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Impulsive control for a Takagi–Sugeno fuzzy model with time-delay and its application to chaotic systems*

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A control approach where the fuzzy logic methodology is combined with impulsive control is developed for controlling some time-delay chaotic systems in this paper. We first introduce impulses into each subsystem with delay of the Takagi–Sugeno (TS) fuzzy IF–THEN rules and then present a unified TS impulsive fuzzy model with delay for chaos control. Based on the new model, a simple and unified set of conditions for controlling chaotic systems is derived by the Lyapunov–Razumikhin method, and a design procedure for estimating bounds on control matrices is also given. Several numerical examples are presented to illustrate the effectiveness of this method.

Keywords: chaotic system, TS fuzzy model, impulsive control, time delay
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1. Introduction

In recent years, fuzzy logic control in dynamical systems has attracted increasing attention due to its applicability in the case of uncertainty. It has become one of the most useful approaches to designing a controller for complex systems, and the Takagi–Sugeno (TS) fuzzy model control design plays an important role in the applications. The TS fuzzy dynamic model was given by Takagi and Sugeno,[1] and represents nonlinear dynamic systems by fuzzy IF–THEN rules in which the local dynamics in different state space regions is represented by linear models. Therefore, the so-called parallel distributed compensation (PDC)[2,3] control design scheme emerged in which an linear feedback control was designed for each corresponding local linear model. The overall controller is nonlinear in general. The manipulation of controlling nonlinear dynamic systems based on the TS fuzzy model always can be divided into two steps. (i) Represent the nonlinear system by a TS fuzzy system; (ii) design a controller for the fuzzy system. Based on the PDC scheme, much research on the control of fuzzy systems has been carried out. In Refs.[4]–[6] many well-known chaotic systems were represented by TS fuzzy models and the local linear controller for the synchronization and the secure communication problems of these chaotic fuzzy systems based on different driving signals were designed. At the same time, impulsive controls[7−16] were used to stabilize linear and nonlinear systems. Impulsive control has been demonstrated to be an effective control method through numerical simulations. Impulsive control is an attractive method in the sense that it may be simpler to implement and involve cheaper control mechanisms. There are many investigations on impulsive fuzzy control or synchronization of different chaotic systems without delays based on TS fuzzy model.[17−20]

Recently, the delayed chaotic system has been considered by many researchers.[19−24] Time-delay chaotic systems are frequently encountered in engineering, biology, economy, and other disciplines.[25,26] The control and synchronization of time-delay systems have received increasing attention.[27,28] However, there are few results in respect of impulsive control of time-delay chaotic systems.

In the present paper, an impulsive control scheme for time-delay chaotic systems based on the TS fuzzy model is introduced which has the advantages of both fuzzy control and impulsive control. Some typical chaotic systems illustratively demonstrate the effectiveness of the proposed control scheme.

The rest of this paper is organized as follows. In Section 2 are given a basic theory on an impulsive differential system with delay, and a description of fuzzy modelling methodology for a TS fuzzy system

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with delay and the impulsive fuzzy control model for controlling a complex system with delay. In Section 3, theoretical stability analysis of impulsive fuzzy chaotic systems is developed, while, in Section 4, several numerical simulations of chaotic systems are carried out based on the proposed method. Finally, in Section 5, some concluding remarks are made.

Notation Throughout this paper, R_+ represents non-negative real numbers, R^n denotes the n-dimensional Euclidean space, $R^{n\times m}$ is the set of $n\times m$ real matrices, $\boldsymbol{I}$ is the identity matrix of appropriate dimensions. The notation $\boldsymbol{X} > 0$ (respectively, $\boldsymbol{X} \geq 0$) for $\boldsymbol{X} \in R^{n\times n}$ means that the matrix $\boldsymbol{X}$ is real symmetric positive definite (respectively, positive semi-definite). $\lambda_{\max}(\boldsymbol{P})$ and $\lambda_{\min}(\boldsymbol{P})$ denote the biggest eigenvalue and the smallest eigenvalue of the matrix $\boldsymbol{P}$ respectively.

