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# Analysis of the decay $B^0 \to \chi_{c1}\pi^0$ with light-cone QCD sum rules*

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In this article, we calculate the contribution from the nonfactorizable soft hadronic matrix element to the decay $B^0 \to \chi_{c1}\pi^0$ with the light-cone quantum chromo-dynamic (QCD) sum rules. The numerical results show that its contribution is rather large and should not be neglected. The total amplitudes lead to a branching fraction which is in agreement with the experimental data marginally.

**Keywords:** light-cone QCD sum rule, $B$-decay, nonfactorizable hadronic matrix element
**PACC:** 1325, 1120D, 1240

## 1. Introduction

Recently, the Belle Collaboration measured the branching fraction for the Cabibbo- and colour-suppressed decay $B^0 \to \chi_{c1}\pi^0$ based on a data sample of $657 \times 10^6$ $B\overline{B}$ events collected at the $\Upsilon(4S)$ resonance with the Belle detector at the KEKB asymmetric-energy $e^+e^-$ collider.[1] The signal is $40 \pm 9$ events with a significance of $4.7\sigma$ including systematic uncertainties, and the branching fraction is about $(1.12 \pm 0.25 \pm 0.12) \times 10^{-5}$.

The decay takes place through the process $b \to dc\bar{c}$ (or more precisely $\bar{b} \to \bar{d}c\bar{c}$, they relate with each other by charge conjunction; in this article, we calculate the amplitudes for the process $b \to dc\bar{c}$, then take the charge conjunction to obtain the branching fraction) at the quark-level.[2] If the leading order tree diagram dominates, the time-dependent CP-violating asymmetries are predicted to be the same as those in the $b \to sc\bar{c}$ decays.[3] The time-dependent CP-violation parameters for the similar decay $B^0 \to J/\psi\pi^0$ have been measured by the Belle[4,5] and Babar[6−8] collaborations. The deviation of the CP-violating asymmetries from these expectations may indicate non-negligible contributions from the penguin amplitudes or new physics.

The quantitative understanding of the $B$-decays depends on our knowledge about the nonperturbative hadronic matrix elements of the operators entering the effective weak Hamiltonian.[2] In recent years, great progress has been made in this aspect, such as the generalized factorization approach,[9,10] the quantum chromo-dynamics (QCD) factorization approach,[11,12] the perturbative QCD,[13,14] the soft-collinear effective theory,[15] etc. Factorization of the hadronic matrix elements has been proved to hold in the leading order in many processes.

The effects of the soft gluons which violate factorization are supposed to be of the order of $O\left(\frac{\Lambda_{\rm QCD}}{m_{\rm b}}\right)$ and neglected in the QCD-improved factorization studies;[11,12] however, no theoretical work has ever proved that they are small quantities. For the colour-suppressed $B$ to charmonia decays, there may be significant impacts of the nonfactorizable soft contributions.

In Ref.[16], Khodjamirian introduced a technique based on the light-cone QCD sum rules to estimate the nonfactorizable soft contributions, where the soft gluon effects are represented by the quark-antiquark-gluon distribution amplitudes of the light mesons, so the hadronic matrix element appears as a part of the hadronic dispersion relation for the correlation function. Thereafter, the light-cone QCD sum rules are applied to the study of the nonfactorizable hadronic matrix elements in the $B$-decays due to the soft gluons exchanges.[17−27]

It is interesting to study the nonfactorizable soft contributions in the decay $B^0 \to \chi_{c1}\pi^0$ with the light-cone QCD sum rules.

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izable contributions from the effective weak Hamiltonian are derived in Section 2; the soft hadronic matrix element $\langle\chi_{c1}\pi^0|\widetilde{\mathcal{O}}(0)|B\rangle$ is calculated with the light-cone sum rules approach in Section 3; numerical results are presented in Section 4; Section 5 is reserved for the conclusion.

## 2. Effective weak Hamiltonian and factorizable contributions

The effective weak Hamiltonian for the $b \to dc\bar{c}$ decay modes can be written as (for a detailed discussion of the effective weak Hamiltonian, one can refer to Ref.[2])

$$H_{\rm w} = \frac{G_{\rm F}}{\sqrt{2}}\Big\{V_{\rm cb}V_{\rm cd}^*\left[C_1(\mu)\mathcal{O}_1 + C_2(\mu)\mathcal{O}_2\right] - V_{\rm tb}V_{\rm td}^*\sum_{i=3}^{10}C_i\mathcal{O}_i\Big\}, \quad (1)$$

where $V_{ij}$'s are the CKM matrix elements, $C_i$'s are the Wilson coefficients calculated at the renormalization scale $\mu \sim O(m_{\rm b})$, and the relevant operators $\mathcal{O}_i$ are given by

$$\begin{aligned}
\mathcal{O}_1 &= (\bar{d}_\alpha b_\beta)_{V-A}(\bar{c}_\beta c_\alpha)_{V-A}\,,\\
\mathcal{O}_2 &= (\bar{d}_\alpha b_\alpha)_{V-A}(\bar{c}_\beta c_\beta)_{V-A}\,,\\
\mathcal{O}_{3(5)} &= (\bar{d}_\alpha b_\alpha)_{V-A}\sum_q(\bar{q}_\beta q_\beta)_{V-A(V+A)}\,,\\
\mathcal{O}_{4(6)} &= (\bar{d}_\alpha b_\beta)_{V-A}\sum_q(\bar{q}_\beta q_\alpha)_{V-A(V+A)}\,, \quad (2)
\end{aligned}$$

