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Can two partially coherent cosh–Gaussian beams generate far fields with the same spectral degree of coherence?*

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Taking partially coherent cosh–Gaussian (ChG) beams as an example of more general partially coherent beams, we have studied the spectral degree of coherence of partially coherent ChG beams in the far field. It is shown that, unlike Gaussian Schell-model (GSM) beams, in the strict sense there do not exist two partially coherent ChG beams which can generate far fields with the same spectral degree of coherence. However, under certain conditions it is possible to find two partially coherent ChG beams with the same spectral degree of coherence in the far field.

Keywords: spectral degree of coherence, radiant intensity, partially coherent cosh–Gaussian beam
PACC: 4225K, 4260F

1. Introduction

Optical coherence and partially coherent beams are a subject of considerable interest from both the theoretical and practical points of view.[1] It was predicted that under certain conditions a class of partially coherent beams, i.e. Gaussian Schell-model (GSM) beams, would produce the same distribution of radiant intensity in the far field as a fully coherent Gaussian laser beam.[2−4] The prediction was confirmed experimentally.[5,6] On the other hand, it was shown that two GSM beams whose beam width w_0 and correlation length σ_0 satisfy a condition given in Ref.[7] will generate far fields with the same spatial coherence properties. A comparison of Refs.[2]–[4] with Ref.[7] shows that for a fixed frequency the conditions for two GSM beams with the same far-field radiant intensity are not the same as those for two GSM beams with the same far-field spectral degree of coherence. Recently, the generation of a variety of partially coherent light beams has attracted much interest.[8−12] It is reasonable to ask if the above results can be extended to the other type of partially coherent light beams. In a recent work[13] we have shown that only under certain conditions is it possible to find equivalent partially coherent cosh–Gaussian (ChG) beams with the same far-field radiant intensity distribution as a fully coherent laser beam. In this paper, taking partially coherent ChG beams as an example of more general partially coherent beams,[14−16] we examine if two partially coherent ChG beams can generate far fields with the same spectral degree of coherence.

2. Spectral degree of coherence and radiant intensity of partially coherent cosh–Gaussian beams in the far field

Consider a two-dimensional partially coherent ChG beam whose cross-spectral density function at the source plane $z = 0$ reads as

$$W^{(0)}(x_1, x_2, \omega) = \sqrt{S^{(0)}(x_1, \omega)}\sqrt{S^{(0)}(x_2, \omega)}\mu^{(0)}(x_1 - x_2, \omega), \quad (1)$$

where the spectral density (i.e., the spectral intensity[7]) $S^{(0)}(x, \omega)$ and the spectral degree of coherence at frequency ω (i.e., the degree of spatial coherence[7]) $\mu^{(0)}(|x_1 - x_2|, \omega)$ of the source read as[1,17]

$$S^{(0)}(x, \omega) = A \exp\left(-\frac{2x^2}{w_0^2}\right) \cosh^2(\Omega_0 x), \quad (2)$$

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$$\mu^{(0)}(x_1 - x_2, \omega) = \exp\left(-\frac{|x_1 - x_2|^2}{2\sigma_0^2}\right), \quad (3)$$

respectively. In Eqs.(2) and (3) A is a positive constant which represents the spectral density at the position $x = z = 0$, w_0 denotes the waist width of the Gaussian amplitude distribution, Ω_0 is a parameter associated with the cosh part, σ_0 is the correlation length. For simplicity of the mathematical manipulation, Eqs.(1)–(3) are presented in the two-dimensional form, but extension to the three-dimensional case is straightforward because in the framework of the paraxial approximation the x and y variables are separable.

