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A self-organizing shortest path finding strategy on complex networks*

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The shortcomings of traditional methods to find the shortest path are revealed, and a strategy of finding the self-organizing shortest path based on thermal flux diffusion on complex networks is presented. In our method, the shortest paths between the source node and the other nodes are found to be self-organized by comparing node temperatures. The computation complexity of the method scales linearly with the number of edges on underlying networks. The effects of the method on several networks, including a regular network proposed by Ravasz and Barabási which is called the RB network, a real network, a random network proposed by Ravasz and Barabási which is called the ER network and a scale-free network, are also demonstrated. Analytic and simulation results show that the method has a higher accuracy and lower computational complexity than the conventional methods.

Keywords: complex networks, self-organization, the shortest path, thermal flux diffusion
PACC: 0565, 0590

1. Introduction

The theory of complex networks is an effective tool to study the complex systems in the real world,[1,2] such as social networks,[3] metabolic networks,[4] citation networks,[5] etc. As topological information on such complex networks is becoming more and more available through computer-aided techniques, there is growing interest in the investigation of properties of such networks.[6−20] At the same time, many achievements of complex network studies are applied to real networks[21] because they can provide meaningful theoretical guidance to solving many problems of real networks.

In complex networks, two remote nodes can be connected by many paths. Among these paths, the shortest one plays an important role in traffic flow on the underlying networks. So far, many algorithms to find the shortest path have been proposed. As a comprehensive diffusion-based method, the break-first-search (BFS) method has high accuracy.[5] However, in the BFS the shortest path is obtained by comprehensively comparing the lengths of various paths, thereby increasing the load on networks[20,22] and may lead to congestion. In order to avoid such circumstances, Yang and Garcia[23] have proposed a direct BFS method in recent years, but in the method all nodes are required to know the statistical information about their neighbouring nodes in advance. Therefore, the load induced by the direct BFS is still heavy on networks.[22] The random walk (RW) algorithm[20] is a local search approach. It can largely reduce the load on networks, but it is slow and inaccurate. Some researchers have applied the RW algorithm to hierarchical networks,[20] random networks,[20] small world networks,[22,24−27] and scale-free networks.[12,23−29] Their results show that the searching speed is much slower than that of BFS and the accuracy depends on the topological features of the networks.[28−31] In order to improve the speed of the RW algorithm, Adamic proposed the highest degree method[22] which is similar to the RW algorithm. In the highest degree method, a walker always walks to its neighbouring node with the highest degree, which makes the method have a higher speed and accuracy than the RW algo-

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rithm on average,[22] but it is still much slower and less accurate than BFS.[31] Jeong and Berman[32] applied the highest degree method to scale-free networks in which the degree distribution obeys a power-law with a tunable exponent γ. Results in Ref.[32] show that the accuracy of the method depends greatly on heterogeneity in the node degree.[32,33]

In the present paper, we introduce a physical method based on the thermal flux diffusing mechanism to find the shortest path on complex networks, which we refer to as self-organizing path finding strategy. In the method, the temperature of a node increases quickly through the path to the neighbouring node with the highest temperature, which is in accordance with the greedy principle of temperature increasing. Therefore, the method has high accuracy and low computational complexity. The method is also faster than BFS in manipulation because it finds the shortest paths from the source node to the other nodes in a parallel fashion. For a network with n nodes and m edges, let the diffusion time be t and integral interval be Δt, then the shortest path from the source node to all the other nodes can be found in time $O(mt/\Delta t)$.

The rest of this paper is organized as follows. In Section 2, we present the thermal flux diffusing model of complex networks. In Section 3, we describe in detail the implementation of our method. In Section 4, we apply it to various networks, and compare its performance with those of other existing methods. Finally, we give the conclusion and discussion in Section 5.

2. Thermal flux diffusing in complex networks

The thermal flux diffusing model of complex networks is described by using a simple network. Without loss of generality, we focus on the simple case in which two nodes have only one edge between them if they are connected as shown in Fig.1, in which node i is connected with node j by the edge l_k.