where we have neglected the Wilson coefficients $C_7$, $C_8$, $C_9$, $C_{10}$ due to their small values. We can reorganize the colour-mismatched quark fields into colour singlet states by Fierz transformation, and express the effective weak Hamiltonian $H_{\rm w}$ in the following form,

$$\begin{aligned}
H_{\rm w} = \frac{G_F}{\sqrt{2}}\Bigg\{&V_{\rm cb}V_{\rm cd}^*\left[\left(C_2(\mu)+\frac{C_1(\mu)}{3}\right)\mathcal{O}_2 + 2C_1(\mu)\widetilde{\mathcal{O}}_2\right]\\
&- V_{\rm tb}V_{\rm td}^*\left[\left(C_3(\mu)+\frac{C_4(\mu)}{3}\right)\mathcal{O}_3 + 2C_4(\mu)\widetilde{\mathcal{O}}_3\right]\\
&-V_{\rm tb}V_{\rm td}^*\left[\left(C_5(\mu)+\frac{C_6(\mu)}{3}\right)\mathcal{O}_5 + 2C_6(\mu)\widetilde{\mathcal{O}}_5\right]\Bigg\}, \quad (3)
\end{aligned}$$

where

$$\begin{aligned}
\widetilde{\mathcal{O}}_2 &= \left(\bar{c}\gamma_\mu(1-\gamma_5)\frac{\lambda_a}{2}c\right)\left(\bar{d}\gamma^\mu(1-\gamma_5)\frac{\lambda_a}{2}b\right),\\
\widetilde{\mathcal{O}}_3 &= \left(\bar{c}\gamma_\mu(1-\gamma_5)\frac{\lambda_a}{2}c\right)\left(\bar{d}\gamma^\mu(1-\gamma_5)\frac{\lambda_a}{2}b\right),\\
\widetilde{\mathcal{O}}_5 &= \left(\bar{c}\gamma_\mu(1+\gamma_5)\frac{\lambda_a}{2}c\right)\left(\bar{d}\gamma^\mu(1-\gamma_5)\frac{\lambda_a}{2}b\right), \quad (4)
\end{aligned}$$

and $\lambda_a'$s are SU(3) Gell–Mann matrices.

The factorizable matrix elements $\langle\chi_{\rm c1}(p)\pi^0(-q)|H_{\rm w}|B(p-q)\rangle$ can be parameterized as $\epsilon^*\cdot qA_f$,

$$\begin{aligned}
\epsilon^*\cdot qA_f = \frac{G_F}{\sqrt{2}}\Bigg\{&-V_{\rm cb}V_{\rm cd}^*\left[C_2(\mu)+\frac{C_1(\mu)}{3}\right]\\
&+V_{\rm tb}V_{\rm td}^*\left[C_3(\mu)+\frac{C_4(\mu)}{3}-C_5(\mu)-\frac{C_6(\mu)}{3}\right]\Bigg\}\\
&\times\langle\chi_{\rm c1}(p)|\bar{c}\gamma_\mu\gamma_5 c|0\rangle\langle\pi^0(-q)|\bar{d}\gamma^\mu b|B(p-q)\rangle\,. \quad (5)
\end{aligned}$$

The $\chi_{\rm c1}$ meson decay constant is defined as $\langle\chi_{\rm c1}(p)|\bar{c}(0)\gamma_\mu\gamma_5 c(0)|0\rangle = f_{\chi_{\rm c1}}m_{\chi_{\rm c1}}\epsilon_\mu^*$. The $B-\pi^0$ form-factor can be parameterized as

$$-\sqrt{2}\langle\pi^0(q)|\bar{d}\gamma_\mu b|B(P)\rangle = f(p^2)(P+q)_\mu - [f(p^2)-f_0(p^2)]\frac{m_B^2-m_\pi^2}{p^2}p_\mu\,, \quad (6)$$

the form-factors $f(p^2)$ and $f_0(p^2)$ can be estimated with the light-cone sum rules approach, here we take the value $f(m^2_{\chi_{c1}}) = 0.62$.[28]

The concise expression for the factorizable matrix elements can be written as

$$A_f = -G_F f_{\chi_{c1}} m_{\chi_{c1}} f(m^2_{\chi_{c1}}) \left\{ V_{cb}V^*_{cd} \left[ C_2(\mu) + \frac{C_1(\mu)}{3} \right] - V_{tb}V^*_{td} \left[ C_3(\mu) - C_5(\mu) + \frac{C_4(\mu) - C_6(\mu)}{3} \right] \right\} . \tag{7}$$

## 3. Light-cone QCD sum rules for the nonfactorizable hadronic matrix element $\langle \chi_{c1}\pi^0|\widetilde{\mathcal{O}}(0)|B\rangle$

