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Synchronization and parameter identification of one class of realistic chaotic circuit*

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In this paper, the synchronization and the parameter identification of the chaotic Pikovsky–Rabinovich (PR) circuits are investigated. The linear error of the second corresponding variables is used to change the driven chaotic PR circuit, and the complete synchronization of the two identical chaotic PR circuits is realized with feedback intensity k increasing to a certain threshold. The Lyapunov exponents of the chaotic PR circuits are calculated by using different feedback intensities and our results are confirmed. The case where the two chaotic PR circuits are not identical is also investigated. A general positive Lyapunov function V, which consists of all the errors of the corresponding variables and parameters and changeable gain coefficient, is constructed by using the Lyapunov stability theory to study the parameter identification and complete synchronization of two non-identical chaotic circuits. The controllers and the parameter observers could be obtained analytically only by simplifying the criterion $\mathrm{d}V/\mathrm{d}t < 0$ (differential coefficient of Lyapunov function V with respect to time is negative). It is confirmed that the two non-identical chaotic PR circuits could still reach complete synchronization and all the unknown parameters in the drive system are estimated exactly within a short transient period.

Keywords: Pikovsky–Rabinovich, parameter identification, chaos, adaptive synchronization
PACC: 0545

1. Introduction

Chaos is observed in the chemical, physical and biological systems. More attention has been paid to the control and/or synchronization of chaos and spatiotemporal chaos[1,2] since the pioneering work carried out by Pecoral and Ott *et al.*[3,4] In the case of control chaos, different schemes are used to control the chaotic system to reach arbitrary periodical orbits or generate multi-scroll chaotic attractors, etc. For example, a problem about how to generate and apply the multi-scroll chaotic attractors has been investigated in detail in Refs.[5] and [7]. On the other hand, a coupling scheme is often used to realize complete synchronization, phase synchronization, generalized synchronization, etc. Up to now, many effective schemes have been proposed to control and synchronize the chaotic systems. For example, in Ref.[8] Luo proposed a scheme of space compression by restricting the variables in appropriate thresholds, and the scheme in which the spatiotemporal chaos could be controlled was proposed in Ref.[9]. Furthermore, one[10] of the authors of the present paper has discussed the mechanism of this scheme. The synchronization of chaos and hyperchaos is very important due to its potential applications in secure communication. In a practical way, the cost, the feasibility and the reliability of the scheme should be considered carefully. Therefore, an intermittent control scheme[11] is better than most of the previous continuous control schemes, and an adaptive control scheme shows more advantages than other schemes in controlling and synchronizing these uncertain chaotic systems.[12−19] As is well known, some parameters of the system could be unknown or the chaotic system is uncertain. Therefore, it is critical to obtain these unknown parameters exactly within a short transient period when a scheme is used to control the chaotic system. Lü *et al*[20] suggested that linear feedback based on the parameter identification be used to control the Chen chaotic system. In Refs.[21] and [22], a parameter observer was constructed to identify the unknown parameters in the chaotic Lorenz system and the Rössler system. Li *et al*[23] proposed an adaptive synchronization scheme to

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estimate the parameters in the uncertain Chua circuit by adding a linear error function of variables on the right-hand side of the first formula, which contains a nonlinear function. As is well known, the chaotic Chua is simple, and it easily controls and realizes synchronization. In fact, it often encounters some difficulties in constructing appropriate parameter observers due to the complexity and diversity of the chaotic system.

In the present paper, the Pikovsky–Rabinovich(PR) circuit, which could generate double-scroll chaotic attractors within an appropriate parameter region,[24−26] is investigated. Our attention is focused mainly on realizing synchronization and parameter estimation of the chaotic PR circuits even if the two chaotic PR circuits are not identical. The procedure for designing any practical controller to realize complete synchronization and parameter identification is discussed in detail by using the Lyapunov stability theory. And it is confirmed that our scheme is effective in realizing synchronization and the parameter estimation of the single chaotic attractor and/or multi-scroll chaotic attractors. The controller and the parameter observers could be constructed carefully according to the nonlinear differential equations by using the Lyapunov stability theory. First, the synchronization of the two identical chaotic PR systems is studied by using the monodirectional coupling scheme, and the controller is simple and linear. Second, we study the synchronization of two non-identical PR circuits with different parameters and all the parameters in the drive system are unknown. It focuses on estimating these unknown parameters in the drive system by using an adaptive synchronization scheme and the coupling is of monodirectional type. It is confirmed that all the unknown parameters could be estimated exactly within a short transient period when the two chaotic systems reach complete synchronization. The discussion and the analysis of our results could give some useful clues to constructing controllers and parameter observers.

