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Scheme for implementing frequency up-conversion between two collective atomic modes*

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This paper presents a scheme for realizing the frequency up-conversion between two collective atomic modes. In the scheme two atomic samples are coupled to a cavity mode. Under the large detuning condition, the two collective atomic modes are coupled via the virtual excitation of the cavity mode and the effective Hamiltonian corresponds to the frequency up-conversion. In the scheme the cavity mode is only virtually excited and thus the process is insensitive to cavity decay.

Keywords: frequency up-conversion, collective atomic mode, virtual excitation
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The interaction between atoms and quantized electromagnetic fields has been a central topic in quantum optics. The Jaynes–Cummings model (JCM),[1,2] which comprises a two-level atom interacting with a single-mode field, is a fully quantum mechanical model. Due to the quantization of the field, the system may exhibit a number of nonclassical effects, including the collapse and revival of the oscillation amplitude,[3] squeezing of the field,[4] and atom–cavity entanglement.[5] Recent experiments with Rydberg atoms and microwave photons in a superconducting cavity have turned the JCM from a theoretical curiosity into a useful and testable enterprise.[6] In the past few decades, much effort has been devoted to generalizations and extensions of the JCM, including multiatom, multilevel, and multimode generalizations.

The cavity QED system is a qualified candidate for entanglement engineering and quantum information processing since atoms are ideal for storing information and photons are suitable for transporting information. Schemes have been proposed for generation of entangled states and implementation of elementary quantum information processors via atom–cavity interaction.[7−11] Three-atom entangled states and two-qubit quantum phase gates have been demonstrated in microwave cavity QED experiments.[12,13] Besides the atom–cavity coupling, the cavity QED system might provide a possibility for engineering two-mode coupling. Recently, schemes have been proposed for implementing frequency up- and down-conversions via the interaction of two cavity modes with a single or more atoms.[14−16]

Previous research on two-mode coupling in cavity QED concentrates on cavity modes. In this paper we propose a scheme for realizing the frequency up-conversion for two collective atomic modes. In the scheme two collections of two-level atoms are coupled to a cavity mode which is initially in the vacuum state. Meanwhile, the first atomic sample is driven by a classical field. Under the large detuning condition, the cavity mode remains in the vacuum state and the two atomic ensembles are coupled via virtual excitation of the cavity mode. If the number of atoms in each ensemble is large enough the atomic ensembles can be treated as bosonic modes and the dynamics of the two collective atomic modes is described by the frequency up-conversion.

We first consider a pair of atomic samples loaded in a single-mode cavity. The first atomic sample is driven by a clasiscal field. Each atom has one excited state $|e\rangle$ and one ground state $|g\rangle$. The Hamiltonian is (assuming $\hbar = 1$)

$$H = H_0 + H_{\rm i}, \tag{1}$$

where

$$H_0 = \omega a^+ a + \omega_0 \sum_{j=1}^{N_1} S_{1,z,j} + \omega_0 \sum_{j=1}^{N_2} S_{2,z,j}, \tag{2}$$

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$$
\begin{aligned}
H_{\mathrm{i}} = & \Omega \sum_{j=1}^{N_1}(S_{1,j}^{-}\mathrm{e}^{\mathrm{i}\omega_{\mathrm{d}}t}+S_{1,j}^{+}\mathrm{e}^{-\mathrm{i}\omega_{\mathrm{d}}t}) \\
& +g\sum_{j=1}^{N_1}(a^{+}S_{1,j}^{-}+aS_{1,j}^{+}) \\
& +g\sum_{j=1}^{N_2}(a^{+}S_{2,j}^{-}+aS_{2,j}^{+}), \qquad (3)
\end{aligned}
$$

where $S_{l,z,j}=\frac{1}{2}(|e_{l,j}\rangle\langle e_{l,j}|-|g_{l,j}\rangle\langle g_{l,j}|)$, $S_{l,j}^{+}=|e_{l,j}\rangle\langle g_{l,j}|$, $S_{l,j}^{-}=|g_{l,j}\rangle\langle e_{l,j}|$, a^{+} and a are the creation and annihilation operators for the cavity mode respectively, ω_0 is the atomic transition frequency, ω is the cavity frequency, g is the atom–cavity coupling strength, and N_l is the number of atoms in the lth atomic sample. The subscript $l=1,2$ labels the atomic samples.