The far-field cross-spectral density function at points P_1 and P_2 specified by the position vectors $r\boldsymbol{s}_1$ and $r\boldsymbol{s}_2$ (see Fig.1) is given by[1]

$$W^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega) = \left(\frac{2\pi k}{r^2}\right)\cos\theta_1 \cos\theta_2 \tilde{W}^{(0)}(-ks_{1x}, ks_{2x}, \omega), \quad (4)$$

where $s_1^2 = s_2^2 = 1$, $\boldsymbol{s} = (s_x,\ s_z)$, $r = \sqrt{x^2 + z^2}$, k is the wave number, and $\tilde{W}^{(0)}(-ks_{1x}, ks_{2x}, \omega)$ is the spatial Fourier transform of $W^{(0)}(x_1,\ x_2,\ \omega)$ of the form

$$\tilde{W}^{(0)}(-ks_{1x}, ks_{2x}, \omega) = \frac{1}{(2\pi)^2}\int\int W^{(0)}(x_1, x_2, \omega) \times \exp\left[ik\left(s_{1x}x_1 - s_{2x}x_2\right)\right]\mathrm{d}x_1\mathrm{d}x_2. \quad (5)$$

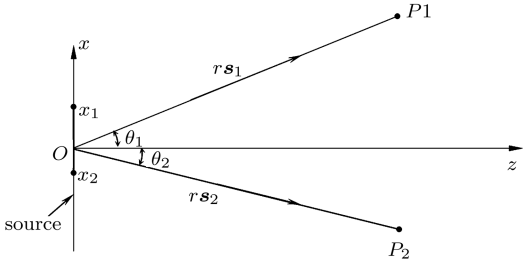


Fig.1. Illustrating the notation.

By substituting Eqs.(1)–(3) into Eq.(5), the integral calculations lead to

$$\tilde{W}^{(0)}(-ks_{1x}, ks_{2x}, \omega) = \frac{Aw_0^2\beta}{8\pi}\exp\left(\frac{\alpha^2\beta^2}{2}\right) \times \exp\left[-\frac{k^2w_0^2\left(1+\beta^2\right)}{8}s_{1x}^2 - \frac{k^2w_0^2\left(1+\beta^2\right)}{8}s_{2x}^2 + \frac{k^2w_0^2\left(1-\beta^2\right)}{4}s_{1x}s_{2x}\right] \times \left\{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right]\cos\left[\frac{kw_0\alpha}{2}\left(s_{1x} - s_{2x}\right)\right] + \cos\left[\frac{kw_0\alpha\beta^2}{2}\left(s_{1x} + s_{2x}\right)\right]\right\}, \quad (6)$$

where

$$\alpha = \Omega_0 w_0, \quad \text{(decentering parameter)}; \quad (7)$$

$$\beta = \left[1 + \left(\frac{w_0}{\sigma_0}\right)^2\right]^{-\frac{1}{2}}, \ \text{(spatial coherence parameter)}. \quad (8)$$

The spectral degree of coherence at frequency ω in the far field is defined as[1]

$$\mu^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega) = \frac{W^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega)}{\sqrt{W^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_1, \omega)}\sqrt{W^{(\infty)}(r\boldsymbol{s}_2, r\boldsymbol{s}_2, \omega)}}. \quad (9)$$

By using Eq.(6), the substitution of Eq.(4) into Eq.(7) yields

$$\mu^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega) = \exp\left[-\frac{k^2w_0^2\left(1-\beta^2\right)}{8}\left(s_{1x} - s_{2x}\right)^2\right] \times \frac{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right]\cos\left[\frac{kw_0\alpha}{2}\left(s_{1x} - s_{2x}\right)\right] + \cos\left[\frac{kw_0\alpha\beta^2}{2}\left(s_{1x} + s_{2x}\right)\right]}{\sqrt{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right] + \cos\left(kw_0\alpha\beta^2 s_{1x}\right)}\sqrt{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right] + \cos\left(kw_0\alpha\beta^2 s_{2x}\right)}}. \quad (10)$$