2. Preliminaries

2.1. Basic theory of impulsive differential system with delay

Given constant $\tau > 0$, we equip the linear space $\mathrm{PC}_\tau = \{\boldsymbol{\psi} : [-\tau, 0] \to R^n | \boldsymbol{\psi}(t^+) = \boldsymbol{\psi}(t), \forall t \in [a, b)$; $\boldsymbol{\psi}(t^-)$ exists in $R^n, \forall t \in (a, b]$, and $\boldsymbol{\psi}(t^-) = \boldsymbol{\psi}(t)$ for all but at most a finite number of points $t \in (a, b]\}$, with the norm $||\cdot||$ defined as $||\boldsymbol{\psi}|| = \sup_{-\tau\leq s\leq 0} ||\boldsymbol{\psi}(s)||$.

Consider the following impulsive system:

$$\begin{aligned} &\boldsymbol{x}'(t) = F(t, \boldsymbol{x}_t),\ t \neq t_k, \\ &\Delta\boldsymbol{x}(t_k) = I_k(t_k, \boldsymbol{x}_{t_k^-}),\quad k \in N = \{1, 2, \ldots\}, \\ &\boldsymbol{x}_{t_0} = \boldsymbol{\phi}, \end{aligned} \tag{1}$$

where $F, I_k : R_+ \times \mathrm{PC}_\tau \to R^n$; $\boldsymbol{\phi} \in \mathrm{PC}_\tau$; $0 \leq t_0 < t_1 < \ldots < t_k < \ldots$, with $\lim\limits_{k\to\infty} t_k = \infty$; $\Delta\boldsymbol{x}(t) = x(t) - x(t^-)$; $\boldsymbol{x}_t, x_{t^-} \in \mathrm{PC}_\tau$ are defined as $\boldsymbol{x}_t(s) = \boldsymbol{x}(t + s)$, and $\boldsymbol{x}_{t^-}(s) = \boldsymbol{x}(t^- + s)$ for $-\tau \leq s \leq 0$, respectively.

We assume that functions $F, I_k, k \in N$ satisfy all necessary conditions for the global existence and uniqueness of solutions for all $t \geq t_0$. Denote the solution of Eq.(1) as $\boldsymbol{x}(t) = \boldsymbol{x}(t, t_0, \boldsymbol{\phi})$ where $\boldsymbol{x}_{t_0} = \boldsymbol{\phi}$. We further assume that all the solutions $(x(t))$ of Eq.(1) are continuous except at $t_k, k \in N$, at which $\boldsymbol{x}(t)$ is right continuous, i.e. $\boldsymbol{x}(t_k^+) = \boldsymbol{x}(t_k), k \in N$.

Definition 2.1[29] Function $V : R_+ \times R^n \to R_+$ is said to belong to class υ_0 if

(i) V is continuous in each of the sets $[t_{k-1}, t_k) \times R^n$ and for each $\boldsymbol{x} \in R^n$, $t \in [t_{k-1}, t_k), k \in N$, $\lim_{(t,\boldsymbol{y})\to(t_k^-,\boldsymbol{x})} V(t, \boldsymbol{y}) = V(t_k^-, \boldsymbol{x})$ exist; and

(ii) $V(t, \boldsymbol{x})$ is locally Lipschitzian in all $\boldsymbol{x} \in R^n$, and for all $t \geq t_0$, $V(t, \boldsymbol{0}) = 0$.

Definition 2.2[29] Given a function $V : R_+ \times R^n \to R_+$, the upper right-hand derivative of V with respect to system (1) is defined as

$$\begin{aligned} &D^+V(t, \boldsymbol{\psi}(0)) \\ &= \limsup_{h\to 0^+} \frac{1}{h}[V(t + h, \boldsymbol{\psi}(0) + hF(t, \boldsymbol{\psi})) \\ &\quad -V(t, \boldsymbol{\psi}(0))], \end{aligned}$$

for $(t, \boldsymbol{\psi}) \in R_+ \times \mathrm{PC}_\tau$.