2. Adaptive synchronization and Parameter identification

The chaotic Pikovsky–Rabinovich model[24−26] could be given as

$$\begin{aligned}\dot{x} &= y - az,\\ \dot{y} &= -x + 2by + cz,\\ \dot{z} &= d(x - z^3 + z),\end{aligned} \tag{1}$$

where a, b, c, and d are parameters, and system (1) becomes chaotic when appropriate values are selected for parameters a, b, c, and d by calculating the Lyapunov exponents of system (1). For example, a double-scroll chaotic attractor is observed in system (1) with the selected parameters: $a = 0.66$, $b = 0.201$, $c = 0.165$, $d = 1/0.047$. In Fig.1, the double-scroll chaotic attractors of system (1) are plotted.

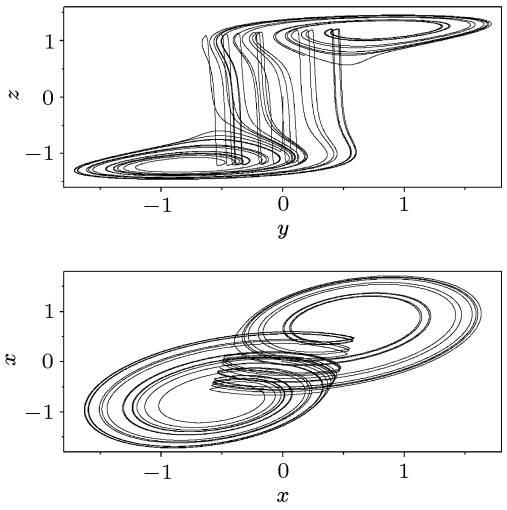


Fig.1. Strange attractor of system (1) with the selected parameters $a = 0.66$, $b = 0.201$, $c = 0.165$, and $d = 1/0.047$.

At first, the monodirectional coupling scheme is used to realize the synchronization of two identical chaotic Pikovsky–Rabinovich models by inputting the linear error of the second corresponding variables of the two chaotic circuits into the driven system, and the driven Pikovsky–Rabinovich model is described by

$$\begin{aligned}&\dot{\hat{x}} = \hat{y} - a\hat{z}, \quad \dot{\hat{y}} = -\hat{x} + 2b\hat{y} + c\hat{z} + u,\\ &\dot{z} = d(\hat{x} - \hat{z}^3 + \hat{z}).\end{aligned} \tag{2}$$

The errors of the corresponding variables between the system (1) and (2) are defined as $e_x = x - \hat{x}$, $e_y = y - \hat{y}$, $e_z = z - \hat{z}$, separately. The two identical chaotic systems could reach complete synchronization when the controller $u = ke_y$ works with the appropriate feedback coefficient k. In Fig.2, the correlation of the Lyapunov exponent spectrum vs feedback coefficient k and the time series of the errors of the corresponding variables for $k = 0.5$ are plotted.