It is noted that GSM beams can be regarded as a special case of partially coherent ChG beams. On substituting $\alpha = 0$ into Eq.(10), the spectral degree of coherence of two-dimensional GSM beams in the far field is given by

$$\mu^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega) = \exp\left[-\frac{\delta^2}{2\Delta^2}\left(s_{1x} - s_{2x}\right)^2\right], \quad (11)$$

where

$$\Delta = \frac{1}{k}\sqrt{\frac{1}{w_0^2} + \frac{1}{\sigma_0^2}}, \quad (12)$$

$$\delta = \frac{w_0}{2\sigma_0}. \quad (13)$$

The radiant intensity in the far field is defined as[1]

$$\begin{aligned} J(\boldsymbol{s}) &= r^2 W^{(\infty)}(r\boldsymbol{s}, r\boldsymbol{s}, \omega) \\ &= 2\pi k \cos^2\theta \tilde{W}^{(0)}(-ks_x, ks_x, \omega). \end{aligned} \quad (14)$$

The substitution of Eq.(6) into Eq.(14) yields

$$\begin{aligned} & J(\boldsymbol{s}) \\ &= \frac{Akw_0^2\beta}{4}\cos^2\theta \exp\left(\frac{\alpha^2\beta^2}{2}\right)\exp\left(-\frac{k^2w_0^2\beta^2}{2}s_x^2\right) \\ &\times\left\{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right] + \cos\left(kw_0\alpha\beta^2 s_x\right)\right\}. \end{aligned} \quad (15)$$

On placing $\alpha = 0$ into Eq.(15), we obtain the far-field radiant intensity of GSM beams of the form

$$J(\boldsymbol{s}) = \left(\frac{Aw_0}{2\Delta}\right)\cos^2\theta \exp\left(-\frac{s_x^2}{2\Delta^2}\right). \quad (16)$$

It is readily seen that Eqs.(11) and (16) are consistent with Eqs.(15) and (2.4) in Refs.[7] and [3] respectively, if two-dimensional GSM beams are considered.

3. Conditions for two partially coherent cosh–Gaussian beams with the same spectral degree of coherence in the far field

From Eq.(10) we see that two partially coherent ChG beams will generate far fields with the same spectral degree of coherence if their beam parameters α, β, w_0 and k satisfy the conditions

$$k^2w_0^2\left(1-\beta^2\right) = \text{const}, \quad (17)$$

$$\alpha^2\left(1-\beta^2\right) = \text{const}, \quad (18)$$

$$kw_0\alpha = \text{const}, \quad (19)$$

$$kw_0\alpha\beta^2 = \text{const}. \quad (20)$$

The conditions (17)–(20) can be rewritten as

$$kw_0 = \text{const}, \quad (21)$$

$$\alpha = \text{const}, \quad (22)$$

$$\beta = \text{const}. \quad (23)$$

It follows from the conditions (21)–(23) that, for a fixed wave number (wavelength λ)[2–4,7] we cannot find two partially coherent ChG beams with the same spectral degree of coherence in the far field because the conditions (21)–(23) uniquely determine a solution of α, β and w_0. This means that for more general partially coherent light beams, like partially coherent ChG beams restricted by the conditions (21)–(23), in the strict sense there do not exist two beams with the same spectral degree of coherence in the far field. However, further discussion can be given when the values of beam parameters α and β fulfill some inequality.

If the inequality

$$\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right] \gg 1 \quad (24)$$

is satisfied, Eq.(10) reduces to

$$\begin{aligned} & \mu^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega) \\ &\approx \exp\left[-\frac{k^2w_0^2\left(1-\beta^2\right)}{8}\left(s_{1x} - s_{2x}\right)^2\right] \\ &\times\cos\left[\frac{kw_0\alpha}{2}\left(s_{1x} - s_{2x}\right)\right]. \end{aligned} \quad (25)$$