In the following, we apply the approach developed in Ref.[16] to the $B \to \pi\pi$ channel to estimate the contribution from the soft-gluon exchanges in the decay $B^0 \to \chi_{c1}\pi^0$. We write down the correlation function $\Pi_\rho(p,q,k)$ firstly,

$$\Pi_\rho(p,q,k) = i^2 \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}(p-q)x} \int \mathrm{d}^4y \mathrm{e}^{\mathrm{i}(p-k)y} \langle 0|T\{J_\rho(y)\widetilde{\mathcal{O}}(0)J_5(x)\}|\pi^0(q)\rangle \, , \tag{8}$$

where $\widetilde{\mathcal{O}} = \widetilde{\mathcal{O}}_2$, the currents $J_\rho = \bar{c}\gamma_\rho\gamma_5 c$ and $J_5 = m_b\bar{b}i\gamma_5 d$ interpolate the mesons $\chi_{c1}$ and $B$, respectively.

The correlation function $\Pi_\rho(p,q,k)$ can be calculated by the operator product expansion approach near the light-cone $x^2 \sim y^2 \sim (x-y)^2 \sim 0$ in perturbative QCD theory. It is a function of three independent momenta chosen to be $q$, $p-k$ and $k$. We introduce the unphysical momentum $k$ in order to avoid the $B$ meson having the same four-momentum before $(p-q)$ and after the decay $(P)$, and thus avoid a continuum of light contributions in the dispersion relation in the $B$-channel. The independent kinematical invariants can be taken as $(p-q)^2$, $(p-k)^2$, $q^2$ , $k^2$, $P^2 = (p-k-q)^2$ and $p^2$. We set $k^2 = 0$ and take $q^2 = m_\pi^2 = 0$, neglecting the small corrections of order $O(m_\pi^2/m_B^2)$. The momentum $p^2$ is kept undefined for the moment in order to make the derivation of the sum rules without restriction. Its value is going to be set later in this section, and chosen to be $p^2 = m^2_{\chi_{c1}}$. The values of $(p-k)^2$, $(p-q)^2$ and $P^2$ should be space-like and large in order to stay far away from the hadronic thresholds in the $B$ and $\chi_{c1}$ channels. All together, we have

$$\begin{aligned} &q^2 = k^2 = 0, \\ &p^2 = m^2_{\chi_{c1}}, \\ &|(p-k)|^2 \gg \Lambda_{\mathrm{QCD}}, \\ &|(p-q)|^2 \gg \Lambda_{\mathrm{QCD}}, \\ &|P|^2 \gg \Lambda_{\mathrm{QCD}}. \end{aligned}$$

The correlation function $\Pi_\rho(p,q,k)$ can be decomposed as

$$\Pi_\rho(p,q,k) = (p-k)_\rho \Pi + q_\rho \widetilde{\Pi}_1 + k_\rho \widetilde{\Pi}_2 + \epsilon_{\rho\alpha\beta\sigma} q^\alpha p^\beta k^\sigma \widetilde{\Pi}_3 \, , \tag{9}$$

due to Lorentz covariance.

According to the basic assumption of quark-hadron duality in the QCD sum rules,[29,30] we insert a complete set of intermediate states with the same quantum numbers as the current operators $J_\mu(y)$ and $J_5(x)$ into the correlation function $\Pi_\rho(p,q,k)$ to obtain the hadronic representation. After isolating the pole terms of the ground state mesons $\chi_{c1}$ and $B$, we obtain the following result,

$$\begin{aligned} \Pi_\rho &= \frac{\langle 0|J_\rho(0)|\chi_{c1}(p-k)\rangle}{m^2_{\chi_{c1}} - (p-k)^2} \langle \chi_{c1}(p-k)|\widetilde{\mathcal{O}}(0)|B(p-q)\rangle|\pi^0(q)\rangle \frac{\langle B(p-q)|J_5(0)|0\rangle}{m_B^2 - (p-q)^2} \\ &\quad + \ldots , \\ &= \frac{f_{\chi_{c1}} m_{\chi_{c1}} \epsilon_\rho}{m^2_{\chi_{c1}} - (p-k)^2} \frac{f_B m_B^2}{m_B^2 - (p-q)^2} \langle \chi_{c1}(p-k)\pi^0(-q)|\widetilde{\mathcal{O}}(0)|B(p-q)\rangle + \ldots , \\ &= \frac{f_{\chi_{c1}} m_{\chi_{c1}}}{m^2_{\chi_{c1}} - (p-k)^2} \frac{f_B m_B^2}{m_B^2 - (p-q)^2} q_\alpha \left[ -g_{\alpha\rho} + \frac{(p-k)_\alpha (p-k)_\rho}{(p-k)^2} \right] A_n + \ldots , \end{aligned} \tag{10}$$

where we have used the definition $\langle\chi_{\rm c1}(p-k)\pi^0(-q)|\widetilde{\mathcal{O}}(0)|B(p-q)\rangle = \epsilon^*\cdot q A_n$, and do not show the contributions from the higher resonances and continuum states above the corresponding thresholds explicitly; they can be written in terms of dispersion integrals and the spectral density can be approximated by the quark-hadron duality ansatz.

