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α preformation and penetration probability for heavy nuclei*

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The α preformation factor and penetration probability have been analyzed for even–even nuclei of Po, Rn, Ra using experimental released energies and α decay half-lives in the frame of the double folding model. It is shown that $N = 126$ is a neutron magic number from α preformation and shell effects play an important role in α preformation. The closer the nucleon number is to the magic number, the more difficult α formation in the parent nucleus is. The preformation factor can supply information on the nuclear structure and the penetration probability mainly determines α decay half-life.

Keywords: preformation factor, penetration probability, magic number
PACC: 2360

1. Introduction

α decay is one of the most important decay modes for heavy nuclei since it can provide information on the nuclear structure such as ground state lifetime, nuclear interaction, nuclear incompressibility, nuclear spin and parity, etc.[1−4] Experimentally, α decay of nuclei is used to identify new nuclides and new elements through an α decay chain from an unknown parent nucleus to a known nuclide.[5,6] Recently, particular interest has been paid to the α decay of super-heavy nuclei with the development of radioactive beams and the new detection technology.

α decay has played an important role in the development of modern physics. From then on the ground state of unstable nuclei has been observed to have different kinds of decay modes such as α decay, β decay, one proton emission, two-proton emission, ^{14}C radioactivity and spontaneous fission. However, the α decay mode is generally used for studies of the decay and synthesis of heavy nuclei. Some newly synthesized super-heavy elements have been identified recently.[7−9] Theoretically, different approaches have been proposed to describe α radioactivity including the shell-model, the cluster-model and the fission-like model,[10−14] primarily motivated by the increasing role of α decay in both the spectrocopy of unstable nuclei and the synthesis of new elements in the study of super-heavy elements. However, for the half-life there still exists a small difference between the experimental and theoretical results, which can be considered to be associated with the preformation factor. The preformation factor is employed in the calculation of the α decay half-life in different approaches.[12,15−17] The decay constant λ is defined as the product of the preformation factor, the penetration probability and the assault frequency. For determining the decay constant, one important problem is how to estimate the preformation factor. The preformation factor has also been used to study the nuclear structure.[18] The nuclear potential was calculated according to the double folding model, which has been successfully used for fusion reactions, scattering and nuclear decays.[19−21]

In this paper, the α preformation factor will be extracted from the experimental α half-lives and will be compared between the different nuclei. The potential barrier of an α-daughter nucleus system is based on the double folding integral with the density-dependence nucleon–nucleon(NN) interaction. The penetration probability is determined using the Wentzel–Kramers–Brillouin (WKB) approximation when the experimental Q_α is used. The decay constant λ is extracted from the experimental α decay half-lives.

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2. The calculation methods

The penetration probability of an emitted α particle is calculated using the WKB approximation by

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_1}^{R_2}\sqrt{2\mu(V(R)-Q_\alpha)}\mathrm{d}R\right], \quad (1)$$

where μ is the reduced mass of the α particle and the daughter nucleus. Q_α is the released energy for α decay. R_1 and R_2 are the two turning points of the WKB approximation integral determined by

$$V(R_1) = Q_\alpha = V(R_2). \quad (2)$$

The total interaction potential $V(R)$ between the α particle and the daughter nucleus is given by

$$V(R) = V_{\mathrm{N}}(R) + V_{\mathrm{C}}(R) + \frac{l(l+1)\hbar^2}{2\mu R^2}, \quad (3)$$

where the last term is the centrifugal potential; μ is the reduced mass of the α-daughter nucleus system, l is the angular momentum carried by the α particle and is equal to zero for the α decay of even–even nuclei according to spin-parity conservation.

In the double folding model the nuclear potential V_{N} is given by

$$V_{\mathrm{N}}(R) = \iint \rho_1(\boldsymbol{r}_1)v(\boldsymbol{s}=|\boldsymbol{R}+\boldsymbol{r}_2-\boldsymbol{r}_1|)\rho_2(\boldsymbol{r}_2)\mathrm{d}\boldsymbol{r}_1\mathrm{d}\boldsymbol{r}_2, \quad (4)$$

where $\rho_1(r_1)$ and $\rho_2(r_2)$ are the matter density distribution functions of the α particle and the daughter nuclei, respectively. The density distribution function is taken to be a Gaussian form for the α particle

$$\rho_1(r) = 0.4229\exp(-0.7024r^2), \quad (5)$$

whose volume integral is equal to 4. The density distribution function of the daughter nucleus can be described by the spherically symmetric Fermi function,

$$\rho_2(r) = \rho_0/[1+\exp((r-c)/a)], \quad (6)$$

where the half-density radius c is expressed as[22,23]

$$c = r_\rho(1-\pi^2a^2/3r_\rho^2) \quad (7)$$

with $r_\rho = 1.13A_{\mathrm{d}}^{1/3}$ and the density diffuseness $a \approx 0.54$ fm, and A_{d} is the mass number of the daughter nucleus. The value of ρ_0 is determined by normalization so that

$$\int \rho_2(\boldsymbol{r})\mathrm{d}\boldsymbol{r} = A_{\mathrm{d}}. \quad (8)$$