Physically, the inequality requires larger decentration (larger α) and worse spatial coherence (smaller β), so that Eq.(10) can simplify to Eq.(25). From Eq.(25) we see that there exist two partially coherent ChG beams with the same spectral degree of coherence in the far field provided that their beam parameters α, β and w_0 satisfy conditions

$$w_0^2\left(1-\beta^2\right) = \text{const}, \quad (26)$$

$$w_0\alpha = \text{const}. \quad (27)$$

On the other hand, we have $\exp[\alpha^2(1-\beta^2)/2] > 1$ because $0 < \beta < 1$. If $\exp[\alpha^2(1-\beta^2)/2] \approx 1$, Eq.(10) can be written as

$$\mu^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega)$$

$$\approx \exp\left[-\frac{k^2 w_0^2\left(1-\beta^2\right)}{8}\left(s_{1x}-s_{2x}\right)^2\right] \times \frac{\cos\left[\frac{kw_0\alpha}{2}\left(s_{1x}-s_{2x}\right)\right] + \cos\left[\frac{kw_0\alpha\beta^2}{2}\left(s_{1x}+s_{2x}\right)\right]}{2\cos\left[\frac{kw_0\alpha\beta^2}{2}s_{1x}\right]\cos\left[\frac{kw_0\alpha\beta^2}{2}s_{2x}\right]}; \quad (28)$$

Equation (28) indicates that for such a case the conditions for two partially coherent ChG beams with the same spectral degree of coherence in the far field are the same as Eqs.(21)–(23). Therefore, there do not exist such two equivalent partially coherent ChG beams.

4. Numerical illustrative examples and analysis

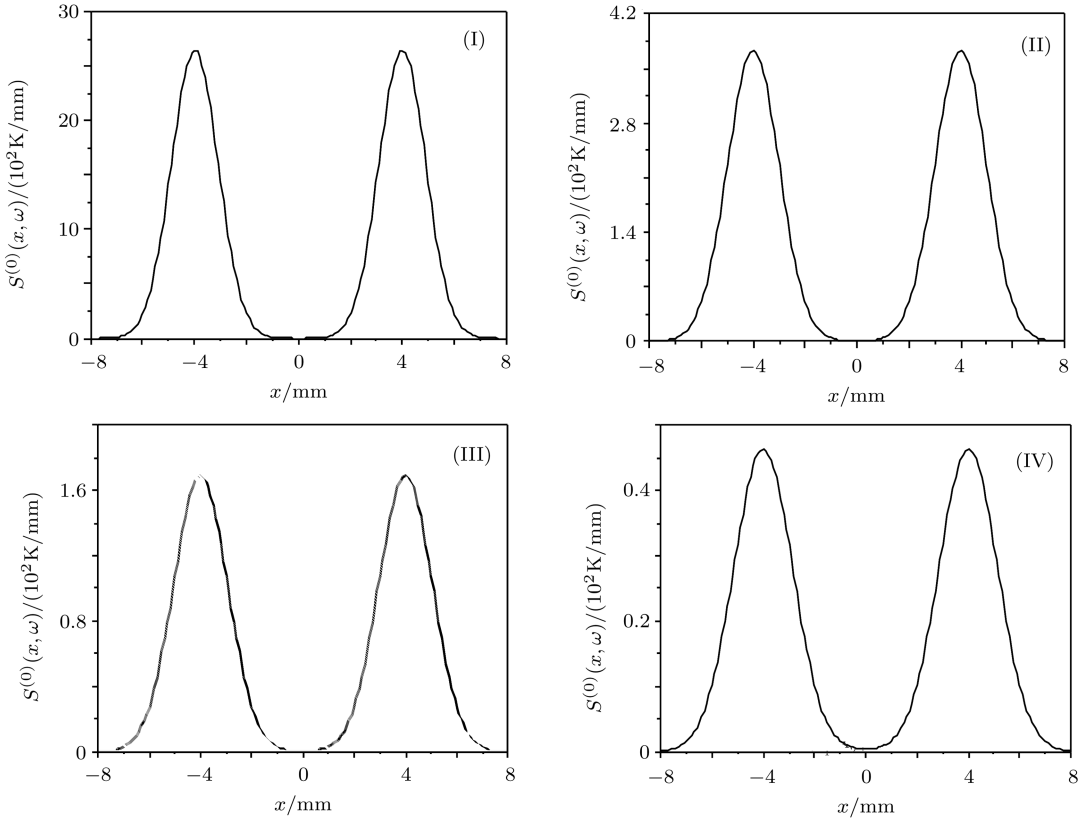
The above theoretical results can be illustrated by numerical examples. In our calculations two symmetric points with respect to the z-axis are chosen, thus $s_{1x} = -s_{2x} = \sin\theta$, and Eq.(10) reduces to

