discrete Parseval theorem for energy conservation, implying that it has a high numerical accuracy. Further, we have performed several numerical tests. The test results show that the DQDHT has a very high numerical accuracy and keeps energy conservation even after thousands of times of repeating the transform either in a spatial domain or in a frequency domain. Finally, as an example, we have applied the DQDHT to the nonlinear propagation of a Gaussian beam through a Kerr medium system with cylindrical symmetry. The calculated results are found to be in excellent agreement with those based on the conventional 2D-FFT algorithm, while the simulation based on the proposed DQDHT takes much less computing time.

Keywords: Hankel transform, Kerr medium, nonlinear propagation
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1. Introduction

The two-dimensional fast Fourier transform (2D-FFT) and its inverse transform are important tools for dealing with light beam propagation problems.[1−5] For a problem with cylindrical symmetry, the Hankel transform (HT) technique is preferred to the 2D-FFT because the former can be transferred into the one-dimensional zero-order HT and its inverse transform (IHT), making the algorithm of HT fast and computing speed comparable to that of the one-dimensional FFT. The HT and the IHT are respectively expressed as follows:[6]

$$g(\rho) = 2\pi \int_0^\infty r f(r)\, \mathrm{J}_0(2\pi\rho r)\, \mathrm{d}r,$$

and

$$f(r) = 2\pi \int_0^\infty \rho g(\rho)\, \mathrm{J}_0(2\pi\rho r)\, \mathrm{d}\rho, \qquad (1)$$

where r is the radial coordinate; ρ is the spatial frequency; J_0 is the zero-order Bessel function of the first kind; $f(r)$ and $g(\rho)$ can be either real or complex functions and are of axial symmetry mathematically, representing the field distributions in a spatial domain and spatial frequency domain, respectively.

It is very difficult to calculate analytically the integral equation (1), thus a numerical algorithm of high performance for HT is needed. Since the quasi-fast HT (QFHT) was proposed by Siegman in 1977,[7] a number of HT numerical algorithms have been suggested.[8−19] In optics, particularly in nonlinear transmission optics, we often need to perform the HT and the IHT algorithms thousands of times starting from a known analytical expression used as an input light field. Thus an efficient and fast numerical HT algorithm is needed. In addition, it is more desirable if the algorithms of the HT and the IHT can directly calculate the value on the symmetry axis. To our knowledge, the zero-order quasi-discrete HT (QDHT) proposed by Yu *et al* [18] and the pth-order quasi-discrete HT (pQDHT) proposed by Guizar–

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Sicairos and Gutierrez–Vega,[19] both based on the Fourier–Bessel expansion, still have high enough numerical accuracies after repeating the transform thousands of times and thus can be applied to the numerical modelling of cylindrically symmetric nonlinear self-focusing.[20] We note that QDHT based on the Fourier–Bessel series has two deficiencies: first, they cannot be used to calculate the values on the symmetry axis directly, the values on the symmetry axis are usually obtained by additional approximate methods or by imminent methods; second, the Fourier–Bessel series is conditionally convergent in a closed interval $[0, R]$ and is divergent at the end R, thus the HT based on the Fourier–Bessel series has a larger error in the region near the end R for the input function which is not zero at the end, implying that the HT algorithm is somewhat inefficient for calculating divergent beam propagations. Here, we propose a new quasi-discrete Hankel transform algorithm based on the Dini series expansion (DQDHT) which is uniformly convergent in the closed interval $[0, R]$. The DQDHT can be used not only to calculate the values on the symmetry axis directly, but also to investigate the cases where it is needed to repeat the transform a large number of times due to its high accuracy, and thus it is more suitable for dealing with light transmission problems.

2. Derivation of the numerical algorithm

Numerical calculations are usually performed in a finite range, such as QFHT[7] and QDHT.[18] Similarly, we assume that $f(r)$ and $g(\rho)$ in Eq.(1) satisfy the following conditions:

$$f(r > b) = 0, \quad \text{and} \quad g(\rho > \beta) = 0, \tag{2}$$

where b and β are the truncated radii of the spatial variable and the spatial frequency variable, respectively.