Definition 2.3[29] The trivial solution of system (1) is said to be globally exponentially stable if there exist some constants $\alpha > 0$ and $M \geq 1$ such that for any initial data $x_{t_0} = \phi$

$$||\boldsymbol{x}(t, t_0, \boldsymbol{\phi})|| \leq M||\phi||\mathrm{e}^{-\alpha(t-t_0)},\ t \geq t_0,$$

where $(t, \boldsymbol{\phi}) \in R_+ \times \mathrm{PC}_\tau$.

Now, we present the Razumikhin-type theorem on global exponential stability for system (1) which was obtained in Ref.[29] based on the Lyapunov–Razumikhin method in the following. The result shows that impulses play an important role in stabilizing delay differential systems.

Lemma 2.1[29] Assume that there exist a function $V \in \upsilon_0$, constants $p, c_1, c_2, \lambda > 0$ and $\alpha > \tau$, such that

(i) $c_1||\boldsymbol{x}||^p \leq V(t, \boldsymbol{x}) \leq c_2||\boldsymbol{x}||^p$ for any $t \in R_+$ and $x \in R^n$;

(ii) $D^+V(t, \boldsymbol{\varphi}(0)) \leq cV(t, \boldsymbol{\varphi}(0))$, for all $t \in [t_{k-1}, t_k), k \in N$, whenever $qV(t, \boldsymbol{\varphi}(0)) \geq V(t + s, \boldsymbol{\varphi}(s))$ for $-\tau \leq s \leq 0$, where $q \geq \max\{\mathrm{e}^{\alpha c}, \mathrm{e}^{2\lambda\alpha}\}$ is a constant;

(iii) $V(t_k, \boldsymbol{\varphi}(0) + I_k(t_k, \boldsymbol{\varphi})) \leq d_kV(t_k^-, \boldsymbol{\varphi}(0))$, where $d_k > 0, \forall k \in N$ are constants;

(iv) $\tau \leq t_k - t_{k-1} \leq \alpha$ and $\ln(d_k) + (\lambda + c)\alpha < -\lambda(t_{k+1} - t_k)$.

Then the trivial solution of system (1) is exponentially stable and the convergence rate is λ/p.

2.2. TS impulsive fuzzy model

The TS fuzzy models[1−3] are described by fuzzy IF–THEN rules, in which each rule locally represents a linear system over a certain region of the state space. The overall system is then an aggregation of these local linear system models.

Specifically, a general TS fuzzy system with delay is described as follows:

Rule i: IF $z_1(t)$ is M_{i1} and $\ldots$ $z_p(t)$ is M_{ip} THEN

$$\begin{cases} \boldsymbol{x}'(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t)),\ i = 1,2,\ldots,r, \\ \boldsymbol{x}_{t_0} = \boldsymbol{\phi}, \end{cases} \tag{2}$$

where M_{ij} is the fuzzy set, r is the number of the IF–THEN rules, $x(t) \in R^n$ is the state vector, $\boldsymbol{A}_i, \boldsymbol{B}_i \in R^{n\times n}$, $z_1(t),\ldots,z_p(t)$ are the premise variables, $\tau_0(t)$ is a time-varying continuous function representing time delay with known constant scalar τ satisfying

$$0 \le \tau_0(t) \le \tau, \tag{3}$$

$\boldsymbol{\phi} \in C_\tau$, $\quad C_\tau = C([-\tau, 0], R^n)$.