In the model, edge l_k is defined as an edge element with density ρ. The diffusing time is t, and the temperature at location s on edge l_k is T_s which is denoted by function $T_s = T_s(s,t)$. The heat flux at location s is q_s, and the heat flux injected from the outside of the network is $\tilde{q}_s$. The rate of heat flux is V_s.

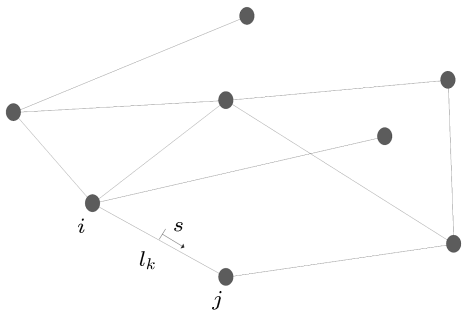


Fig.1. Thermal flux diffusing model of a complex network.

According to the first law of thermodynamics that the thermal energy is conserved,[34] we specify this as the edge element, and then the thermal dynamic equation of the point at location s on the edge is

$$\rho c\left(\frac{\partial T_s}{\partial t}+V_s\frac{\partial T_s}{\partial l_k}\right)+\frac{\partial T_s}{\partial l_k}q_s=\tilde{q}_s, \tag{1}$$

where c is a heat coefficient. Next, Fourier's law is used to link the heat flux q_s to the thermal gradient,[20] and q_s can be denoted as

$$q_s=-K\frac{\partial T_s}{\partial l_k}, \tag{2}$$

where K is the conductivity coefficient of edges. Using expression (2), Eq.(1) can be written as

$$\rho c\left(\frac{\partial T_s}{\partial t}+V_i\frac{\partial T_s}{\partial l_k}\right)-\frac{\partial}{\partial l_k}\left(K_k\frac{\partial T_s}{\partial l_k}\right)=\tilde{q}_s. \tag{3}$$

For a network with $m$edges, we consider the network as a volume. Pre-multiplying Eq.(3) by a virtual change in temperature δT_s, the dynamic equation of the network can be written as

$$\begin{aligned}&\int_m\left(\rho c(\delta T_s)\left(\frac{\partial T_s}{\partial t}+V_s\frac{\partial T_s}{\partial l_k}\right)\right.\\&\left.-\frac{\partial}{\partial l_k}\left[(\delta T_s)\left(K_k\frac{\partial T_s}{\partial l_k}\right)\right]\right)\mathrm{d}\,(m)\\&=\int_m(\delta T_s)\tilde{q}_s\mathrm{d}\,(m).\end{aligned} \tag{4}$$

Then, in Fig.1, the temperature at any location s on edge l_k is

$$T_s=\frac{1}{2}\left\{\begin{array}{c}(1-s)\\(1+s))\end{array}\right\}^{\mathrm{T}}\left\{\begin{array}{c}T_i\\T_j\end{array}\right\}=\boldsymbol{S}_k^{\mathrm{T}}\boldsymbol{T}_k, \tag{5}$$

where $\boldsymbol{S}_k$ is defined as the temperature distribution function of the edge l_k. $T_s = T_i$ when $s = -1$, and $T_s = T_j$ when $s = 1$. The derivative of expression (5) with respect to time can be written as

$$\dot{T}_s=\frac{\partial T_s}{\partial t}=\boldsymbol{S}_k^{\mathrm{T}}\dot{\boldsymbol{T}}_k. \tag{6}$$

δT_s has the form

$$\delta T_s = \boldsymbol{S}_k^{\mathrm{T}} \delta \boldsymbol{T}_k. \tag{7}$$

Let $\boldsymbol{B} = \dfrac{\partial(\boldsymbol{S}_k T_s)}{\partial l_k}$. Now, the variational statement of Eq.(4) can be combined with expression (5) through Eq.(7) to yield