Now we carry out the operator product expansion near the light-cone to obtain the representation at the level of quark-gluon degrees of freedom. Let us write down the propagator of a massive quark in the external gluon field in the Fock–Schwinger gauge firstly,[31]

$$S_{ij}(x,y) = {\rm i}\int\frac{{\rm d}^4k}{(2\pi)^4}{\rm e}^{-{\rm i}\,k(x-y)}\left\{\frac{\not k+m}{k^2-m^2}\delta_{ij} - \int_0^1 {\rm d}v\, g_{\rm s}\, G^{\mu\nu}_{ij}(vx+(1-v)y)\right.$$
$$\left.\times\left[\frac{1}{2}\frac{\not k+m}{(k^2-m^2)^2}\sigma_{\mu\nu} - \frac{1}{k^2-m^2}v(x-y)_\mu\gamma_\nu\right]\right\}, \tag{11}$$

where $G^{\mu\nu}_a$ is the gluonic field strength, $g_{\rm s}$ denotes the strong coupling constant.

Substituting the above $b$ and $c$ quark propagators into the correlation function $\Pi_\rho$, and completing the corresponding integrals, we can obtain the hadronic spectral density at the level of quark-gluon degrees of freedom. In the calculation, the following three-particle $\pi^0$ light-cone distribution amplitudes are useful,

$$\begin{aligned}
-\sqrt{2}\langle 0|\bar d(0)\sigma_{\mu\nu}\gamma_5 G_{\alpha\beta}(vy)d(x)|\pi^0(q)\rangle &= {\rm i}\,f_{3\pi}\left[q_\alpha q_\mu g_{\beta\nu} + q_\beta q_\nu g_{\alpha\mu} - q_\beta q_\mu g_{\alpha\nu}\right.\\
&\left. -\, q_\alpha q_\nu g_{\beta\mu}\right]\int\mathcal{D}\alpha_i\phi_{3\pi}(\alpha_i){\rm e}^{-{\rm i}\,q(x\alpha_1+yv\alpha_3)}\,,\\
-\sqrt{2}\langle 0|\bar d(0) i\gamma_\mu\tilde G_{\alpha\beta}(vy)d(x)|\pi^0(q)\rangle &= f_\pi q_\mu\frac{q_\alpha x_\beta - q_\beta x_\alpha}{qx}\int\mathcal{D}\alpha_i\widetilde{\phi}_\parallel(\alpha_i){\rm e}^{-{\rm i}\,q(x\alpha_1+yv\alpha_3)}\\
&+ f_\pi(g^\perp_{\mu\alpha}q_\beta - g^\perp_{\mu\beta}q_\alpha)\int\mathcal{D}\alpha_i\widetilde{\phi}_\perp(\alpha_i){\rm e}^{-{\rm i}\,q(x\alpha_1+yv\alpha_3)}\,,\\
-\sqrt{2}\langle 0|\bar d(0)\gamma_\mu\gamma_5 G_{\alpha\beta}(vy)d(x)|\pi^0(q)\rangle &= f_\pi q_\mu\frac{q_\alpha x_\beta - q_\beta x_\alpha}{qx}\int\mathcal{D}\alpha_i\phi_\parallel(\alpha_i){\rm e}^{-{\rm i}\,q(x\alpha_1+yv\alpha_3)}\\
&+ f_\pi(g^\perp_{\mu\alpha}q_\beta - g^\perp_{\mu\beta}q_\alpha)\int\mathcal{D}\alpha_i\phi_\perp(\alpha_i){\rm e}^{-{\rm i}\,q(x\alpha_1+yv\alpha_3)}\,,
\end{aligned}\tag{12}$$

where

$$\tilde G_{\alpha\beta} = \frac{1}{2}\epsilon_{\alpha\beta\rho\sigma}G^{\rho\sigma},\quad \mathcal{D}\alpha_i = {\rm d}\alpha_1{\rm d}\alpha_2{\rm d}\alpha_3\delta(1-\alpha_1-\alpha_2-\alpha_3),\quad g^\perp_{\alpha\beta} = g_{\alpha\beta} - \frac{x_\alpha q_\beta + x_\beta q_\alpha}{qx}.$$

The twist-3 and twist-4 light-cone distribution amplitudes can be parameterized as

$$\begin{aligned}
\phi_{3\pi}(\alpha_i) &= 360\alpha_1\alpha_2\alpha_3^2\left[1+\frac{\omega_3}{2}(7\alpha_3-3)\right],\\
\phi_\perp(\alpha_i) &= 30\delta^2(\alpha_1-\alpha_2)\alpha_3^2\left[\frac{1}{3}+2\epsilon(1-2\alpha_3)\right],\\
\phi_\parallel(\alpha_i) &= 120\delta^2\epsilon(\alpha_1-\alpha_2)\alpha_1\alpha_2\alpha_3\,,\\
\widetilde{\phi}_\perp(\alpha_i) &= 30\delta^2\alpha_3^2(1-\alpha_3)\left[\frac{1}{3}+2\epsilon(1-2\alpha_3)\right],\\
\widetilde{\phi}_\parallel(\alpha_i) &= -120\delta^2\alpha_1\alpha_2\alpha_3\left[\frac{1}{3}+\epsilon(1-3\alpha_3)\right],
\end{aligned}\tag{13}$$

the nonperturbative parameters in the light-cone distribution amplitudes can be estimated with the QCD sum rules.[34–36]