Each consequent linear system represented by $\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t))$ is called a subsystem. The following final output of the fuzzy system can be obtained by using the singleton fuzzifier, product fuzzy inference and weighted average defuzzifier:

$$\begin{aligned} \boldsymbol{x}'(t) &= \sum_{i=1}^{r} \frac{w_i(\boldsymbol{z}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t))\}}{\sum_{i=1}^{r} w_i(\boldsymbol{z}(t))} \\ x_{t_0} &= \phi, \end{aligned} \tag{4}$$

where $\boldsymbol{z}(t) = (z_1(t),\ldots,z_p(t))$, and $w_i(t) = \prod_{i=1}^{r} M_{ij}(z_j(t))$. For any t, $M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in M_{ij}, satisfying the following conditions:

$$\sum\nolimits_{i=1}^{r} w_i(\boldsymbol{z}(t)) > 0, \quad w_i(\boldsymbol{z}(t)) \ge 0, \quad i = 1,2,\ldots,r.$$

By introducing $h_i(\boldsymbol{z}(t)) = \dfrac{w_i(\boldsymbol{z}(t))}{\sum_{k=1}^{r} w_k(\boldsymbol{z}(t))}$ instead of $w_i(\boldsymbol{z}(t))$, Eq.(4) can be rewritten as

$$\begin{aligned} \boldsymbol{x}'(t) &= \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t))\}, \\ \boldsymbol{x}_{t_0} &= \boldsymbol{\phi}. \end{aligned} \tag{5}$$

Note that

$$\sum\nolimits_{i=1}^{r} h_i(\boldsymbol{z}(t)) = 1, \quad h_i(\boldsymbol{z}(t)) \ge 0, \quad i = 1,2,\ldots,r,$$

for all t , where $h_i(\boldsymbol{z}(t))$ can be regarded as the normalized weights of the IF–THEN rules.

Now, we consider the following system with delay:

$$\begin{aligned} \boldsymbol{x}'(t) &= f(t, \boldsymbol{x}_t), \\ \boldsymbol{x}_{t_0} &= \boldsymbol{\phi}, \end{aligned} \tag{6}$$

where $\boldsymbol{x}(t)$ is the state vector, $f : R_+ \times C_\tau \to R^n$ is continuous, and $\phi \in C_\tau$.

Suppose that Eq.(6) can be represented by the TS fuzzy model (4). Let $\{t_k\}_1^\infty$ be a discrete time set and $0 \le t_0 < t_1 < \ldots < t_k < \ldots$, with $\lim\limits_{k\to\infty} t_k = \infty$; $\Delta\boldsymbol{x}(t_k) = \boldsymbol{x}(t_k) - \boldsymbol{x}(t_k^-)$ is the 'jump' of the state vector at t_k, then

$$\begin{aligned} &\boldsymbol{x}'(t) = f(t, \boldsymbol{x}_t),\ t \ne t_k, \\ &\Delta\boldsymbol{x}(t_k) = U(t_k, \boldsymbol{x}(t_k^-)),\ k \in N = \{1,2,\ldots\}, \\ &\boldsymbol{x}_{t_0} = \boldsymbol{\phi}, \end{aligned} \tag{7}$$

is an impulsive differential system with delay. Similar to the construction of the impulsive controlled continuous system, if we choose the impulsive controller in the form of linear feedback, the impulsive controlled fuzzy model can be expressed as follows:

Rule i: IF $z_1(t)$ is M_{i1} and $\ldots$ $z_p(t)$ is M_{ip} THEN

$$\begin{cases} \boldsymbol{x}'(t) = \boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t)),\ t \ne t_k \\ \Delta\boldsymbol{x}(t) = D_i\boldsymbol{x}(t^-),\ t = t_k, k \in N, \\ \boldsymbol{x}_{t_0} = \boldsymbol{\phi}, \end{cases} \tag{8}$$

where $\boldsymbol{D}_i$ is the coefficient of the impulsive control. Then the final output of the system is shown below:

$$\begin{aligned} &\boldsymbol{x}'(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\{\boldsymbol{A}_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t))\},\ t \ne t_k, \\ &\Delta\boldsymbol{x}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{z}(t^-))\boldsymbol{D}_i\boldsymbol{x}(t^-), \quad t = t_k, k \in N, \\ &\boldsymbol{x}_{t_0} = \boldsymbol{\phi}. \end{aligned} \tag{9}$$