Suppose $\delta_{\mathrm{c}}=\omega_0-\omega_{\mathrm{c}}\gg g\sqrt{N_l(\bar{n}+1)}$, where $\bar{n}$ is the mean photon number of the cavity field. Then the atomic systems do not exchange energy with the cavity mode. However, due to the virtual excitation of the cavity mode each atom undergoes a photon-number dependent Stark shift and is coupled with other atoms. Under the condition $\delta_{\mathrm{d}}=\omega_0-\omega_{\mathrm{d}}\gg\Omega$, the classical field only induces a Stark shift. Then the effective Hamiltonian is[7,8]

$$
\begin{aligned}
H_{\mathrm{e}} = & \lambda_{\mathrm{d}}\sum_{j=1}^{N_1}(|e_{1,j}\rangle\langle e_{1,j}|-|g_{1,j}\rangle\langle g_{1,j}|) \\
& +\lambda_{\mathrm{c}}\sum_{j=1}^{N_1}(|e_{1,j}\rangle\langle e_{1,j}|-|g_{1,j}\rangle\langle g_{1,j}|)a^{+}a \\
& +\lambda_{\mathrm{c}}\sum_{j=1}^{N_2}(|e_{2,j}\rangle\langle e_{2,j}|-|g_{2,j}\rangle\langle g_{2,j}|)a^{+}a \\
& +\lambda_{\mathrm{c}}\sum_{j,k=1}^{N_1}S_{1,j}^{+}S_{1,k}^{-}+\lambda_{\mathrm{c}}\sum_{j,k=1}^{N_2}S_{2,j}^{+}S_{2,k}^{-} \\
& +\lambda_{\mathrm{c}}\sum_{p=1}^{N_1}\sum_{q=1}^{N_2}(S_{1,p}^{+}S_{2,q}^{-}+S_{1,p}^{-}S_{2,q}^{+}), \qquad (4)
\end{aligned}
$$

where $\lambda_{\mathrm{d}}=\Omega^2/\delta_{\mathrm{d}}$ and $\lambda_{\mathrm{c}}=g^2/\delta_{\mathrm{c}}$. As the photon-number operator commutes with the effective Hamiltonian, the photon-number is conserved during the interaction. Assuming that the cavity is initially in the vacuum state, we have $a^{+}a=0$ and the effective Hamiltonian reduces to

$$
\begin{aligned}
H_{\mathrm{e}} = & \lambda_{\mathrm{d}}\sum_{j=1}^{N_1}(|e_{1,j}\rangle\langle e_{1,j}|-|g_{1,j}\rangle\langle g_{1,j}|) \\
& +\lambda_{\mathrm{c}}\left[\sum_{j,k=1}^{N_1}S_{1,j}^{+}S_{1,k}^{-}+\sum_{j,k=1}^{N_2}S_{2,j}^{+}S_{2,k}^{-}\right] \\
& +\lambda_{\mathrm{c}}\sum_{p=1}^{N_1}\sum_{q=1}^{N_2}(S_{1,p}^{+}S_{2,q}^{-}+S_{1,p}^{-}S_{2,q}^{+}). \qquad (5)
\end{aligned}
$$

We can rewrite the effective Hamiltonian as

$$
\begin{aligned}
H_{\mathrm{e}} = & 2\lambda_{\mathrm{d}}\sum_{j=1}^{N_1}|e_{1,j}\rangle\langle e_{1,j}| \\
& +\lambda_{\mathrm{c}}\left[\sum_{j,k=1}^{N_1}S_{1,j}^{+}S_{1,k}^{-}+\sum_{j,k=1}^{N_2}S_{2,j}^{+}S_{2,k}^{-}\right] \\
& +\lambda_{\mathrm{c}}\sum_{p=1}^{N_1}\sum_{q=1}^{N_2}(S_{1,p}^{+}S_{2,q}^{-}+S_{1,p}^{-}S_{2,q}^{+}), \qquad (6)
\end{aligned}
$$

where we have discarded the constant energy $-\lambda_{\mathrm{d}}\sum_{j=1}^{N_1}(|e_{1,j}\rangle\langle e_{1,j}|+|g_{1,j}\rangle\langle g_{1,j}|)$, which can only induce a trivial common phase. Set

$$
\begin{aligned}
b_l &= \frac{1}{\sqrt{N_l}}\sum_{j=1}^{N_l}|g_{l,j}\rangle\langle e_{l,j}|, \\
b_l^{+} &= \frac{1}{\sqrt{N_l}}\sum_{j=1}^{N_l}|e_{l,j}\rangle\langle g_{l,j}|, \\
n_l &= \sum_{j=1}^{N_l}|e_{l,j}\rangle\langle e_{l,j}|, \qquad (7)
\end{aligned}
$$

then we have

$$
\begin{aligned}
&\left[b_l,b_l^{+}\right]=1-\frac{2}{N_l}n_l, \\
&\left[n_l,b^{+}\right]=b^{+}, \\
&[n_l,b]=-b. \qquad (8)
\end{aligned}
$$

Let $\bar{n}_l$ be the average number of atoms in the lth atomic sample being in the state $|e\rangle$. Under the conditions $N_l\gg 1$ and $N_l\gg\bar{n}_l$, the collective atomic operators b_l and b_l^{+} can be regarded as bosonic operators and each atomic system can be regarded as a bosonic system. Then the effective Hamiltonian can be rewritten as

$$
H_{\mathrm{e}}=H_{\mathrm{e},0}+H_{\mathrm{e,i}}, \qquad (9)
$$

where

$$
H_{\mathrm{e},0}=(2\lambda_{\mathrm{d}}+N_1\lambda_{\mathrm{c}})b_1^{+}b_1+N_2\lambda_{\mathrm{c}}b_2^{+}b_2, \qquad (10)
$$

$$
H_{\mathrm{e,i}}=\sqrt{N_1N_2}\lambda_{\mathrm{c}}(b_1^{+}b_2+b_1b_2^{+}). \qquad (11)
$$