$$\mu^{(\infty)}(\theta, \omega)$$

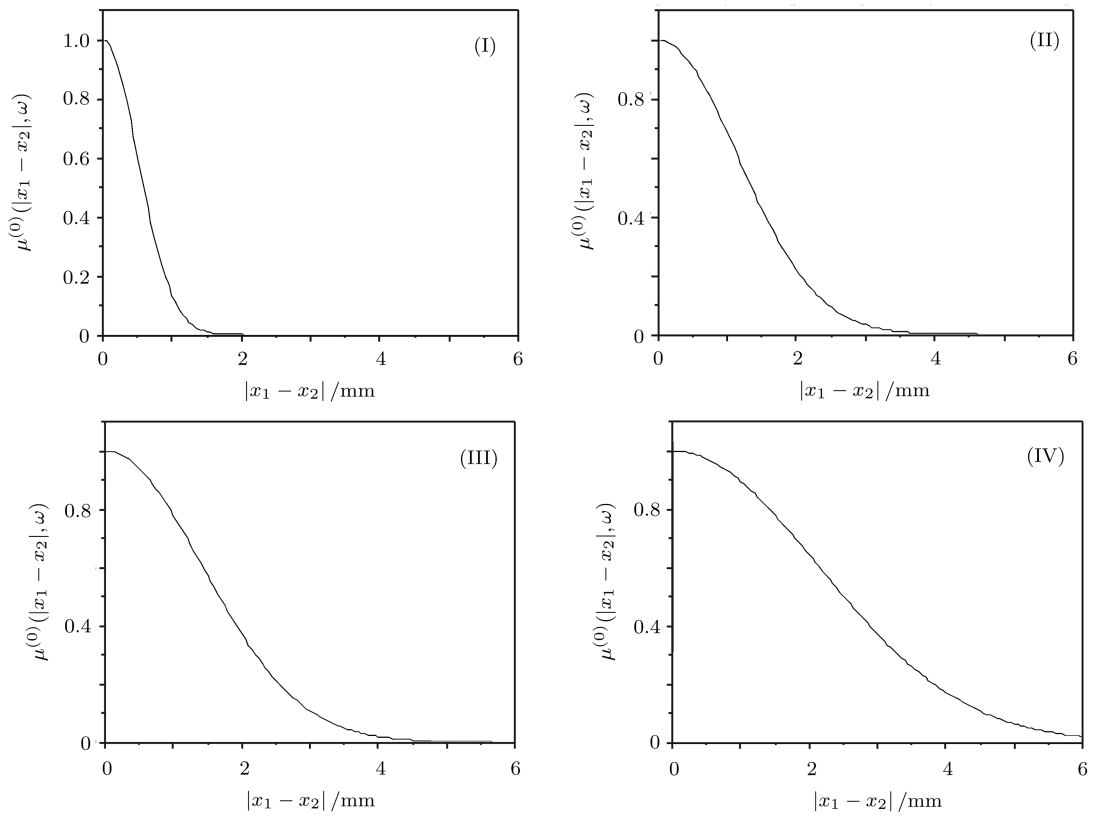
$$= \exp\left[-\frac{k^2 w_0^2\left(1-\beta^2\right)}{2}\sin^2\theta\right] \times \frac{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right]\cos\left(kw_0\alpha\sin\theta\right)+1}{\exp\left[\frac{\alpha^2}{2}\left(1-\beta^2\right)\right]+\cos\left(kw_0\alpha\beta^2\sin\theta\right)}, \quad (29)$$

where $\mu^{(\infty)}(\theta, \omega) = \mu^{(\infty)}(r\boldsymbol{s}_1, r\boldsymbol{s}_2, \omega)$.