2.3. Some other lemmas

Lemma 2.2[30] Given a positive definite matrix $\boldsymbol{G} \in R^{n\times n}$, for any $\boldsymbol{x}, \boldsymbol{y} \in R^n$, then

$$2\boldsymbol{x}^{\mathrm{T}}\boldsymbol{y} \le \boldsymbol{x}^{\mathrm{T}}\boldsymbol{G}^{-1}\boldsymbol{x} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{G}\boldsymbol{y}.$$

Lemma 2.3[31] Suppose that matrices $\{\boldsymbol{M}_i\}_{i=1}^r \in R^{n\times m}$ and a positive-semidefinite matrix $\boldsymbol{P} \in R^{n\times n}$ are given. If $\sum_{i=1}^r h_i = 1$ and $0 \le h_i \le 1$, then

$$\left(\sum_{i=1}^{r} h_i\boldsymbol{M}_i\right)^{\mathrm{T}} \boldsymbol{P} \left(\sum_{i=1}^{r} h_i\boldsymbol{M}_i\right) \le \sum_{i=1}^{r} h_i\boldsymbol{M}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{M}_i.$$

3. Design of fuzzy impulsive controller

In this section, we use the stability results presented in Section 2 to design a fuzzy impulsive controller for the TS fuzzy model (9).

Theorem 3.1. Suppose that there exist matrix $\boldsymbol{P} > 0$, positive constants $q, c, \lambda > 0$ and $\alpha > \tau$ such that

(a)

$$\begin{bmatrix} \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i - (c-q)\boldsymbol{P} & \boldsymbol{P}\boldsymbol{B}_i \\ \boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{P} \end{bmatrix} \leq 0, \quad i = 1, 2, \ldots, r, \tag{10}$$

where $q \geq \max\{\mathrm{e}^{\alpha c}, \mathrm{e}^{2\lambda\alpha}\}$;

(b) $\tau \leq t_k - t_{k-1} \leq \alpha, k \in N$ and

$$\ln(d) + (\lambda + c)\alpha < -\lambda(t_{k+1} - t_k), \tag{11}$$

where $d = \dfrac{\lambda_{\max}(\boldsymbol{P})}{\lambda_{\min}(\boldsymbol{P})} \max\limits_{1\leq i\leq r} ||\boldsymbol{I} + \boldsymbol{D}_i||^2$. Then the trivial solution of system (9) is exponentially stable and the convergence rate is $\dfrac{\lambda}{2}$.

Proof Construct a Lyapunov function candidate as

$$V(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t), \tag{12}$$

and $\boldsymbol{P} > 0$ is to be determined.

It is easy to see

$$\lambda_{\min}(\boldsymbol{P})||\boldsymbol{x}(t)||^2 \leq V(\boldsymbol{x}(t)) \leq \lambda_{\max}(\boldsymbol{P})||\boldsymbol{x}(t)||^2, \tag{13}$$

so we can choose $c_1 = \lambda_{\min}(\boldsymbol{P})$, and $c_2 = \lambda_{\max}(\boldsymbol{P})$.

The time derivative of $V(\boldsymbol{x}(t))$ is taken along state trajectory (9), yielding

$$\begin{aligned} V'(\boldsymbol{x}(t)) &= \boldsymbol{x}^{\mathrm{T}}(t)\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i)\boldsymbol{x}(t) \\ &\quad + 2\boldsymbol{x}^{\mathrm{T}}(t)\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{P}\boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t)). \end{aligned} \tag{14}$$

By Lemma 2.2, we have

$$\begin{aligned} &2\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t)) \\ &\leq \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{B}_i\boldsymbol{P}^{-1}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t) \\ &\quad + \boldsymbol{x}^{\mathrm{T}}(t-\tau_0(t))\boldsymbol{P}\boldsymbol{x}(t-\tau_0(t)). \end{aligned} \tag{15}$$