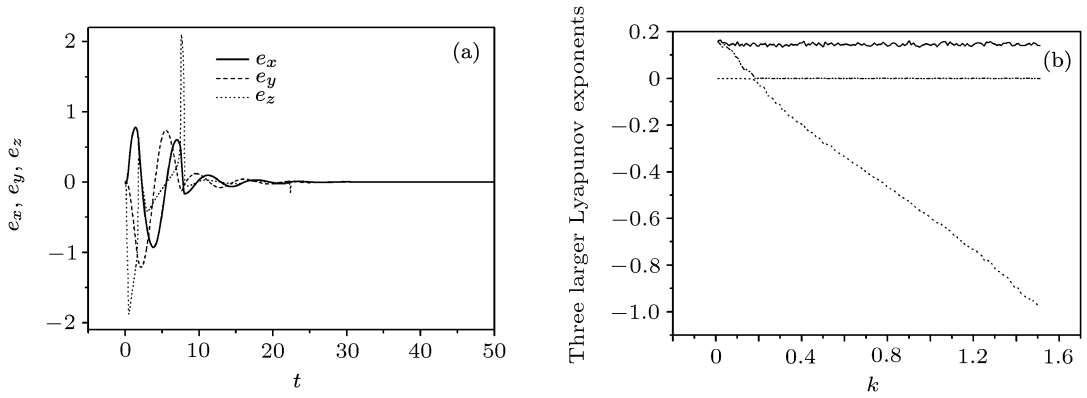


Fig.2. Evolutions of errors of corresponding variables in the two identical chaotic systems with controller $u = 0.5e_y$ (a), and the Lyapunov exponent spectrum vs feedback coefficient k (b).

The curves in Fig.2(a) confirm that the errors of the corresponding variables from the two identical chaotic systems decrease to zero and thus the two chaotic systems reach complete synchronization subjected to the linear controller $u = 0.5e_y$. Extensive numerical studies show that a shorter transient period is required to reach complete synchronization when a bigger feedback coefficient k is used. The Lyapunov exponent spectrum in Fig.2(b) confirms that the driven chaotic system (2) could synchronize the drive chaotic system (1) with increasing feedback coefficient k to a certain threshold.

Clearly, complete synchronization between the two identical chaotic systems is achieved under the action of controller $u = ke_y$ with an appropriate feedback coefficient. In fact, some parameters of the drive system are unknown and the two chaotic systems may be non-identical (parameters are not the same). Therefore, it is important to estimate the unknown parameters of the drive system and realize the synchronization between the two chaotic systems. In the present paper, we discuss the case where four parameters in the chaotic PR circuits are not identical. Our aim is to estimate all the unknown parameters by using adaptive synchronization based on the Lyapunov stability theory. The driven system is given as

$$\dot{\hat{x}} = \hat{y} - \hat{a}\hat{z} + u_1, \quad \dot{\hat{y}} = -\hat{x} + 2\hat{b}\hat{y} + c\hat{z} + u_2,$$
$$\dot{z} = \hat{d}(\hat{x} - \hat{z}^3 + \hat{z}) + u_3. \tag{3}$$

The errors for parameters are defined as $e_a = a - \hat{a}$, $e_b = b - \hat{b}$, $e_c = c - \hat{c}$, $e_d = d - \hat{d}$. The positive Lyapunov function $V(e_x, e_y, e_z, e_a, e_b, e_c, e_d) = V$ is described by

$$V = 0.5(e_x^2 + e_y^2 + e_z^2) + (e_a^2 + e_b^2 + e_c^2 + e_d^2)/20k, \tag{4}$$

where k is a constant and it is the feedback coefficient (or gain coefficient) in the following controller. It is critical to select appropriate forms for the three controllers u_1, u_2 and u_3 to satisfy the following condition:

$$\dot{V} = \mathrm{d}V/\mathrm{d}t = e_x\dot{e}_x + e_y\dot{e}_y + e_z\dot{e}_z + (e_a\dot{e}_a + e_b\dot{e}_b + e_c\dot{e}_c + e_d\dot{e}_d)/10k < 0. \tag{5}$$

The two chaotic systems reach complete synchronization only when the criteria of expression (4) and inequality (5) are satisfied well.