Now consider the problem of eigenvalue of the second kind of homogeneous boundary condition. Transform functions $f(r)$ and $g(\rho)$ in Eq.(1) can be rewritten in the form of the Dini series, respectively[21]

$$f(r) = \frac{2}{b^2} \sum_{n=0}^{\infty} f_n \mathrm{J}_0^{-2}(\alpha_n) \mathrm{J}_0\left(\frac{\alpha_n r}{b}\right),$$

and

$$g(\rho) = \frac{2}{\beta^2} \sum_{n=0}^{\infty} g_n \mathrm{J}_0^{-2}(\alpha_n) \mathrm{J}_0\left(\frac{\alpha_n \rho}{\beta}\right), \tag{3}$$

where the coefficients f_n and g_n are given respectively as

$$f_n = \int_0^b r f(r) \mathrm{J}_0\left(\frac{\alpha_n r}{b}\right) \mathrm{d}r = \frac{1}{2\pi} g\left(\frac{\alpha_n}{2\pi b}\right),$$

and

$$g_n = \int_0^\beta \rho g(\rho) \mathrm{J}_0\left(\frac{\alpha_n \rho}{\beta}\right) \mathrm{d}\rho = \frac{1}{2\pi} f\left(\frac{\alpha_n}{2\pi\beta}\right), \tag{4}$$

where α_n $(n = 0, 1, 2, \ldots)$ are the real nonnegative roots of the zero-order Bessel function derivative $\mathrm{J}_0'(x)$, with $\alpha_0 = 0$.

Let $S = 2\pi b\beta$, $r = \alpha_n b/S$ and $\rho = \alpha_n \beta/S$. Given N satisfying $\alpha_N = S$, the coefficients f_n and g_n in expression (4) are identically equal to zero for $n > N$ according to condition (2). Consequently, the sum of infinite terms in expression (3) is turned into the sum of $N+1$ terms. Thus, substituting Eq.(4) into expression (3), we obtain the corresponding formulae of the discrete HT and IHT as

$$g\left(\frac{\alpha_m}{2\pi b}\right) = \frac{1}{\pi\beta^2} \sum_{n=0}^{N} f\left(\frac{\alpha_n}{2\pi\beta}\right) \mathrm{J}_0^{-2}(\alpha_n) \mathrm{J}_0\left(\frac{\alpha_n \alpha_m}{S}\right),$$

$$f\left(\frac{\alpha_n}{2\pi\beta}\right) = \frac{1}{\pi b^2} \sum_{m=0}^{N} g\left(\frac{\alpha_m}{2\pi b}\right) \mathrm{J}_0^{-2}(\alpha_m) \mathrm{J}_0\left(\frac{\alpha_n \alpha_m}{S}\right). \tag{5}$$

In order to reduce the multiplication times in expressions (5), we define the vectors $G(m)$ and $F(n)$ as

$$G(m) = g\left(\frac{\alpha_m}{2\pi b}\right) \left|\mathrm{J}_0^{-1}(\alpha_m)\right| \beta,$$

and

$$F(n) = f\left(\frac{\alpha_n}{2\pi\beta}\right) \left|\mathrm{J}_0^{-1}(\alpha_n)\right| b, \tag{6}$$

and thus expression (5) is simplified into

$$G(m) = \sum_{n=0}^{N} c_{mn} F(n),$$

and

$$F(n) = \sum_{m=0}^{N} c_{nm} G(m), \tag{7}$$

where c_{nm} are the elements of an $(N+1)$-dimensional transformation matrix $\boldsymbol{C}$ and given by

$$c_{nm} = \frac{2}{S} \left|\mathrm{J}_0^{-1}(\alpha_n)\right| \left|\mathrm{J}_0^{-1}(\alpha_m)\right| \mathrm{J}_0\left(\frac{\alpha_n \alpha_m}{S}\right). \tag{8}$$