From expression (3), when $qV(t, \boldsymbol{x}(t)) \geq V(\boldsymbol{x}(t-\tau_0(t)))$, substituting inequality (15) into expression (14) yields

$$\begin{aligned} V'(\boldsymbol{x}(t)) \leq\ & \boldsymbol{x}^{\mathrm{T}}(t)\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i \\ &+ \boldsymbol{P}\boldsymbol{B}_i\boldsymbol{P}^{-1}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P} + q\boldsymbol{P})\boldsymbol{x}(t). \end{aligned} \tag{16}$$

By the condition of Theorem 3.1(a) and the Schur complement formula, we have

$$\boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i + \boldsymbol{P}\boldsymbol{B}_i\boldsymbol{P}^{-1}\boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P} + q\boldsymbol{P} \leq c\boldsymbol{P}. \tag{17}$$

Hence

$$V'(\boldsymbol{x}(t)) \leq cV(\boldsymbol{x}(t)), \tag{18}$$

when $qV(t, \boldsymbol{x}(t)) \geq V(\boldsymbol{x}(t-\tau_0(t)))$.

From Lemma 2.3 and condition (b) of this theorem, one can see

$$\begin{aligned} V(\boldsymbol{x}(t_k)) &= \left(\sum\nolimits_{i=1}^{r} h_i(\boldsymbol{z}(t_k^-))(\boldsymbol{I} + \boldsymbol{D}_i)\boldsymbol{x}(t_k^-)\right)^{\mathrm{T}} \boldsymbol{P} \left(\sum\nolimits_{i=1}^{r} h_i(\boldsymbol{z}(t_k^-))(\boldsymbol{I} + \boldsymbol{D}_i)\boldsymbol{x}(t_k^-)\right) \\ &\leq \sum\nolimits_{i=1}^{r} h_i(\boldsymbol{z}(t_k^-))\boldsymbol{x}(t_k^-)^{\mathrm{T}}(\boldsymbol{I} + \boldsymbol{D}_i)^{\mathrm{T}}\boldsymbol{P}(\boldsymbol{I} + \boldsymbol{D}_i)\boldsymbol{x}(t_k^-) \\ &\leq \lambda_{\max}(\boldsymbol{P})\sum\nolimits_{i=1}^{r} h_i(\boldsymbol{z}(t_k^-))||\boldsymbol{I} + \boldsymbol{D}_i||^2||\boldsymbol{x}(t_k^-)||^2 \\ &\leq \frac{\lambda_{\max}(\boldsymbol{P})}{\lambda_{\min}(\boldsymbol{P})}\sum\nolimits_{i=1}^{r} h_i(\boldsymbol{z}(t_k^-))||\boldsymbol{I} + \boldsymbol{D}_i||^2 V(\boldsymbol{x}(t_k^-)) \leq dV(\boldsymbol{x}(t_k^-)). \end{aligned} \tag{19}$$

In terms of inequalities (11), (13), (18) and (19), all of the conditions of Lemma 2.1 are satisfied, so the trivial solution of system (9) is exponentially stable and the convergence rate is $\dfrac{\lambda}{2}$.

Let $P = I$, we can obtain the following theorem.

Theorem 3.2 Let $\lambda_{\mathrm{A}} = \max_{1\leq i\leq r} \lambda_{\max}(\boldsymbol{A}_i + \boldsymbol{A}_i^{\mathrm{T}})$, $d = \max_{1\leq i\leq r} ||\boldsymbol{I} + \boldsymbol{D}_i||^2$, $b = \max_{1\leq i\leq r} ||\boldsymbol{B}_i||$. Suppose that there exist positive constants c, λ and $\alpha > \tau$ such that for $\tau \leq t_k - t_{k-1} \leq \alpha, k \in N$,

$$\ln(d) + (\lambda + c)\alpha < -\lambda(t_{k+1} - t_k), \tag{20}$$

where $c = \lambda_{\mathrm{A}} + 2b\sqrt{q}$, and $q \geq \max\{\mathrm{e}^{\alpha c}, \mathrm{e}^{2\lambda\alpha}\}$.