After carrying out the operator product expansion near the light-cone, we obtain the following expression for the correlation function $\Pi$ (for technical details, one can consult Ref.[20]),

$$\begin{aligned}
\Pi &= \frac{m_b f_{3\pi}}{2\pi^2}\int_0^1{\rm d}v\int_0^1{\rm d}t\int\mathcal{D}\alpha_i\frac{\phi_{3\pi}(\alpha_i)q\cdot(p-k)\left[tvq\cdot k + 2(1-t)(1-v)q\cdot p\right]}{\{m_b^2-[p-q(1-\alpha_1)]^2\}\{\widetilde{m}_c^2-(p-k-v\alpha_3 q)^2\}}\\
&+\frac{m_b^2 f_\pi}{4\pi^2}\int_0^1{\rm d}v\int_0^1{\rm d}t\int\mathcal{D}\alpha_i\frac{q\cdot(p-k)}{\{m_b^2-[p-q(1-\alpha_1)]^2\}\{\widetilde{m}_c^2-(p-k-v\alpha_3 q)^2\}}\\
&\times\left\{(1-2t)\left[2(1-v)\phi_\perp(\alpha_i)-(1-2v)\phi_\parallel(\alpha_i)\right]+2(1-v)\widetilde{\phi}_\perp(\alpha_i)-\widetilde{\phi}_\parallel(\alpha_i)\right\},
\end{aligned}\tag{14}$$

where $\widetilde{m}_c^2 = \dfrac{m_c^2}{t(1-t)}$.

In order to suppress the contributions from the high resonances and continuum states, we perform an $n$-th derivative with respect to the momentum $(p-k)^2$ to obtain stable $n$-th moment sum rules and Borel transform with respect to the momentum $(p-q)^2$ in the $B$-channel to obtain the Borel sum rules, then match with Eq.(10). Finally, we obtain the sum rule for the nonfactorizable soft matrix element,

$$\begin{aligned}
A_n = & \frac{2m_{\chi_{c1}}}{f_B f_{\chi_{c1}} m_B^2\left[m_{\chi_{c1}}^2 - P^2\right]} \int_{4m_c^2}^{s_0} \mathrm{d}s \left\{ \frac{f_{3\pi} m_b}{4\pi^2} \int_{\mathrm{t_i}}^{t_\mathrm{f}} \mathrm{d}t \int_{\alpha_0}^{1} \mathrm{d}\alpha \int_{\alpha_c}^{\alpha} d\beta \right. \\
& \times \phi_{3\pi}(1-\alpha, \alpha-\beta, \beta) \left[ \frac{t\alpha_c}{2\beta}\left(P^2 - s - \frac{m_b^2 - m_{\chi_{c1}}^2}{\alpha}\right) - (1-t)\left(1-\frac{\alpha_c}{\beta}\right)\frac{m_b^2 - m_{\chi_{c1}}^2}{\alpha}\right] \\
& + \frac{f_\pi m_b^2}{8\pi^2} \int_{\mathrm{t_i}}^{t_\mathrm{f}} \mathrm{d}t \int_{\alpha_0}^{1} \mathrm{d}\alpha \int_{\alpha_c}^{\alpha} \mathrm{d}\beta \left[ 2\left(1-\frac{\alpha_c}{\beta}\right)\widetilde{\phi}_\perp - \widetilde{\phi}_\parallel \right. \\
& \left. \left. + \ (1-2t)\left[2\left(1-\frac{\alpha_c}{\beta}\right)\phi_\perp - \left(1-\frac{2\alpha_c}{\beta}\right)\phi_\parallel\right]\right](1-\alpha, \alpha-\beta, \beta)\right\} \\
& \times \exp\left(\frac{\alpha m_B^2 + m_{\chi_{c1}}^2(1-\alpha) - m_b^2}{\alpha M^2}\right)\left(\frac{m_{\chi_{c1}}^2 + Q_0^2}{s+Q_0^2}\right)^{n+1} \frac{1}{\alpha\beta}\,,
\end{aligned} \tag{15}$$

where

$$\begin{aligned}
& t_\mathrm{i} = \frac{1}{2}\left(1 - \sqrt{1 - \frac{4m_c^2}{s}}\right), \quad t_\mathrm{f} = \frac{1}{2}\left(1 + \sqrt{1 - \frac{4m_c^2}{s}}\right), \\
& \alpha_c = \frac{t(1-t)s - m_c^2}{t(1-t)(s-P^2)}, \quad \alpha_0 = \frac{m_b^2 - m_{\chi_{c1}}^2}{s_B - m_{\chi_{c1}}^2},
\end{aligned} \tag{16}$$

and $Q_0^2 = 4m_c^2\xi$. In the calculation, $P^2$ is chosen to be a large space-like squared momentum ($|P^2| \sim m_b^2$) in order to stay far away from the hadronic thresholds. The value of $\alpha_c$ is a small positive quantity but not always small enough to be safely neglected. We perform the following approximation for the $\beta$ integral,