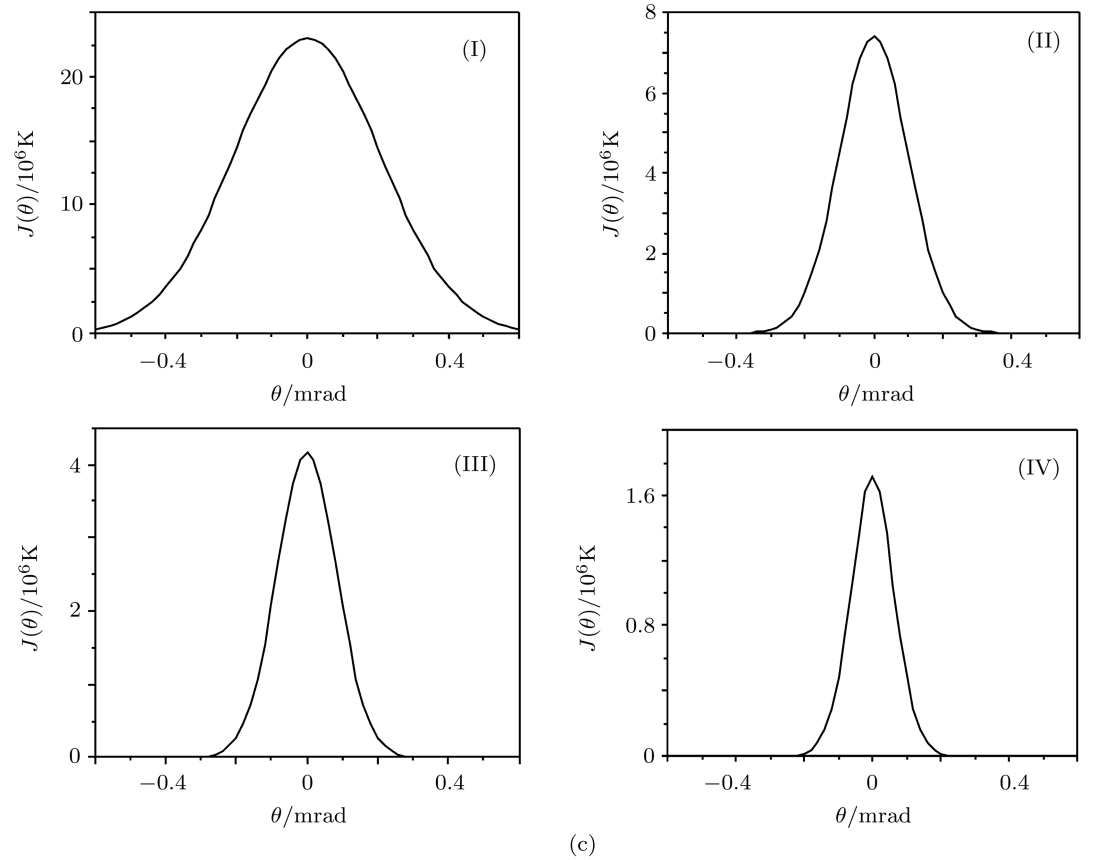
Figure 2 gives (a) spectral densities $S^{(0)}(x, \omega)$ and (b) spectral degree of coherence $\mu^{(0)}(|x_1 - x_2|, \omega)$ at the source plane $z = 0$, and (c) far-field radiant intensities $J_\omega(\theta)$ of four partially coherent ChG beams, whose beam parameters are listed in Table 1. From Figs.2(a)–2(c) and Table 1 we see that among them three partially coherent ChG beams satisfy inequality



(a)



(b)



(c)

Fig.2. (a) Spectral densities $S^{(0)}(x, \omega)$ at the source plane $z = 0$, (b) Spectral degrees of coherence $\mu^{(0)}(|x_1 - x_2|, \omega)$ at the source plane $z = 0$, and (c) Far-field radiant intensities $J(\theta)$ normalized by $K{=}Aw_0$ of four partially coherent ChG beams.