To preserve the self-consistency of expression (7), the matrix $\boldsymbol{C}$ defined by expression (6) needs to be an orthogonal matrix, namely it satisfies $\boldsymbol{CC} = \boldsymbol{I}$ ($\boldsymbol{I}$ is the corresponding identity matrix) or its determinant $\det[C] = \pm 1$. While C is the function of the single parameter S, i.e. $\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{S})$. So, it is very important to correctly choose S to make $\boldsymbol{C}$ an orthogonal matrix. According to QDHT[18] and pQDHT[19] methods based on the Fourier–Bessel expansion, it can be found that for $S = j_{N+1}$(J_n is nth zero of the zero-order Bessel function), $\boldsymbol{C}$ is also an approximate orthogonal matrix. Further analysis demonstrates that the accuracy will be improved by the following modified expression

$$S = 2\left|\mathrm{J}_0^{-1}(\alpha_k)\right| \sqrt{1 + \sum_{n=1}^{N} \mathrm{J}_0^{-2}(\alpha_n)\, \mathrm{J}_0^2\left(\frac{\alpha_k \alpha_n}{\mathrm{J}_{N+1}}\right)}, \quad (k = 0, 1, \ldots, N), \tag{9}$$

which can be validated by calculating the minimum value of $||\det[\boldsymbol{C}]| - 1|$. For $N = 10$, and $k = 0$ or 2, the value of $||\det[\boldsymbol{C}]| - 1|$ from expression (9) is 8.57×10^{-4} or 8.16×10^{-5}, while the value from $S = j_{N+1}$ is 2.21×10^{-2}. This shows that S from expression (9) makes the matrix $\boldsymbol{C}$ approach an orthogonal matrix much more. k is a parameter for adjusting the accuracy of the result and only related to the sample point number N. For a given N, we find that $||\det[\boldsymbol{C}]| - 1|$ reaches its minimum when $k = \mathrm{Int}(N/4)$ (Int(.) denotes the getting integer operator) by the numerical method. Thereby, in the following numerical calculation, k is exclusively taken as $\mathrm{Int}(N/4)$.

An important feature of the DQDHT is that it is energy preserving, i.e. in the algorithm the Parseval theorem is satisfied

$$\int_0^\infty r\left|f(r)\right|^2 \mathrm{d}r = \int_0^\infty \rho\left|g(\rho)\right|^2 \mathrm{d}\rho. \tag{10}$$

To prove this, substituting expression (3) into Eq.(10), using expression (7), and performing some algebraic calculation, we can obtain the discrete expression for the Parseval theorem as follows:

$$\frac{2}{S^2} \sum_{n=0}^{N} \left|F(n)\right|^2 = \frac{2}{S^2} \sum_{m=0}^{N} \left|G(m)\right|^2, \tag{11}$$

which means that DQDHT can be transformed repeatedly and the conservation of energy is satisfied in both the space domain and the spatial frequency domain.

3. Test and verification of the algorithm

The numerical accuracy of DQDHT is tested by codes written in C^{++}, where $N(N+1)/2$ double precision memory unit can be saved by storing data in the triangle matrix, due to $\boldsymbol{C}$ being real symmetric matrixes. The zeros of the Besssel function and its derivative can be calculated by directly solving the nonlinear equation or referring to Ref.[22].

First, we test the DQDHT by using the continuous function $f(r) = r^2 \exp(-\pi r^2)$ as an input function. Its exact expression of zero-order HT in the interval $[0, \infty)$ is given as follows:

$$g(\rho) = \exp\left(-\pi\rho^2\right)\left(1 - \pi\rho^2\right) \ / \ \pi. \tag{12}$$

Thereby, we define the absolute error in the space frequency domain as $|g - g^*|$, where $g^*(\rho)$ is the transformation value of DQDHT and we represent the maximum absolute error as MaxE, the minimum as MinE, and the absolute value of the mean error $|g - g^*|/N$ as MeanE. Taking $b = \beta = (S/2\pi)^{0.5}$, the absolute errors are shown in Table 1. It can be seen that all values of the absolute errors are very small, suggesting that the numerical accuracy is rather high for DQDHT using a continuous function as an input function, even when the point number N is small. Additionally, the numerical accuracy can quickly be improved as N increases.