Then the trivial solution of system (9) is exponentially stable and the convergence rate is $\dfrac{\lambda}{2}$.

Proof Let $V(\boldsymbol{x}(t)) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{x}(t) = ||\boldsymbol{x}(t)||^2$, the time derivative of $V(\boldsymbol{x}(t))$ is taken along state trajectory (9), yielding

$$\begin{aligned} V'(\boldsymbol{x}(t)) &= \boldsymbol{x}^{\mathrm{T}}(t)\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))(\boldsymbol{A}_i^{\mathrm{T}}+\boldsymbol{A}_i)\boldsymbol{x}(t) + 2\boldsymbol{x}^{\mathrm{T}}(t)\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))\boldsymbol{B}_i\boldsymbol{x}(t-\tau_0(t)) \\ &\le \lambda_{\mathrm{A}} V(\boldsymbol{x}(t)) + 2||\boldsymbol{x}(t)||\sum_{i=1}^{r} h_i(\boldsymbol{z}(t))||\boldsymbol{B}_i||\cdot||\boldsymbol{x}(t-\tau_0(t))|| \\ &\le \lambda_{\mathrm{A}} V(\boldsymbol{x}(t)) + 2b\sqrt{V(\boldsymbol{x}(t))}\sqrt{V(\boldsymbol{x}(t-\tau_0(t)))} \\ &\le \lambda_{\mathrm{A}} V(\boldsymbol{x}(t)) + 2b\sqrt{q}V(\boldsymbol{x}(t)) = cV(\boldsymbol{x}(t)), \end{aligned} \tag{21}$$

when $qV(t,\boldsymbol{x}(t)) \ge V(\boldsymbol{x}(t-\tau_0(t)))$ and $t \in [t_{k-1}, t_k), k \in N$.

Like the proof of inequality (19), we derive

$$V(\boldsymbol{x}(t_k)) \le dV(\boldsymbol{x}(t_k^-)). \tag{22}$$

From inequalities (20), (21) and (22), we can obtain Theorem 3.2 immediately.

Remark 3.1. The inequality (10) is a non-linear matrix inequality (NLMI). Let $\theta = c - q$, then we will have θ and $\boldsymbol{P}$ by solving the following problem, i.e. by minimizing θ, subject to $\boldsymbol{P} > 0$ and

$$\begin{bmatrix} \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i - \theta\boldsymbol{P} & \boldsymbol{P}\boldsymbol{B}_i \\ \boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{P} \end{bmatrix} \le 0,\ i = 1, 2, \ldots, r. \tag{23}$$

Algorithm

☆ In order to solve inequality (23), by choosing any positive scalar θ big enough in advance, by solving the linear matrix inequality $\begin{bmatrix} \boldsymbol{A}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A}_i - \theta\boldsymbol{P} & \boldsymbol{P}\boldsymbol{B}_i \\ \boldsymbol{B}_i^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{P} \end{bmatrix} < 0,\ (i = 1, 2, \ldots, r)$, based on the LMI technique, and by using the binary-search method for constant θ, we can obtain suitable values of θ and $\boldsymbol{P}$.

☆ Choosing a suitable form of the number q and solving the equation $c - q = \theta$ to find the value of c.

☆ Choose a suitable impulsive interval α according to the needs of the actual applications and a constant λ. In terms of the condition (b) of Theorem 3.1, compute the bound on d and estimate the bound on $\boldsymbol{I} + \boldsymbol{D}_i$. We can use the following inequality:

$$||\boldsymbol{I} + \boldsymbol{D}_i|| \le \sqrt{\frac{d\lambda_{\min}(\boldsymbol{P})}{\lambda_{\max}(\boldsymbol{P})}} \tag{24}$$

in order to find the control matrix $\boldsymbol{D}_i$.

Remark 3.2. We can also choose a suitable form of q in Theorem 3.2 to solve the equation $c = \lambda_{\mathrm{A}} + 2b\sqrt{q}$ to obtain the constant c.

4. Experimental results

In this section, we illustrate the effectiveness of our results by showing several simulation results through employing the Lorenz system and Chen system with delays.