The NN interaction v between two nucleons in Eq.(4) is given frequently by M3Y interaction, which is designed to reproduce the G-matrix elements in an oscillator basis.[24] The density-dependent NN interaction is given by

$$v(s,\rho,E) = F(\rho,E)v^{(\mathrm{M3Y})}(s,E), \quad (9)$$

$$v^{(\mathrm{M3Y})}(s,E) = 7999\frac{\mathrm{e}^{-4s}}{4s} - 2134\frac{\mathrm{e}^{-2.5s}}{2.5s} + \hat{J}_{00}(E)\delta(s), \quad (10)$$

where the zero-range pseudo term $\hat{J}_{00}(E)$ is written as

$$\hat{J}_{00}(E) \approx -276[1-0.005(E/A)]\ \mathrm{MeV}\cdot\mathrm{fm}^3. \quad (11)$$

The density-dependence factor is given by

$$F(\rho,E) = C[1-\beta(E)\rho_1^{2/3}][1-\beta(E)\rho_2^{2/3}]. \quad (12)$$

Equation (12) has been successfully used for explaining scattering and α cluster radioactivity. The theoretical results of the nuclear incompressibility with $C = 2.06$, $\beta = 1.6257$ fm^2 and $n = 2/3$ also agree well with experimental data.[25] In Eq.(12) ρ_1 and ρ_2 are the density distributions of the α particle and the daughter nucleus, respectively.

The Coulomb potential is calculated between the α particle and the residual daughter nucleus by a point-like plus uniform model, which is given by

$$V_{\mathrm{C}}(R) = Z_1Z_2\mathrm{e}^2\begin{cases}\dfrac{1}{R} & (R>R_{\mathrm{C}}),\\ \dfrac{1}{2R_{\mathrm{C}}}\left[3-\left(\dfrac{R}{R_{\mathrm{C}}}\right)^2\right] & (R<R_{\mathrm{C}}).\end{cases}, \quad (13)$$

where the spherical charge distribution for the residual daughter nucleus and the point particle for the emitted α particle are assumed and $e^2 = 1.44$ MeV$\cdot$fm; $R_{\mathrm{C}} = c_\alpha + c_{\mathrm{d}}$ with c_α and c_{d} being calculated by using Eq.(7); Z_1 and Z_2 represent the atomic numbers of the emitted α particle and the residual daughter nucleus, respectively.

The decay constant λ is extracted from the experimental α decay half-life $T_{1/2}$

$$\lambda = \frac{\ln 2}{T_{1/2}}. \quad (14)$$

The assault frequency is calculated from the classical method[26]

$$\nu = \frac{1}{2R}\sqrt{\frac{2E_\alpha}{M_\alpha}}, \quad (15)$$

where $E_\alpha = Q_\alpha\left(1-\frac{M_\alpha}{M}\right)$. M_α and M are the masses of the α particle and parent nucleus, respectively. R is the radius of the parent nucleus given by

$$R = (1.28A^{1/3} + 0.8A^{-1/3} - 0.76)\ \text{fm}, \qquad (16)$$

where A is the mass number of the parent nucleus. The preformation factor is obtained as

$$P_0 = \frac{\lambda}{\nu P} \qquad (17)$$

with λ, ν, P determined by Eqs.(14), (15) and Eq.(1), respectively.