Table 1. Absolute errors of DQDHT for $f(r)$.

N	10	20	30	40
MinE	2.7×10^{-9}	1.7×10^{-16}	8.3×10^{-18}	1.3×10^{-18}
MaxE	1.9×10^{-7}	5.2×10^{-14}	2.8×10^{-16}	1.9×10^{-16}
MeanE	5.1×10^{-8}	9.4×10^{-15}	9.7×10^{-17}	7.8×10^{-17}

Figure 1 shows a comparison between the exact values of function $g(\rho)$ and the corresponding results for performing the continuous DQDHT 101 times

when $N = 100$ and also between their errors. The curve denotes the analytical results, and the solid circle denotes the DQDHT results. It can easily be found that the corresponding MinE, MaxE and MeanE are respectively 2.17×10^{-17}, 1.80×10^{-13} and 6.80×10^{-15}, and the results of DQDHT are in excellent agreement with the exact values of function $g(\rho)$. In addition, on a personal computer with Intel Mobile Pentium IV (1.5 GHz) and EMS memory 512 MB, it takes 0.031 s to finish 101 times of performing DQDHT. Hence, the DQDHT can keep very high numerical accuracy after many times of performing the transformation, and it takes much less computing time as well.

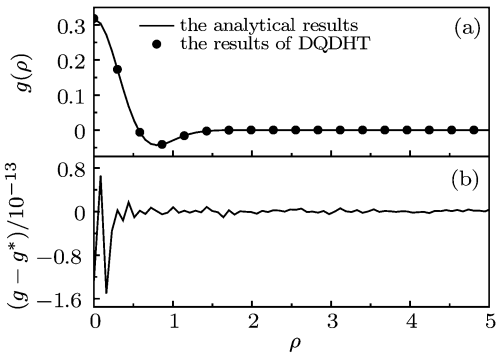


Fig.1. Comparisons between the exact values from function $g(\rho)$ and the corresponding results from 101 times of continuously performing DQDHT (a) and between their errors (b).

Secondly, we take a top-hat function as an input function, and $f(r) = 1$ $(0 \le r \le b)$. Its exact expression of zero-order HT is

$$g(\rho) = \begin{cases} \pi b^2, & (\rho = 0), \\ bJ_1(2\pi b\rho)/\rho, & (\rho > 0). \end{cases} \tag{13}$$

A comparison between the analytical results from expression (11) and the results from the DQDHT with the top-hat function for $b = 5$, $N = 200$ is shown in Fig.2(a), and a comparison of absolute error between the analytical results and the 200-point QDHT and DQDHT as a function of ρ is plotted in Fig.2(b). The lines denote the analytical results and the symbols represent the results from QDHT and DQDHT. In Fig.2(a), it is obvious that the analytical results and the results from DQDHT are in good agreement with each other. In Fig.2(b), it can be seen that the absolute errors are very small, in addition, the solid circle curve is under the sold square curve in the entire spatial frequency interval, indicating that the DQDHT based on Dini series expansion is more accurate than the QDHT based on Fourier–Bessel series expansion. For the former, the absolute error is below about 10^{-5} while that for the latter is above about 10^{-3}. It can also be seen that due to the fact that the value of the spatial spectral function is approximately equal to zero at ρ=17.13, the absolute error for the DQDHT has a minimum value about 10^{-10}.