$$\int_{\alpha_c}^{\alpha} \mathrm{d}\beta G(s,t,\alpha,\beta) = \left\{\int_0^{\alpha} - \int_0^{\alpha_c}\right\} \mathrm{d}\beta G(s,t,\alpha,\beta)\,, \tag{17}$$

where $G$ is an abbreviation for the integral functions and can be written as

$$\begin{aligned}
G(s,t,\alpha,\beta) = & A\phi_{3\pi}(1-\alpha,\alpha-\beta,\beta) + B\widetilde{\phi}_\parallel(1-\alpha,\alpha-\beta,\beta) + C\widetilde{\phi}_\perp(1-\alpha,\alpha-\beta,\beta) \\
& + D\phi_\parallel(1-\alpha,\alpha-\beta,\beta) + E\phi_\perp(1-\alpha,\alpha-\beta,\beta)\,,
\end{aligned}$$

$A$, $B$, $C$, $D$, $E$ are formal notations. We can expand the light-cone distribution amplitudes $\phi_{3\pi}$, $\widetilde{\phi}_\parallel$, $\widetilde{\phi}_\perp$, $\phi_\parallel$ and $\phi_\perp$ in terms of Taylor series of $\beta$, for example,

$$\begin{aligned}
\phi_{3\pi}(1-\alpha,\alpha-\beta,\beta) = & \phi_{3\pi}(1-\alpha,\alpha-\beta,\beta)|_{\beta=0} + \frac{\partial}{\partial\beta}\phi_{3\pi}(1-\alpha,\alpha-\beta,\beta)|_{\beta=0}\beta \\
& + \frac{1}{2}\frac{\partial^2}{\partial\beta^2}\phi_{3\pi}(1-\alpha,\alpha-\beta,\beta)|_{\beta=0}\beta^2 + \dots\,,
\end{aligned} \tag{18}$$

and continue $P^2$ into the time-like region analytically, $P^2 = m_B^2$, then complete the integral $\int_0^{\alpha_c} \mathrm{d}\beta G(s,t,\alpha,\beta)$. This procedure ensures the disappearance of the unphysical momentum $k$ from the ground state contribution and enables the extraction of the physical matrix element due to the simultaneous conditions, $P^2 = m_B^2$ and $(p-q)^2 = m_B^2$.

The explicit expression for the physical hadronic matrix element is lengthy due to the re-summation of all the Taylor series of $\beta$, here we show only the leading terms explicitly,

$$
\begin{aligned}
A_n = & \frac{2m_{\chi_{c1}}}{f_B f_{\chi_{c1}} m_B^2 \left[m_{\chi_{c1}}^2 - m_B^2\right]} \int_{4m_c^2}^{s_0} \mathrm{d}s \left\{ \frac{f_{3\pi} m_b}{4\pi^2} \int_{t_i}^{t_f} \mathrm{d}t \int_{\alpha_0}^{1} \mathrm{d}\alpha \int_0^{\alpha} \mathrm{d}\beta \right. \\
& \times \phi_{3\pi}(1-\alpha, \alpha-\beta, \beta) \left[ \frac{t\alpha_c}{2\beta} \left( m_B^2 - s - \frac{m_b^2 - m_{\chi_{c1}}^2}{\alpha} \right) - (1-t)\left(1 - \frac{\alpha_c}{\beta}\right) \frac{m_b^2 - m_{\chi_{c1}}^2}{\alpha} \right] \\
& + \frac{f_\pi m_b^2}{8\pi^2} \int_{t_i}^{t_f} \mathrm{d}t \int_{\alpha_0}^{1} \mathrm{d}\alpha \int_0^{\alpha} \mathrm{d}\beta \left[ 2\left(1 - \frac{\alpha_c}{\beta}\right) \widetilde{\phi}_\perp - \widetilde{\phi}_\parallel \right. \\
& \left. \left. + \ (1-2t) \left[ 2\left(1 - \frac{\alpha_c}{\beta}\right) \phi_\perp - \left(1 - \frac{2\alpha_c}{\beta}\right) \phi_\parallel \right] \right] (1-\alpha, \alpha-\beta, \beta) \right\} \\
& \times \exp\left( \frac{\alpha m_B^2 + m_{\chi_{c1}}^2 (1-\alpha) - m_b^2}{\alpha M^2} \right) \left( \frac{m_{\chi_{c1}}^2 + Q_0^2}{s + Q_0^2} \right)^{n+1} \frac{1}{\alpha\beta} \\
& - \frac{2m_{\chi_{c1}}}{f_B f_{\chi_{c1}} m_B^2 \left[m_{\chi_{c1}}^2 - m_B^2\right]} \int_{4m_c^2}^{s_0} \mathrm{d}s \left\{ \frac{f_{3\pi} m_b}{4\pi^2} \int_{t_i}^{t_f} \mathrm{d}t \int_{\alpha_0}^{1} \mathrm{d}\alpha \int_0^{\alpha_c} \mathrm{d}\beta \right. \\
& \times \phi_{3\pi}(1-\alpha, \alpha-\beta, \beta) \mid_{\beta=0} \left[ \frac{t\alpha_c}{2\beta} (m_B^2 - s - \frac{m_b^2 - m_{\chi_{c1}}^2}{\alpha}) \right. \\
& \left. - (1-t)(1 - \frac{\alpha_c}{\beta}) \frac{m_b^2 - m_{\chi_{c1}}^2}{\alpha} \right] \\
& + \frac{f_\pi m_b^2}{8\pi^2} \int_{t_i}^{t_f} \mathrm{d}t \int_{\alpha_0}^{1} \mathrm{d}\alpha \int_0^{\alpha_c} \mathrm{d}\beta \left[ 2(1 - \frac{\alpha_c}{\beta}) \widetilde{\phi}_\perp - \widetilde{\phi}_\parallel \right. \\
& \left. \left. + \ (1-2t) \left[ 2\left(1 - \frac{\alpha_c}{\beta}\right) \phi_\perp - \left(1 - \frac{2\alpha_c}{\beta}\right) \phi_\parallel \right] \right] (1-\alpha, \alpha-\beta, \beta) \mid_{\beta=0} \right\} \\
& \times \exp\left( \frac{\alpha m_B^2 + m_{\chi_{c1}}^2 (1-\alpha) - m_b^2}{\alpha M^2} \right) \left( \frac{m_{\chi_{c1}}^2 + Q_0^2}{s + Q_0^2} \right)^{n+1} \frac{1}{\alpha\beta} + \dots \ . \qquad (19)
\end{aligned}
$$