$$\begin{aligned}&\int_m \rho c \left((\boldsymbol{\delta} T_k)^{\mathrm{T}} \boldsymbol{S}_k \boldsymbol{S}_k^{\mathrm{T}} \dot{T}_k + (\boldsymbol{\delta} T_k)^{\mathrm{T}} \boldsymbol{S}_k V_s \boldsymbol{B} T_k \right) \mathrm{d}\,(m) \\ &- \int_m (\boldsymbol{\delta} T_k)^{\mathrm{T}} K_k \boldsymbol{B}^{\mathrm{T}} \boldsymbol{B} T_k \mathrm{d}\,(m) \\ &= \int_k (\boldsymbol{\delta} T_k)^{\mathrm{T}} \boldsymbol{S}_k \tilde{q} \mathrm{d}(m), \end{aligned} \tag{8}$$

where V_s, ρ and c are assumed to be constant over the edge for simplicity. $\boldsymbol{T}_k$, $\dot{\boldsymbol{T}}_k$ and $\delta \boldsymbol{T}_k$ are nodal quantities and do not vary over the edge, so that they may also be removed from the integral. All quantities are premultiplied by the arbitrary vector $\delta \boldsymbol{T}_k$, thus it may be dropped from the resulting equation. Thus, Eq.(8) may be reduced into the form of a dynamical equation as follows:

$$\boldsymbol{C}_k^{\boldsymbol{t}} \dot{T}_k(t) + \mathbf{K}_k^{\boldsymbol{t}} \boldsymbol{T}_k(t) = \boldsymbol{Q}_k(t), \tag{9}$$

where

$$\boldsymbol{C}_k^{\boldsymbol{t}}(t) = \rho c \int_m \boldsymbol{S}_k \boldsymbol{S}_k^{\mathrm{T}} \mathrm{d}\,(m)\,, \tag{10}$$

$$\boldsymbol{K}_k^{\boldsymbol{t}}(t) = \int_m (\rho c \boldsymbol{S} V_s \boldsymbol{B} - K_k \boldsymbol{B}^{\mathrm{T}} \boldsymbol{B}) \mathrm{d}\,(m)\,, \tag{11}$$

$$\begin{aligned}\boldsymbol{Q}_k(t) &= \int_m \boldsymbol{S}_k \tilde{q} \mathrm{d}\,(m) \\ &= \int_m -\boldsymbol{S}_k K_k \boldsymbol{B} T_{ck}(t) \mathrm{d}\,(m)\,. \end{aligned} \tag{12}$$

In expression (12), $\boldsymbol{T}_{ck}(t) = \left[T_{ci}(t)\ T_{cj}(t) \right]$, where $T_{ci}(t)$ is the temperature load on node i from the outside of the network at time t (here, except on the source node, all temperature loads equal zero, i.e. $T_{ci}(t) = 0$). The node temperature at any time t can be obtained by reading node temperature vector $\boldsymbol{T}_k$. For a network with n nodes and m edges, the computational complexity of constructing the model is $O(m)$. Let the diffusing time be t and integral interval be Δt, then the number of steps of iteration is $O(t/\Delta t)$. Hence, all node temperatures can be obtained in time $O(mt/\Delta t)$.

3. Self-organizing path finding strategy

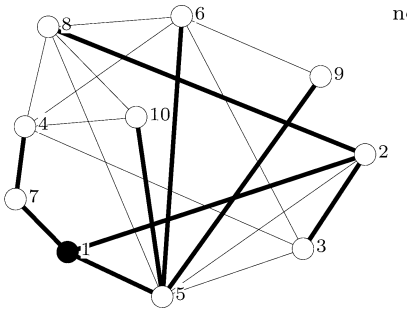
The cross-sectional area of the edge between nodes i and j is set to be S. At a diffusing time moment t_p, we define q_{ij} as the transient thermal flux from node i to node j. $\Delta t/t_p \leq 1/5$ should be satisfied to ensure the accuracy of discrete simulation.[34]

$$q_{ij} = -K \left(\frac{T_i(t_{\mathrm{p}}) - T_j(t_{\mathrm{p}})}{S l_k} \right) A_{ij}. \tag{13}$$

The node temperature vector $\boldsymbol{T}_k$ can be obtained by solving Eq.(9), and the transient thermal flux matrix $\boldsymbol{q}_t$ composed of element q_{ij} can be calculated from expression (13). $q_{ij} > 0$ indicates that the thermal flux is transmitted from node j to node i, and $q_{ij} < 0$ otherwise.