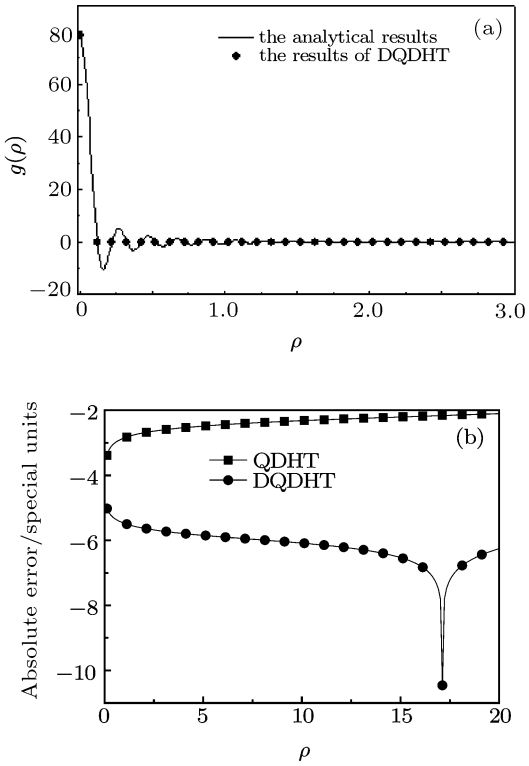


Fig.2. Comparisons between analytical results from HT and results from DQDHT, with a top-hat function used as an input function, for $b = 5$, $N = 200$ (a), and between their absolute errors from the analytical results and the results of QDHT and DQDHT (b).

4. Application of DQDHT to beam propagation

In an optical system, the light beam usually propagates through varieties of elements and media. It is always necessary to divide the numerical simulation of the propagation process into lots of calculation steps among which the output results of the previous step are treated as the input value for the immediately next step, especially for numerical simulation of the optical beam propagation process in a periodic or nonlinear medium, where thousands of repetitions of the numerical calculation are needed. For the generic two-dimensional (2D) simulation of optical propagation, the 2D fast Fourier transform (2DFFT) and its inverse

Fourier transform are commonly applied. In propagation of optical beams through systems with cylindrical symmetry, the Hankel transform and its inverse transform, instead of the 2D Fourier transform and its inverse transform, are applied due to the fact that the former takes much less computing time. However, a higher numerical accuracy of the computation of HT and IHT is needed in relation to that of 2DFT and 2DIFT. That is, an efficient and accurate numerical HT algorithm is crucial for HT applied to optical field propagation and especially to its nonlinear propagation where the HT and the IHT need to be computed thousands of times starting from a known analytical expression used as an input function.

What follows is that a Super–Gaussian beam with spatial chirp is taken as an example to show that the DQDHT given above is very suitable for the numerical simulation of optical beam propagation. Assume that the analytical expression used as an input function is

$$E(r) = E_0 \exp\left[-\frac{1}{2}\left(\frac{r^2}{w^2}\right)^m + \mathrm{i}\, cr^2\right], \qquad (14)$$

where E_0, w, m and c denote the amplitude, the spatial width, the order and the chirp of the Super–Gaussian light beam, respectively. In the following numerical calculation, we take $E_0 = 1$, $w = 1$, $m = 20$, $c = 0.01$, the radius of the calculation region $b = 5$, and the number of the sample points $N = 256$. The variations of the intensity and energy with HT pair number are shown in Figs.3 and 4. After a number of HT pairs, it can be seen from Fig.3 that the values of on-axis intensity accord well with its initial values (Fig.3(a)), and that energy conservation still holds (Fig.3(b)). In order to further demonstrate that the DQDHT above

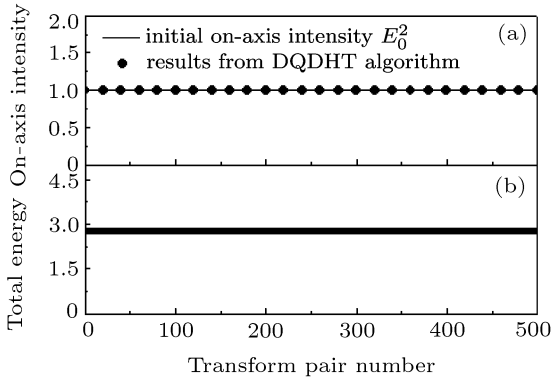


Fig.3. Variations of on-axis intensity and total energy of a chirped 20-order Super-Gaussian light beam with HT pair number. Panel (a) shows a comparison between the on-axis intensity from DQDHT and the initial one; and panel (b) indicates the variation of the total energy of the light beam with HT pair number.

is very accurate, in Fig.4(a) a comparison is made between the intensity distribution after 500 HT pairs and the exact intensity distribution of a chirped 3-order Super–Gaussian light beam. It is found that both results are in good agreement with each other. Additionally, Fig.4(b) shows the absolute errors between them, and it is obvious that the errors are rather small, indicating that the DQDHT is very accurate.