In performing the $\beta$ integral $\int_0^{\alpha_c}$, we need only the values of the light-cone distribution amplitudes $\phi_{3\pi}$, $\widetilde{\phi}_\parallel$, $\widetilde{\phi}_\perp$, $\phi_\parallel$, $\phi_\perp$ and their derivations at zero momentum fraction i.e. $\beta = 0$, there are no problems with negative partons (quarks and gluons) momentum fractions. The analytical continuation of $P^2$ to its positive value ends up with an unavoidable theoretical uncertainty, if only a few terms of the Taylor series are taken. However, with the re-summation to all orders of $\beta$ in Eq.(19), the assumption of quark-hadron duality is still applicable in the case of heavy meson final states.

## 4. Numerical results

The input parameters are taken as $V_{\rm cd} = -0.230$, $V_{\rm cb} = 41.2 \times 10^{-3}$, $V_{\rm tb} = 1.0$, $V_{\rm td} = 8.1 \times 10^{-3}$, $m_{\chi_{c1}} = 3.511\,\mathrm{GeV}$, $m_B = 5.2795\,\mathrm{GeV}$,[37] $f_B = 0.18\,\mathrm{GeV}$, $s_B = 35 \pm 2\,\mathrm{GeV}^2$,[32] $f_{\chi_{c1}} = 0.335\,\mathrm{GeV}$,[33] $s_0 = (16.0 \pm 0.5)\,\mathrm{GeV}^2$,[30] $f_\pi = 0.13\,\mathrm{GeV}$, $m_b = (4.7 \pm 0.1)\,\mathrm{GeV}$, $m_c = (1.35 \pm 0.05)\,\mathrm{GeV}$, $f_{3\pi} = (0.45 \pm 0.15) \times 10^{-2}\,\mathrm{GeV}^2$, $\omega_3 = -1.5 \pm 0.7$, $\delta^2 = (0.18 \pm 0.06)\,\mathrm{GeV}^2$, $\varepsilon = \dfrac{21}{8}(0.2 \pm 0.1)$[34−36] at the energy scale about $\mu = 1\,\mathrm{GeV}$.

The parameters $n$ and $M^2$ must be carefully chosen to warrant the high resonances and continuum states being suppressed and obtain a reliable perturbative QCD calculation. The stable region for the Borel parameter $M^2$ is found in the interval $M^2 = 11 \pm 2\,\mathrm{GeV}^2$, which is known from the $B$ channel QCD sum rules.[32] In the charmonium channels, we usually perform the $n$-th derivative and take $n$-th moment sum rules to satisfy the stability criteria,[30] the ideal interval is $n = 4 - 6$. The parameter $\xi$ is usually allowed to take values larger than 1 for the $P$-wave charmonia, we observe that the best interval is $\xi = 1.8 - 2.6$.

The nonfactorizable soft contributions come from

the three-particle twist-3 and twist-4 $\pi^0$ light-cone distribution amplitudes, however, our present knowledge about these distribution amplitudes is rather poor. The uncertainties of the nonperturbative parameters $f_{3\pi}$, $\omega_3$, $\delta^2$ and $\varepsilon$ are large, about 33%–50%, of the nonfactorizable soft contributions $A_n$ can be simplified into the following form,

$$A_n = y_1 f_{3\pi} + y_2 f_{3\pi}\omega_3 + y_3\delta^2 + y_4\delta^2\varepsilon\,, \tag{20}$$

where the $y_i$ are numerical coefficients not shown explicitly. The uncertainties originating from the nonperturbative parameters $f_{3\pi}$, $\omega_3$, $\delta^2$ and $\varepsilon$ are rather large, even out of control. We take the same treatment as Refs.[20]–[23] and [26], and neglect the corresponding uncertainties; however, it weakens the predictive ability. The uncertainties originating from other parameters ($s_B$, $s_0$, $m_b$, $m_c$, $n$, $\xi$, etc) are less than 22%, so we take them into account with the formula $\Delta = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 (x_i - \bar{x}_i)^2}$, where the $f$ denotes the nonfactorizable soft contributions, and $x_i$ denotes the input parameters.