In order to describe in detail the implementation of our method, we consider a simple random network shown in Fig.2(a). We define node 1 as the source node and set its temperature to be 1000 °C, and the temperatures of other nodes are initialized to be 0 °C. Simulation parameters are taken to be $K = 100$ W/m·°C, $\rho = 144$ kg/m^3, $c = 0.2$ J/kg·°C, the length of the edges in the model is set to be 1 m, and the cross-sectional area of the edges 1 m^2, t_p 1.5 h, and Δt 0.25 h. The element q_{ij} in matrix $\boldsymbol{q}_t$ is calculated from expression (13), and the result is shown in Fig.2(b). According to the greedy principle of temperature increasing, the element with a maximal value in vector $\boldsymbol{q}_i$ indicates the neighbouring node that transmits the maximal thermal flux to node i. The edge between them is on one of the shortest paths from node i to the source node. Figure 2(a) shows a connected diagram composed of thick edges, which reveals the shortest paths from the source node to the other nodes.

Actually, there is more than one shortest path from the source node to some nodes in Fig.2(a). For example, there are two shortest paths from node 1 to node 3: one is 1→2→3 (denoted by node ID) and the other is 1→5→3. In Fig.2(b), accordingly, there are two equal maximal elements in each of the thermal flux vectors of nodes 3 and 8. In this case, node 1 selects one of them at random to indicate one link of its shortest path. One point should be noted, i.e. different simulation parameters will lead to different values of the elements in the thermal flux matrix, but this has no influence on the results.



node number 1 2 3 4 5 6 7 8 9 10

$$\boldsymbol{q}_t = \begin{bmatrix} 0 & 24.885 & 0 & 0 & 26.625 & 0 & 22.13 & 0 & 0 & 0 \\ -24.885 & 0 & 5.597 & 0 & 0.886 & 0 & 0 & 6.36 & 0 & 0 \\ 0 & -8.597 & 0 & -0.23 & -3.68 & 1.34 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1.72 & 0 & 0 & 1.356 & -11.23 & 2.243 & 0 & 0.786 \\ -26.625 & -0.886 & 5.597 & 0 & 0 & 4.785 & 0 & 6.36 & 4.246 & 4.685 \\ 0 & 0 & -1.34 & -1.356 & -4.785 & 0 & 0 & -0.785 & 1.032 & 0 \\ -22.13 & 0 & 0 & 7.23 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -8.36 & 0 & -2.243 & -4.34 & 0.785 & 0 & 0 & 0 & 1.258 \\ 0 & 0 & 0 & 0 & -4.246 & -1.24 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1.786 & -4.685 & 0 & 0 & -1.258 & 0 & 0 \end{bmatrix}$$

Fig.2. Results obtained by applying our method to a simple random network. Panel (a) shows a random network in which the connected network is composed of thick edges, revealing the shortest paths from the source node (denoted by a solid circle) to other nodes, and panel (b) indicates node transient thermal flux matrix $\boldsymbol{q}_t$ (kW/m^2), where the element framed with a hexagon in each node vector indicates the link selected by a node.

Compared with the RW method and the highest degree method, our method has advantages, i.e. high speed and accuracy. Unlike the shortest path obtained by a comprehensive comparison among various path lengths in BFS, in our method the shortest path is obtained by a local comparison among node temperatures, thereby significantly reducing the complexity of computation. Moreover, such processes can be implemented in parallel, which makes our method faster than BFS in computation.

4. Simulation

In order to test the performance of our method on networks with various topologies, we first apply it to a regular RB network[35] with two hierarchical levels proposed by Ravasz and Barabási and a real network (the Santa Fe Institute collaboration network[36] which has a large diameter). Then, for a homogeneous network we use the Poisson random network model given by Erdös and Rényi.[37] Finally, scale-free network[38] in which the degree distribution obeys a power-law with an exponent ranging from 2.1 to 2.9 is used to test the accuracy of our method. The simulation parameters used in the applications are the same as those described in Section 3.

4.1. RB network

We first apply our method to an RB[35] network of 25 nodes with two hierarchical levels proposed by Ravasz and Barabási. The central node 1 is defined as the source node. The shortest paths from the source node to the other nodes are shown in Fig.3.