3. Results and discussion

In the present calculation we select the even–even nuclei of Po, Rn and Ra to study the penetration probability and the preformation factor. Figure 1 shows the penetration probability as a function of neutron number N. Overall, the penetration probabilities decrease with increasing neutron number up to the spherical shell closure $N = 126$, and then increase rapidly with the neutron number. A maximum probability is shown for all three selected isotopes when N is equal to 128, indicating that α emission is much easier for nuclei with two neutrons outside the shell closure. Moreover, the maximum penetration probabilities are nearly identical at $N = 128$, after which the penetration probability decreases again with increasing N. At the same neutron number it increases with increasing proton number because the proton numbers of Po isotopes are closer to the proton magic number $Z = 82$ than those of Rn and Ra. The closer the nucleon number is to the magic number, the more difficult it is for the α particle to penetrate the parent nuclei. The variation range of the penetration probability is from 10^{-28} to

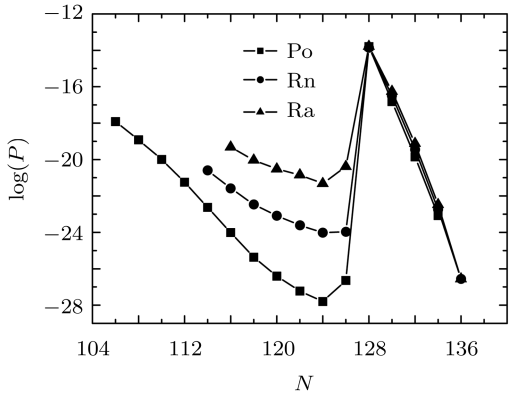


Fig.1. Decimal logarithms of penetration probabilities of Po, Rn and Ra isotopes.

10^{-14}, which is much larger as compared with the preformation factor and the assault frequency that change only slightly. According to Eqs.(14) and (17) the penetration probability mainly determines α decay half-life.

Figure 2 shows the preformation factor as a function of N. One can see that the preformation factors change slightly in the Po, Ro and Ra isotopes. The preformation factors generally decrease with increasing N up to $N = 126$, a minimum always exists for Po, Rn and Ra isotopes when N is equal to 126, and then the preformation factors increase sharply with N. The preformation factors of Po isotopes are basically smaller than those of Rn and Ra isotopes. Both $N = 126$ and $Z = 82$ are known as magic numbers. The closer the nucleon number is to shell closure, the harder it is for the α cluster to form in the parent nuclei. The dramatic change of the preformation factor around the magic number indicates that shell effects play an important role in the α formation mechanism in the parent nuclei. On the other hand, the preformation factor can reflect the shell effects.

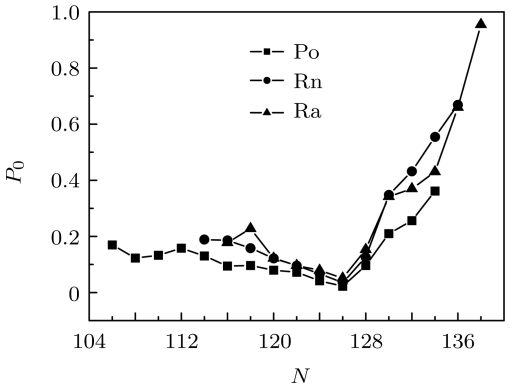


Fig.2. α preformation factors of even–even Po, Rn and Ra isotopes.

Recently, isotopes of elements 112, 114, 116 and ^{294}118 have been produced by irradiations of ^{233,238}U, ^{242}Pu, ^{248}Cm and ^{249}Cf targets with a ^{48}Ca beam at various energies in fusion–evaporation reactions.[7−9] Various models including density-dependent M3Y (DDM3Y),[2] the generalized liquid drop model (GLDM)[27,28] and Viola–Seaborg systematics (VSS)[29] focus on these elements. Now the penetration probability and preformation factor are calculated using their experimental data. The results are shown in Table 1. One can see that the range of the penetration probability is from 10^{-18} to 10^{-22}, corresponding to the orders of half-lives from milliseconds to seconds. The range of penetration prob-

ability is relatively narrow. The preformation factors are relatively smaller than those of the spherical heavy nuclei, particularly that of the $^{294}118$ element, which is the smallest among all the calculated results. These results indicate that the super-heavy nuclei are weakly bound, and their α preformation is difficult in the mother nucleus. Such nuclei will quickly decay through α emission when synthesized. The penetration probabilities also dominate the α half-lives of super-heavy elements.