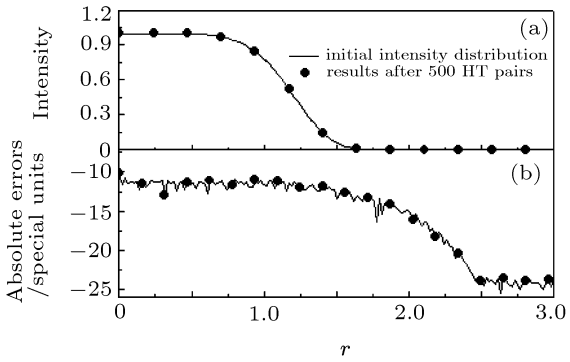


Fig.4. (a) Comparison between intensity distribution after 500 HT pairs and initial intensity, distribution of the chirped 3-order Super–Gaussian light beam; and (b) between their absolute errors.

Furthermore, to demonstrate that the proposed DQDHT is suitable for the simulation of nonlinear propagation, we take the numerical simulation of the self-focusing of a Gaussian light beam propagating through a nonlinear Kerr medium for example. In the nonlinear Kerr medium, the complex amplitude of the total optical field is governed by the following nonlinear Schrödinger equation in the paraxial approximation:[1−5,20,23]

$$2\,\mathrm{i}\,k_0 \frac{\partial A}{\partial\, z} + \nabla_\perp^2 A + 2k_0^2 \frac{n_2}{n_0} |A|^2 A = 0, \qquad (15)$$

where A is the complex amplitude of optical field, $\nabla_\perp^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ denotes the transverse Laplace operator, z is the coordinate variable of propagation axis, n_0 is the linear refractive index, n_2 is the nonlinear coefficient, k_0 is the wavenumber in the vacuum. The second and third terms on the left-hand side of Eq.(15) account for diffraction and Kerr nonlinearity, respectively. Generally, the symmetric split-step fast Fourier transform method (SSFFT) is used to numerically simulate the nonlinear propagation, in which the algorithm of each step can be written as[23]

$$A(x, y; z + h) = \Im^{-1} \exp\left(\hat{D}\frac{h}{2}\right) \Im \exp\left(\hat{N}h\right) \Im^{-1} \times \exp\left(\hat{D}\frac{h}{2}\right) \Im A(x, y; z), \qquad (16)$$

where $\Im$ and $\Im^{-1}$ denote the 2DFFT and its inverse Fourier transform, respectively, h denotes the step length of calculation, $\hat{D}$ and $\hat{N}$ represent the linear and the nonlinear operators, respectively, which are defined as

$$\hat{D} = \mathrm{i}\,\frac{1}{2k_0}\nabla_\perp^2, \quad \text{and} \quad \hat{N} = \mathrm{i}\,k_0\frac{n_2}{n_0}\,|A|^2\,. \tag{17}$$

For the case of cylindrical symmetry, the Fourier transform and the inverse Fourier transform can both be replaced with the Hankel transform. Thereby, the complex amplitude of the output optical field can be worked out from that of the incident optical field by repeating the operation in expression (16).