Taking into account the next-to-leading order Wilson coefficients calculated in the naive dimensional regularization scheme[2] for $\mu = \overline{m_b}(m_b) = 4.40\,\mathrm{GeV}$ and $\Lambda^{(5)}_{\overline{\rm MS}} = 225\,\mathrm{MeV}$,

$$\begin{aligned}
&C_1(\overline{m_b}(m_b)) = 1.082\,,\\
&C_2(\overline{m_b}(m_b)) = -0.185\,,\\
&C_3(\overline{m_b}(m_b)) = 0.014\,,\\
&C_4(\overline{m_b}(m_b)) = -0.035\,,\\
&C_5(\overline{m_b}(m_b)) = 0.009\,,\\
&C_6(\overline{m_b}(m_b)) = -0.041\,,
\end{aligned} \tag{21}$$

finally we obtain the numerical ratio $R$ between the contributions from the nonfactorizable and factorizable hadronic matrix elements, as shown in Fig.1,

$$\begin{aligned}
R =& \frac{\sqrt{2}C_1(\mu)\langle\chi_{\rm c1}(p)\pi^0(-q)|\widetilde{\mathcal{O}}(0)|B(p-q)\rangle}{-\left[C_2(\mu)+\frac{C_1(\mu)}{3}\right] f_{\chi_{\rm c1}} m_{\chi_{\rm c1}} f(m^2_{\chi_{\rm c1}})}\\
=& 0.68^{+0.20}_{-0.16}\,.
\end{aligned} \tag{22}$$

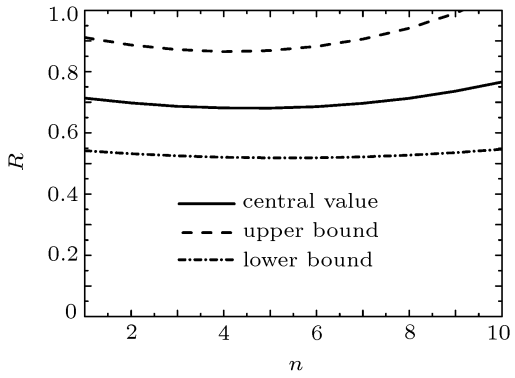



**Fig.1.** The ratio $R$ between the contributions from the nonfactorizable and factorizable hadronic matrix elements, as a function of $n$.

The nonfactorizable soft contributions are rather large and they must be included for analysing the branching fraction. The total amplitudes lead to the branching fraction,

$$Br(B^0 \to \chi_{\rm c1}\pi^0) = 1.41^{+0.35}_{-0.25} \times 10^{-5}\,, \tag{23}$$

which is in agreement with the experimental data $(1.12 \pm 0.25 \pm 0.12) \times 10^{-5}$ marginally.[1]

The factorizable contributions of the operator $\mathcal{O}_2(0)$ can be analysed in the same way with the correlation function $\widehat{\Pi}_\rho(p,q,k)$,

$$\begin{aligned}
\widehat{\Pi}_\rho =& \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}(p-q)x} \int \mathrm{d}^4y \mathrm{e}^{\mathrm{i}(p-k)y} \langle 0|T\{J_\rho(y)\mathcal{O}_2(0)J_5(x)\}|\pi^0(q)\rangle\\
=& \int \mathrm{d}^4x \mathrm{e}^{-\mathrm{i}(p-q)x} \langle 0|T[\overline{d}(0)\gamma^\mu b(0)J_5(x)]|\pi^0(q)\rangle \int \mathrm{d}^4y \mathrm{e}^{\mathrm{i}(p-k)y} \langle 0|T[J_\rho(y)J_\mu(0)]|0\rangle\\
\propto& (p-k)_\rho f\left((p-k)^2\right) \times f^2_{\chi_{\rm c1}} + \ldots\,.
\end{aligned} \tag{24}$$

The contributions of the soft gluons can be absorbed into the $B \to \pi^0$ form-factor $f\left((p-k)^2\right)$ or the decay constant $f_{\chi_{\rm c1}}$, and are quite different from the sum rules for the operator $\widetilde{\mathcal{O}}(0)$, where the contributions of the soft gluons are nonfactorizable, so we are free of double-counting.

## 5. Conclusion

In this article, we calculate the contributions from the nonfactorizable soft hadronic matrix element $\langle\chi_{\rm c1}(p)\pi^0(-q)|\widetilde{\mathcal{O}}(0)|B(p-q)\rangle$ to the decay $B^0 \to \chi_{\rm c1}\pi^0$ with the light-cone QCD sum rules. The numeri-

cal results show that its contribution is rather large and should not be neglected. The total amplitudes lead to a branching fraction which is marginally in agreement with the experimental data.

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