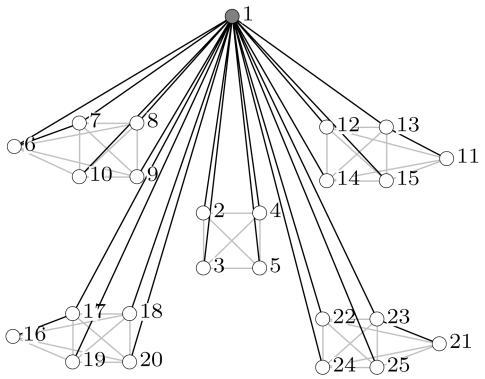


Fig.3. Results obtained by applying our method to an RB network of 25 labelled nodes with two hierarchical levels. A connective network composed of the edges in black colour reveals the shortest paths from the source node to the other nodes.

As shown in Fig.3, the shortest path from the source node to any node may be detected correctly with our method. The largest path length between nodes in the network is 2 (from nodes 1 to 6, 11, 16, and 21, respectively). For the regular and hierarchical topology of the network, each of the thermal flux vectors of nodes 6, 11, 16, and 21 has four equal maximal elements, which can be selected at random to indicate a link of its shortest path.

4.2. Santa Fe Institute collaboration network

Now we consider a real collaboration network of scientists at the Santa Fe Institute, an interdisci-

plinary research centre in Santa Fe, New Mexico. The 271 vertices in this network represent scientists resident at the Santa Fe Institute during any part of the calendar years 1999 or 2000 and their collaborators.[36] An edge is drawn between a pair of scientists if they coauthored one or more articles in the same time period. The largest component of the collaboration network consists of 118 scientists. We select this network as a test network because it has a large diameter. We define the node in grey colour as the source node. A connective network that reveals the shortest paths from the source node to the other nodes is shown in Fig.4. The shortest paths found by our proposed method accord with the real ones.

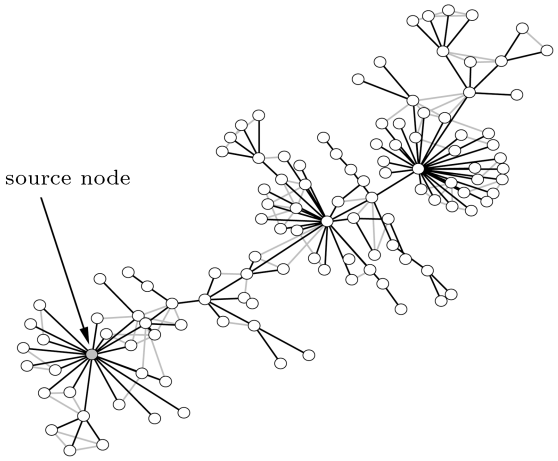


Fig.4. Results obtained by applying our method to the Santa Fe collaboration network, where the source node is in grey colour. The shortest paths from the source node to the other nodes are denoted by the edges in black colour.

4.3. Poisson random network and scale-free network

For a homogeneous random network we use the model given by Erdös and Rényi (ER network).[37] We consider a network with N nodes where two nodes are connected with a connection probability p. The largest connected component in the ER network is shown to always exist for $p > 1/N$.[39] The scale-free network is generated by using the method given by Newman[38] in which the degree distribution obeys a power-law with an exponent γ ranging from 2.1 to 2.9. The largest connected component always exists for $2 < \lambda < 3.48$.[38] The degree distribution of this largest connected component is nearly the same as that in the original network. The widely used D_{avg} (the average distance)[35−38] is used here as a metric to evaluate the searching performance. The shorter the D_{avg} is, the better the performance will be.[37]

Figure 5 shows the performance of our proposed method for homogeneous random networks with 500, 1000, 1500, 2000, and 2500 nodes separately. The connection probability (p) is taken to be 0.004, i.e. $p = 0.004$. We also make a comparison between our results and the results obtained from the two traditional methods (the typical RW method[20] and the Highest Degree[16] method).