$$\dot{e}_x = e_y - az + \hat{a}\hat{z} - u_1 = e_y - \hat{a}e_z - ze_a - u_1, \tag{6}$$
$$\dot{e}_y = -e_x + 2by - 2\hat{b}\hat{y} + cz - \hat{c}\hat{z} - u_2 = -e_x + 2e_by + 2\hat{b}e_y + ze_c + \hat{c}e_z - u_2, \tag{7}$$
$$\dot{e}_z = d(x + z^3 - z) - \hat{d}(\hat{x} + \hat{z}^3 + \hat{z}) - u_3 = e_d(x - z^3 + z) + \hat{d}[e_x - e_z(z^2 + \hat{z}^2 + z\hat{z} - z)] - u_3, \tag{8}$$

$$\begin{aligned}\dot{V} &= e_a(\dot{e}_a/10k - ze_x) + e_b(\dot{e}_b/10k + 2ye_y) + e_c(\dot{e}_c/10k + ze_y) + e_d[\dot{e}_d/10k + e_z(x - z^3 + z)] \\ &\quad + (\hat{d} - \hat{a})e_ze_x - u_1e_x + 2\hat{b}e_y^2 + \hat{c}e_ze_y - u_2e_y - \hat{d}e_z^2(z^2 + \hat{z}^2 + z\hat{z} - 1) - u_3e_z \\ &= e_a(\dot{e}_a/10k - ze_x) + e_b(\dot{e}_b/10k + 2ye_y) + e_c(\dot{e}_c/10k + ze_y) + e_d[\dot{e}_d/10k + e_z(x - z^3 + z)] \\ &\quad + [(\hat{d} - \hat{a})e_z - u_1]e_x - (k - 2\hat{b})e_y^2 + (ke_y + \hat{c}e_z - u_2)e_y - \hat{d}e_z^2(z^2 + \hat{z}^2 + z\hat{z}) + (\hat{d}e_z - u_3)e_z.\end{aligned} \tag{9}$$

The criterion of expression Eq.(5) is satisfied when the following prerequisites of inequality (11), expressions (12) and (13a)–(13b) are available:

$$k \geq 2\hat{b} \tag{11}$$

$$u_1 = (\hat{d} - \hat{a})e_z; \quad u_2 = ke_y + \hat{c}e_z; \quad u_3 = \hat{d}e_z \tag{12}$$

$$e_a(\dot{e}_a/10k - ze_x) = 0; \quad e_b(\dot{e}_b/10k + 2ye_y) = 0; \tag{13a}$$

$$e_c(\dot{e}_c/10k + ze_y) = 0;$$
$$e_d[\dot{e}_d/10k + e_z(x - z^3 + z)] = 0. \tag{13b}$$

As a result, expression (9) is approached by

$$\dot{V} = -(k - 2\hat{b})e_y^2 - \hat{d}(z^2 + \hat{z}^2 + z\hat{z})e_z^2 < 0. \tag{14}$$

Clearly, the two non-identical chaotic systems reach complete synchronization, and then the parameter observers are given as

$$\dot{a} = \dot{\hat{a}} + 10kze_x;$$
$$\dot{b} = \dot{\hat{b}} - 20kye_y;$$
$$\dot{c} = \dot{\hat{c}} - 10kze_y;$$
$$\dot{d} = \dot{\hat{d}} - 10ke_z(x - z^3 + z). \tag{15}$$

The parameter observes for the drive chaotic system (1) could be simplified and described by the following expression (16) when the parameters of the drive system, a, b, c and d are constants:

$$\dot{\hat{a}} = -10kze_x; \quad \dot{\hat{b}} = 20kye_y;$$
$$\dot{\hat{c}} = 10kze_y; \quad \dot{\hat{d}} = 10ke_z(x - z^3 + z). \tag{16}$$

The changeable parameters $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ in the driven chaotic system come close to the unknown parameters a, b, c and d in the drive system when the two chaotic systems reach complete synchronization, and thus all the unknown parameters are identified exactly. And the coefficient k determines the transient period to estimate the unknown parameters exactly.

As a result, the unknown parameters are estimated when the two chaotic systems reach complete synchronization according to expressions (11) to (16). And the transient period to identify the unknown parameters and to reach the synchronization becomes shorter when a bigger feedback coefficient k is used. The error evolutions of the corresponding variables with time and the identified results for the unknown parameters in the drive system (1) are given with $k = 10$ in Fig.3 and $k = 20$ in Fig.4, separately. In our numerical simulation studies, the initial parameters for the driven system are selected as $\hat{a} = 0.6$, $\hat{b} = 0.2$, $\hat{c} = 0.15$ and $\hat{d} = 20$ in time steps of 0.001.