To this end, we numerically calculate expression (16) by the 2DFFT and the proposed DQDHT separately and make a comparison between the obtained results. In the numerical calculation, the incident light beam is modelled as a Gaussian light beam with $m = 1$, $E_0^2 = 1.0$, $w = 2.5$, and $c = 0$. Set the wavelength of the optical field $\lambda = 1.32$, the linear refraction index $n_0 = 1$ and the nonlinear coefficient $n_2 = 0.02$. The propagation distance z and the propagation step number are taken to be 25λ and 500 respectively. The calculation regions are sampled as 512×512 in 2DFFT and 256 in DQDHT, separately. The obtained results are shown in Figs.5 and 6. From Fig.5, it is clearly seen that at a distance of 21.75λ, i.e. $z = 21.75\lambda$, the Gaussian light beam collapses due to the nonlinear self-focusing effect. The results based on the 2DFFT and the inverse Fourier transform are found to be in good agreement with those based on the proposed DQDHT. In addition, for a given computer, the time consumed by the above numerical calculation based on 2DFFT is 764.68 s, while the time consumed by that based on DQDHT is only 3.76 s. This suggests that for the simulation of optical beam nonlinear propagation in a system with cylindrical symmetry, substituting DQDHT for 2DFFT can greatly enhance the efficiency of the numerical calculation.

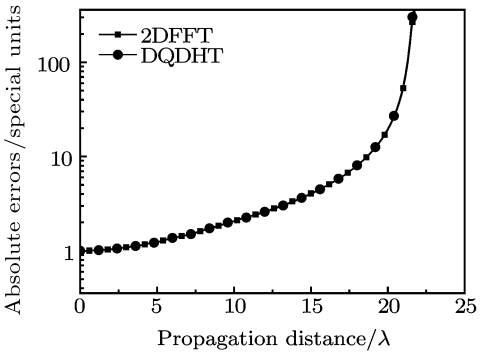


Fig.5. Variation of the on-axis intensity of the optical beam with propagation distance.

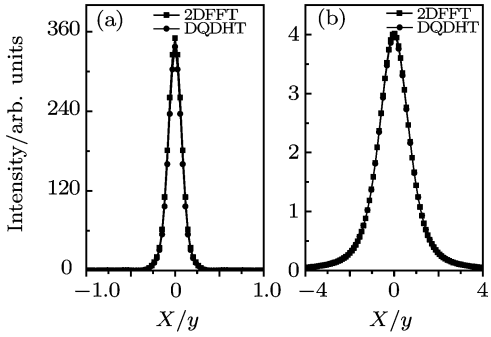


Fig.6. Intensity distributions (a) on the focus ($z = 21.75\lambda$) and (b) in front of the focus ($z = 21\lambda$).

5. Conclusions

Based on Dini series expansion, a discrete expression for the zero-order Hankel transform in an arbitrary finite-range is derived, and thereby a DQDHT is proposed. By making use of the property that the first zero of the zero-order Bessel function derivative is zero, the proposed DQDHT can be applied to directly working out the on-axis values starting from a known analytical expression used as an input function with cylindrical symmetry. In addition, except for the truncated treatment of the input function, no other approximation is made, which leads the DQDHT to possess higher numerical accuracy and keep the discrete form of the Parseval theorem of energy conservation holding, and makes it suitable for thousands of repetitions of HT and IHT algorithms.

The numerical accuracy of the DQDHT is tested in a frequency domain by using two different input functions (a continuous function and a top-hat function) and in a spatial domain by using a chirped Super–Gaussian function. The results show that the DQDHT has a rather high numerical accuracy and can keep energy conservation holding even after thousands of times of performing the transform either in the frequency domain or in the spatial domain. At the same time, the numerical simulation of a Gaussian light beam propagating through a nonlinear Kerr medium system with cylindrical symmetry is performed, based on the proposed DQDHT and the conventional 2DFFT algorithm, separately. Both results are found to be in excellent agreement with each other, while the simulation based on the proposed DQDHT takes much less computing time.

The general fast HT algorithm reported in the literature usually has a larger approximation in the truncated area. So it can neither directly be used to calculate the values on the symmetric axis nor keep the energy conservation holding simultaneously. However, the DQDHT proposed by us can not only directly be used to calculate the values on the symmetric axis, but also can keep the energy conservation holding and require less computing time as well, therefore it is more suitable for the numerical simulation of an optical beam propagating through systems with cylindrical symmetry.

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