Table 1. The results for even–even nuclei of 112, 114, 116 and 118 isotopes.

nuclei	Q_α/(MeV)	$T_{1/2}$/s	$\nu \times 10^{21}/\mathrm{s}^{-1}$	P	P_0	References
^{260}Sg	9.920	0.95×10^{-2}	1.440	3.448×10^{-19}	0.147	[30]
^{266}Sg	8.836	25.7	1.348	2.779×10^{-22}	0.072	[31]
^{264}Hs	10.590	0.54×10^{-3}	1.480	4.230×10^{-18}	0.205	[32]
^{266}Hs	10.381	2.3×10^{-3}	1.461	1.375×10^{-18}	0.150	[33]
^{270}Ds	11.242	0.1×10^{-3}	1.513	4.213×10^{-17}	0.109	[33]
$^{284}112$	9.35	9.8	1.355	1.180×10^{-22}	0.442	[34]
$^{286}114$	10.35	0.16	1.423	2.008×10^{-20}	0.152	[7]
$^{288}114$	10.09	0.80	1.401	4.207×10^{-21}	0.147	[7]
$^{290}116$	11.0	0.015	1.459	2.411×10^{-19}	0.131	[7]
$^{292}116$	10.80	0.018	1.443	8.060×10^{-20}	0.331	[7]
$^{294}118$	11.81	1.8×10^{-3}	1.505	5.546×10^{-18}	0.046	[9]

4. Conclusion

Nuclear potentials have been calculated by the double folding model with density-dependence NN interaction. The penetration probability is obtained in the frame of the WKB approximation using the experimental released energy and the decay constant is extracted from the experimental α decay half-life. The α preformation factors and penetration probabilities of even–even nuclei near $N = 126$ are studied systematically. The penetration probability can dramatically determine the half-life of cluster emission. The change of preformation factor with neutron number shows that shell effects play an important role for α preformation. It is difficult to form an α cluster in the parent nuclei when the nucleon number is close to the magic number. The preformation factor reflects the fact that $N = 126$ is a neutron magic number. The calculation extends to the super-heavy nuclei. It is shown that the super-heavy nuclei are weakly bound. They are quickly decayed through emitting α clusters when synthesized. The present calculation demonstrates its success in studying the preformation factor systematically. Clearly, odd A and odd–odd nuclei, which are considered as the angular momentum for spin-parity conservation in the decay process, will be included systematically for studying cluster preformation in future work.

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References

[1] Delin D S, Peltonen S and Suhonen J 2006 *Phys. Rev.* C **73** 014315
[2] Roy Chowdhury P, Samanta C and Basu D N 2006 *Phys. Rev.* C **73** 014612
[3] Seif W M 2006 *Phys. Rev.* C **74** 034302
[4] Basu D N 2004 *J. Phys.* G: *Nucl. Part. Phys.* **30** B35
[5] Hofmann S and Münzenberg G 2000 *Rev. Mod. Phys.* **72** 733
[6] Oganessian Yu Ts, Yeremin A V, Popeko A G, Bogomolov S L, Buklanov G V, Chelnokov M L, Chepigin V I, Gikal B N, Gorshkov V A, Gulbekian G G, Itkis M G, Kabachenko A P, Lavrentev A Yu, Malyshev O N, Rohac J, Sagaidak

R N, Hofmann S, Saro S, Giardina G and Morita K 1999 *Nature* (London) **400** 242

[7] Oganessian Yu Ts, Utyonkov V K, Lobanov Yu V, Abdullin F Sh, Polyakov A N, Shirokovsky I V, Tsyganov Y S, Gulbekian G G, Bogomolov S L, Gikal B N, Mezentsev A N, Iliev S, Subbotin V G, Sukhov A M, Voinov A A, Buklanov G V, Subotic K, Zagrebaev V I, Itkis M G, Patin J B, Moody K J, Wild J F, Stoyer M A, Stoyer N J, Shaughnessy D A, Kenneally J M, Wilk P A, Lougheed R W, Ilkaev R I and Vesnovskii S P 2004 2004 *Phys. Rev.* C **70** 064609

[8] Oganessian Yu Ts, Utyonkov V K, Lobanov Yu V, Abdullin F Sh, Polyakov A N, Shirokovsky I V, Tsyganov Y S, Gulbekian G G, Bogomolov S L, Gikal B N, Mezentsev A N, Iliev S, Subbotin V G, Sukhov A M, Voinov A A, Buklanov G V, Subotic K, Zagrebaev V I, Itkis M G, Patin J B, Moody K J, Wild J F, Stoyer M A, Stoyer N J, Shaughnessy D A, Kenneally J M, Wilk P A, Lougheed R W, Ilkaev R I and Vesnovskii S P 2005 *Phys. Rev.* C **71** 029902(E)