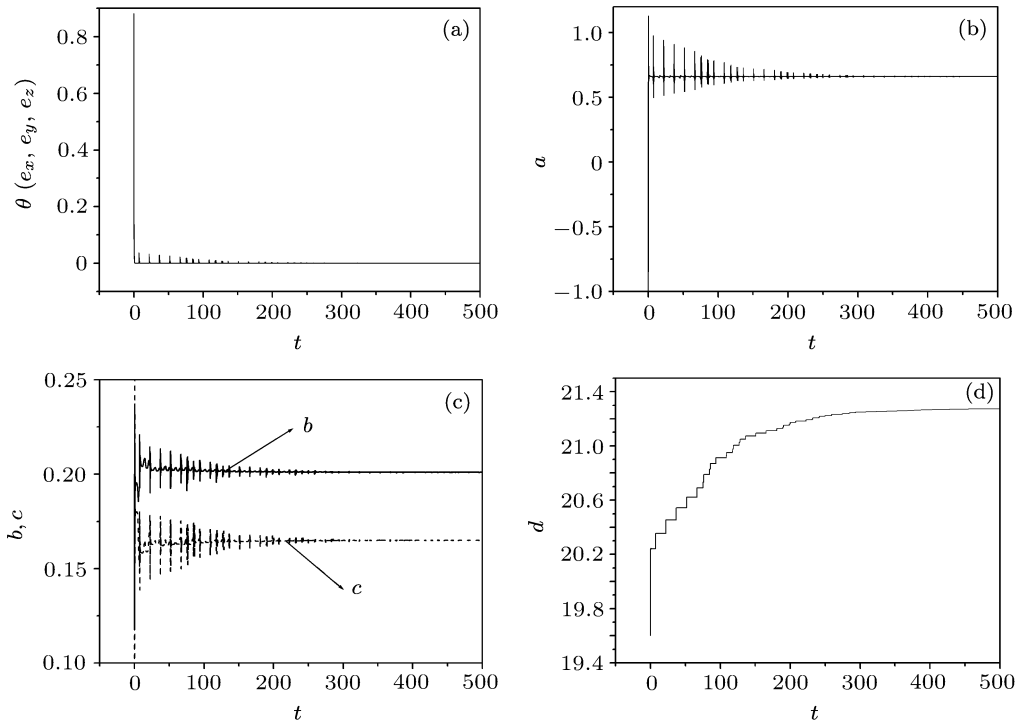


Fig.3. Evolutions of the error function $\theta(e_x,\ e_y,\ e_z) = \sqrt{e_x^2 + e_y^2 + e_z^2}$ with time and the identified results for the unknown parameters in drive system (1) with $k = 10$.