$\exp[\alpha^2(1-\beta^2)/2] \gg 1$, but this is not the case for the fourth beam. As a result, the three partially coherent ChG beams have the same spectral degree of coherence in the far field, and the fourth beam has not, as shown in Fig.3, although the fourth beam fulfills the conditions (26)–(27). It is of interest to note that, as can be seen from Fig.2(c), for such a case the far-field radiant intensity distributions of partially coherent ChG beams are different. In fact, from Eq.(15) and inequality $\exp[\alpha^2(1-\beta^2)/2] \gg 1$ the far-field radiant intensity is expressed as

$$J(\theta) = J(0)\cos^2\theta \exp\left(-\frac{\sin^2\theta}{2\Delta^2}\right). \tag{30}$$

The conditions under which two partially coherent ChG beams will generate far fields with the same radiant intensity read as[13]

$$J(0) = \frac{Aw_0}{4\Delta}\exp\left(\frac{\alpha^2}{2}\right) = \text{const}, \tag{31}$$

$$\Delta = \frac{1}{k}\sqrt{\frac{1}{w_0^2}+\frac{1}{\sigma_0^2}} = \text{const}. \tag{32}$$

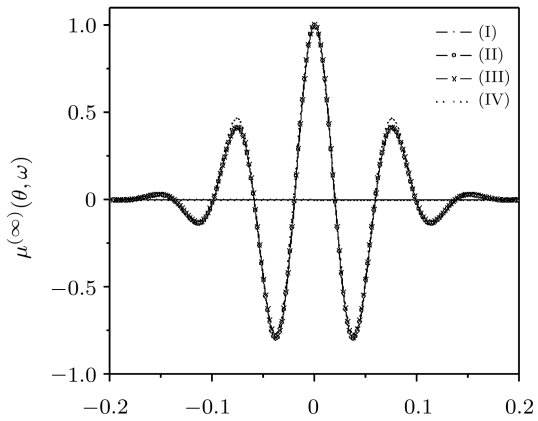


Fig.3. Far-field spectral degrees of coherence of four partially coherent ChG beams.

A comparison shows that the conditions (31) and (32) are different from the conditions (26)–(27).

Table 1. Optical beam parameters relating to Figs.2 and 3 ($\lambda = 0.6328$ μm).

	(I) PC-ChG beam	(II) PC-ChG beam	(III) PC-ChG beam	(IV) PC-ChG beam
$\exp[\alpha^2(1-\beta^2)/2]$	9304.37	403.43	145.76	25.93
w_0/mm	1.8	2.0	2.1	2.3
β	0.27	0.5	0.56	0.68
α	4.44	4	3.81	3.48
σ_0/mm	0.50	1.15	1.42	2.13

5. Concluding remarks

In this paper, by means of the theory of optical coherence it has been shown that in the strict sense there do not exist two partially coherent ChG beams which may produce far fields with the same spectral degree of coherence. However, under the condition $\exp[\alpha^2(1-\beta^2)/2] \gg 1$ we can find two partially coherent ChG beams with the same spectral degree of coherence in the far field if the conditions (26)–(27) are satisfied, but for such a case the radiant intensity distribution in the far field of the two partially coherent beams is different because the conditions (31)–(32) are not fulfilled. If $\exp[\alpha^2(1-\beta^2)/2] \approx 1$, there do not exist two partially coherent ChG beams with the same spectral degree of coherence in the far field. Therefore, the same far-field spectral degree of coherence of two partially coherent ChG beams does not mean the same far-field radiant intensity because the conditions (26)–(27) and (31)–(32) are different.

Our study can be regarded as an extension of Ref.[7] because the GSM beam can be treated as a special case of partially coherent ChG beams by letting $\alpha = 0$.

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