The time evolution of the whole system is determined by Schrödinger's equation:

$$
\mathrm{i}\frac{\mathrm{d}|\psi(\mathrm{t})\rangle}{\mathrm{d}t}=H_{\mathrm{e},0}|\psi(t)\rangle. \qquad (12)
$$

Performing the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_{\mathrm{e},0}t}|\psi'(t)\rangle, \tag{13}$$

we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(\mathrm{t})\rangle}{\mathrm{d}t} = H'_{\mathrm{e,i}}|\psi'(t)\rangle, \tag{14}$$

where

$$H'_{\mathrm{e,i}} = \sqrt{N_1N_2}\lambda_{\mathrm{c}}(\mathrm{e}^{\mathrm{i}\delta t}b_1^+b_2 + \mathrm{e}^{-\mathrm{i}\delta t}b_1b_2^+), \tag{15}$$

where $\delta = 2\lambda_{\mathrm{d}} + N_1\lambda_{\mathrm{c}} - N_2\lambda_{\mathrm{c}}$. With the choice $2\lambda_{\mathrm{d}} = N_2\lambda_{\mathrm{c}} - N_1\lambda_{\mathrm{c}}$, we have

$$H'_{\mathrm{e,i}} = \sqrt{N_1N_2}\lambda_{\mathrm{c}}(b_1^+b_2 + b_1b_2^+). \tag{16}$$

The Hamiltonian $H'_{\mathrm{e,i}}$ describes the frequency up-conversion process, in which the creation of one quantum in one mode is accompanied with the annihilation of one quantum in the other mode and the total quantum number is conserved. The evolution of the two collective atomic modes is given by

$$S(\xi) = \mathrm{e}^{(\xi b_1^+b_2 + \xi^* b_1b_2^+)}, \tag{17}$$

where $\xi = -\mathrm{i}\sqrt{N_1N_2}\lambda_{\mathrm{c}}t$, with t being the interaction time.

The frequency up-conversion can be used to produce important entangled states for the two collective atomic modes. Suppose that the two modes are initially in the Fock state $|1\rangle_a|0\rangle_b$. After coupling the two modes evolve to

$$|\psi'(t)\rangle = \cos\theta\,|1\rangle_a|0\rangle_b - \mathrm{i}\sin\theta\,|0\rangle_a|1\rangle_b, \tag{18}$$

where $\theta = \sqrt{N_1N_2}\lambda_{\mathrm{c}}t$. With the choice $t = \pi/(4\sqrt{N_1N_2}\lambda_{\mathrm{c}})$, we obtain the maximally entangled state

$$|\psi'(t)\rangle = \frac{1}{\sqrt{2}}(|1\rangle_a|0\rangle_b - \mathrm{i}\,|0\rangle_a|1\rangle_b). \tag{19}$$

The frequency up-conversion process can also be used to transfer the state from one collective atomic mode to the other mode or exchange the quantum states between these two modes.[17]

We now briefly address the experimental feasibility of the proposed scheme. In recent cavity QED experiments with Rydberg atoms,[12,13] the atomic radiative time is $T_{\mathrm{r}} = 3\times10^{-2}$ s, the cavity damping time is about $T_{\mathrm{c}} = 10^{-3}$ s, and the coupling constant is $g = 2\pi\times25$ kHz. Set $N_1 \sim N_2 \sim 100$ and $\delta \sim 100$ g. The interaction time needed to produce the maximally entangled state for the two modes is $t = 5\times10^{-6}$ s. The infidelity induced by the atomic decay is about $t/T_{\mathrm{r}} \simeq 1.67\times10^{-4}$. In the scheme, the cavity is only virtually excited and the effective damping time of the cavity is about $T'_{\mathrm{c}} = T_{\mathrm{c}}\delta^2/(N_1g^2) = 10^{-1}$ s. The infidelity due to the cavity decay is about $t/T'_{\mathrm{c}} = 5\times10^{-5}$. Thus the total infidelity is about 2.17×10^{-4}.

In summary, we have proposed a scheme for implementing frequency up-conversion for two collective atomic modes. In the scheme the atomic samples dispersively interact with a cavity mode initially in the vacuum state. The virtual excitation of the cavity mode leads to coupling between the atomic samples. Under certain conditions, the atomic samples can be regarded as bosonic systems and the coupling can be described by the frequency up-conversion. In comparison with the schemes of Refs.[14], [15] and [16], the present scheme discusses the implementation of the frequency up-conversion for two collective atomic modes, rather than two cavity modes. The cavity mode always remains in the vacuum state during the process and thus the present scheme is more insensitive to cavity decay.

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