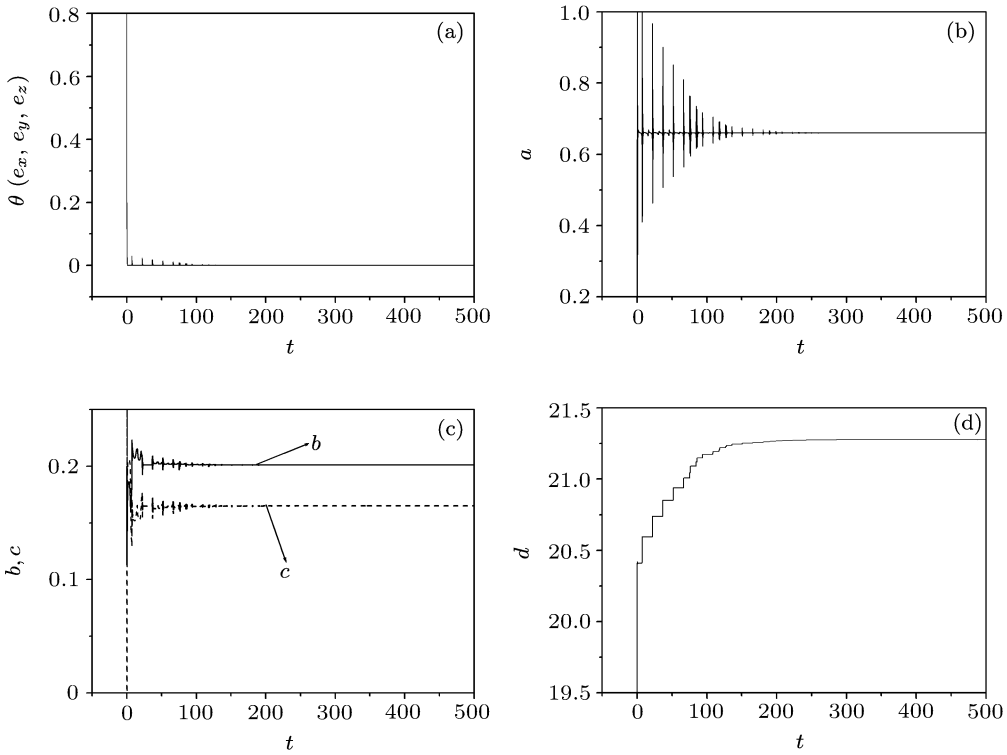


Fig.4. Evolutions of the error function $\theta(e_x,\ e_y,\ e_z) = \sqrt{e_x^2 + e_y^2 + e_z^2}$ with time and the identified results for the unknown parameters in drive system (1) with $k = 20$.

The numerical results in Figs.3 and 4 confirm our theoretical analysis as done above, and it is confirmed that the two non-identical chaotic systems could still reach complete synchronization and all the unknown parameters in the drive system could be estimated at a high accuracy. Extensive numerical results confirm that the transient period is shorter when a bigger feedback coefficient k is used, and the results are independent of the initial values of the system.

Above all, parameters identification and adaptive complete synchronization are realized by using appropriate parameter observers and controllers, which could be constructed by using the Lyapunov stability theory. As is well known, a single chaotic attractor and multi-scroll chaotic attractors could be generated from appropriate nonlinear differential equations within appropriate parameter regions. According to our scheme and guidance, appropriate controllers and parameter observers could be constructed analytically by using the Lyapunov stability theory. Clearly, this scheme could also be used to realize a generalized synchronization and the errors of corresponding variables could be defined as $e_x = x - f(\hat{x})$, $e_y = y - f(\hat{y})$, and $e_z = z - f(\hat{z})$, where the function $f(\hat{x})$, $f(\hat{y})$ and $f(\hat{z})$ could be nonlinear and/or linear.

3. Conclusion

In this paper, complete synchronization between two identical and/or non-identical chaotic Pikovsky–Rabinovich (PR) circuits is investigated based on the Lyapunov stability theory. In the case of two identical chaotic PR systems, complete synchronization is realized when a linear controller works on the driven system and the conditional Lyapunov exponents are calculated to find the critical threshold for feedback intensity k. In the case of synchronization of two non-identical chaotic PR systems, an appropriate positive Lyapunov function V is constructed, and appropriate parameter observers and linear controllers are selected analytically to make $\mathrm{d}V/\mathrm{d}t$ become negative. As a result, all unknown parameters in the drive system are estimated when the two non-identical chaotic systems reach complete synchronization. Comparing our results with most of the previous results, our study carefully shows a way to construct a general but appropriate positive Lyapunov function and find appropriate parameter observers and controllers to realize complete synchronization of two non-identical chaotic systems. The Lyapunov function involves all the errors of the corresponding variables and parameters of

the two chaotic systems, and a controllable gain coefficient is introduced into the Lyapunov function to adjust the transient period for parameter estimation. The four unknown parameters are all estimated while in some previous studies only partial unknown parameters of the chaotic system were estimated. The parameter observers and controllers could be constructed exactly for the chaotic systems, thereby generating a single chaotic attractor or multi-scroll attractors, that is to say, the parameter controllers and the parameter observers are often determined by the nonlinear chaotic model. Furthermore, the scheme proposed in this paper could also be used to estimate unknown parameters in other chaotic systems and realize generalized synchronization as well.

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