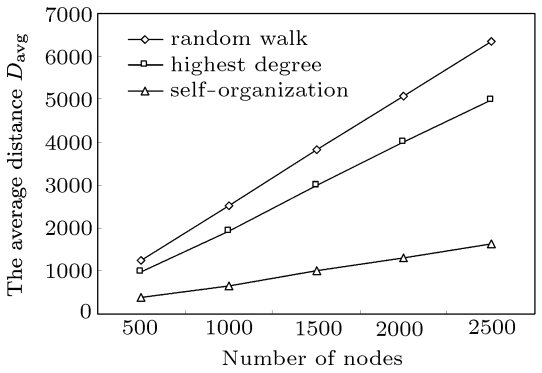


Fig.5. Curves for D_{avg} obtained in these ER networks versus number of nodes. The three curves show the results obtained by the self-organizing method (triangles), the highest degree method (squares) and the RW method (diamonds) separately.

In such homogeneous networks, the temperatures of nodes distribute naturally according to the length of their shortest paths from the source node. Hence, as Fig.5 shows, the self-organization method performs best in the networks from 500 nodes to 2500 nodes in size. As network size increases, our method becomes more efficient than the RW and highest degree methods.

Next we compare the methods by showing how they perform on scale-free networks with power-law exponents ranging from 2.1 to 2.9, as shown in Fig.6. We observe that the scale-free networks become homogeneous in node degree as the value of exponent γ increases. Then, the probability of the walker in the RW and the highest degree walking to the node with a larger degree is reduced, so the accuracy of the two methods decreases dramatically as γ increases. For using the greedy principal of temperature increasing, our method has a high accuracy on networks with various topologies. Because the results obtained by our method are the same as those of BFS, there is no need to show the results of BFS again in Figs.5 and 6.

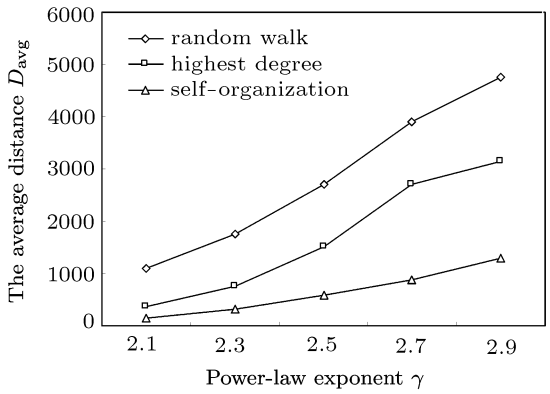


Fig.6. Comparison among the methods in the scale-free network on 2000 nodes, with different values of power-law exponent. The average distance is obtained for each method in the scale-free network as a function of power-law exponent. All path lengths systematically increase with the increase of the value of power-law exponent γ due to the fact that the average degree of the network decreases with γ.

5. Conclusion and discussion

In this paper, we present a method for finding the self-organizing shortest path on complex networks based on thermal flux diffusing. As a fully localized diffusion-based method, our method has higher speed and accuracy than the RW method and the highest degree method. Unlike the shortest path obtained by a comprehensive comparison among various path lengths in BFS, in our method the shortest path is obtained by a local comparison of node temperatures, thereby significantly reducing the complexity of computation. Moreover, such processes can be implemented in parallel, which makes our method faster than BFS in computation. Finally, we give several illustrative applications, including a regular hierarchical network, a real network, a random network and a scale-free network. The results show that our method is fast and accurate on various topology complex networks.

However, shortcomings to our method also exist. First, for the successful implementation of our method, all the nodes must reach a certain temperature higher than 0 °C, so t must run over a certain time. Second, the value of $t/\Delta t$ is an important parameter which can influence the accuracy and speed in computation. On one hand, if the value of $t/\Delta t$ is too small, the result would be inaccurate because it cannot satisfy the condition of numerical simulation.[29] On the other hand, since all node temperatures can be obtained in time $O(mt/\Delta t)$, large $t/\Delta t$ will reduce the speed. Our future work is to improve the thermal flux diffusion model to avoid the influence of the above simulation parameters and apply this method to real networks, such as biology networks, metabolic networks, communication networks, etc.

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