[9] Oganessian Yu Ts, Utyonkov V K, Lobanov Yu V, Abdullin F Sh, Polyakov A N, Shirokovsky I V, Tsyganov Yu S, Gulbekian G G, Bogomolov S L, Gikal B N, Mezentsev A N, Iliev S, Subbotin V G, Sukhov A M, Voinov A A, Buklanov G V, Subotic K, Zagrebaev V I, Itkis M G, Patin J B, Moody K J, Wild J F, Stoyer M A, Stoyer N J, Shaughnessy D A, Kenneally J M, Wilk P A, Lougheed R W, Ilkaev R I and Vesnovskii S P 2002 *JINR Communication* **D7** 287; Lawrence Livermore National Laboratory 2003 *Rep. UCRL-ID-151619*

[10] Poenaru D N, Greiner W, Depta K, Ivascu M, Mazilu D and Sandulescu A 1986 *At. Data Nucl. Data Tables* **34** 423

[11] Buck B, Merchant A C and Perez S M 1993 *At. Data Nucl. Data Tables* **54** 53

[12] Xu C and Ren Z Z 2005 *Nucl. Phys.* A **753** 174

[13] Xu C and Ren Z Z 2005 *Nucl. Phys.* A **760** 303

[14] Basu D N 2003 *Phys. Lett.* B **566** 90

[15] Xu C and Ren Z Z 2006 *Phys. Rev.* C **74** 014304

[16] Royer G and Moustabchir R 2001 *Nucl. Phys.* A **683** 182

[17] Zhang H F, Zuo W, Li J Q and Royer G 2006 *Phys. Rev.* C **74** 017304

[18] Mohr P 2006 *Phys. Rev.* C **73** R031301

[19] Gontchar I I, Hinde D J, Dasgupta M and Newton J O 2004 *Phys. Rev.* C **69** 024610

[20] Khoa Dao T, Satchler G R and von Oertzen W 1997 *Phys. Rev.* C **56** 954

[21] Guzmán F, Goncalevs M, Tavares O A P and Duarte S B 1999 *Phys. Rev.* C **59** R2339

[22] Srivastava D K, Ganguly N K and Hodgson P E 1974 *Phys. Lett.* B **51** 439

[23] Srivastava D K and Ganguly N K 1983 *Phys. Lett.* B **124** 6

[24] Bertsch G, Borysowicz J, Mcmanus H and Love W G 1977 *Nucl. Phys.* A **284** 399

[25] Basu D N 2005 *Int. J. Mod. Phys.* E **14** 739

[26] Gambhir Y K, Bhagwat A and Gupta M 2005 *Phys. Rev.* C **71** 037301

[27] Royer G and Gherghescu R A 2002 *Nucl. Phys.* A **699** 479

[28] Royer G, Zbiri K and Bonilla C 2004 *Nucl. Phys.* A **730** 35

[29] Sobiczewski A, Patyk Z and Cwiok S S 1989 *Phys. Lett.* B **224** 1

[30] Audi G, Wapstra A H and Thibault C 2003 *Nucl. Phys.* A **729** 337

[31] Düllmann Ch E, Bruchle W, Dressler R, Eberhardt K, Eichler B, Eichler R, Gaggeler H W, Ginter T N, Glaus F, Gregorlch K E, Ho?man D C, Jager E, Jost D T, Kirbach U W, Lee D M, Nitsche H, Patin J B, Pershina V, Piguet D, Qin Z, Schadel M, Schausten B, Schimpf E, Schott H J, Sovema S, Sudowe R, Thorle P, Timokhin S N, Trautmann N, Turler A, Vahle A, Wirth G, Yakushev A B and Zielinski P M 2002 *Nature* **418** 859

[32] Audi G and Wapstra A H 2003 *Nucl. Phys.* A **729** 3

[33] Hofmann S, Heβberger F P, Ackermann D, Antalic S, Cagarda P, Ćwiok S, Kindler B, Kojouharova J, Lommel B, Mann R, Münzenberg G, Popeko A G, Saro S, Schött H J and Yeremin A V 2001 *Eur. Phys. J.* A **10** 5

[34] Oganessian Yu Ts, Utyonkov V K, Lobanov Yu V, Abdullin F Sh, Polyakov A N, Shirokovsky I V, Tsyganov Yu S, Gulbekian G G, Bogomolov S L, Gikal B N, Mezentsev A N, Iliev S, Subbotin V G, Sukhov A M, Ivanov O V, Buklanov G V, Subotic K, Itkis M G, Moody K J, Wild J F, Stoyer N J, Stoyer M A and Lougheed R W 2000 *Phys. Rev.* C **62** R041604