Example 4.1. Lorenz system with delay[24] is given as

$$\begin{aligned} x_1'(t) &= \beta(x_2(t-\tau_0(t)) - x_1(t)), \\ x_2'(t) &= \eta x_1(t) - x_2(t) - x_1(t)x_3(t), \\ x_3'(t) &= x_1(t)x_2(t) - \gamma x_3(t-\tau_0(t)), \end{aligned} \tag{25}$$

where β, γ and η are constants. Assume that $x_1(t) \in [-\zeta, \zeta]$ and $\zeta > 0$. Let the premise variable be $x_1(t)$, the system (25) can be exactly represented by following TS model:

$$\begin{aligned} &\text{Rule1 : IF } \ x_1(t) \ \text{ is } \ M_1, \text{THEN } \ x'(t) = A_1x(t) + B_1x(t-\tau_0(t)), \\ &\text{Rule2 : IF } \ x_1(t) \ \text{ is } \ M_2, \text{THEN } \ x'(t) = A_2x(t) + B_2x(t-\tau_0(t)). \end{aligned} \tag{26}$$

Here

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad \boldsymbol{A}_1 = \begin{bmatrix} -\beta & 0 & 0 \\ \eta & -1 & -\zeta \\ 0 & \zeta & 0 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -\beta & 0 & 0 \\ \eta & -1 & \zeta \\ 0 & -\zeta & 0 \end{bmatrix}, \quad \boldsymbol{B}_1 = \boldsymbol{B}_2 = \begin{bmatrix} 0 & \beta & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\gamma \end{bmatrix},$$

$M_1(x_1(t)) = \frac{1}{2}\left(1+\frac{x_1(t)}{\zeta}\right)$, and $M_2(x_1(t)) = \frac{1}{2}\left(1-\frac{x_1(t)}{\zeta}\right)$.

In the present paper, $\beta = 10$, $\gamma = 8/3$, and $\eta = 28$. By Eq.(9), choosing $h_i(x_1(t)) = M_i(x_1(t))$, $i = 1, 2$, we can obtain an impulsive fuzzy control model of system (25).

According to Theorem 3.1, using the algorithm given above and through computation, we can obtain $\theta = 44.1687$, $\boldsymbol{P} = \begin{bmatrix} 131.1666 & -100.1625 & -0.0000 \\ -100.1625 & 224.3585 & 0.0000 \\ -0.0000 & 0.0000 & 101.9042 \end{bmatrix}$, and $\frac{\lambda_{\max}(\boldsymbol{P})}{\lambda_{\min}(\boldsymbol{P})} = 4.2833$.

If we choose $\tau_0(t) = 0.044$, $\boldsymbol{\phi}(s) = (8, 4, 1)^{\mathrm{T}}$, and $-0.044 \le s \le 0$, the chaotic behaviour is shown in Fig.1(a). Setting $\tau = 0.044$ and $\alpha = 0.045 = t_k - t_{k-1}$, and choosing $q = \mathrm{e}^{\alpha c}$, we can solve the equation $c - \mathrm{e}^{\alpha c} = 44.1687$ to obtain $c = 57.4136$. From inequality (11), we obtain $d < 0.0755$, by using inequality (14), we have $||\boldsymbol{I} + \boldsymbol{D}_i|| < 0.1328$, $i = 1, 2$. Especially, by choosing $\boldsymbol{D}_1 = \boldsymbol{D}_2 = \mathrm{diag}(-0.8675, -0.8675, -0.8675)$, $0 < \lambda < 0.044$, $||\boldsymbol{I} + \boldsymbol{D}_i|| = 0.1325$, then $d = 0.0752$. By using $c = 57.4136$, $q = \mathrm{e}^{\alpha c} = 13.2449$ and $\boldsymbol{P}$, and by performing computation, we can find all of the conditions of Theorem 3.1 are satisfied. The simulation of impulsive fuzzy control is given in Fig.1(b).

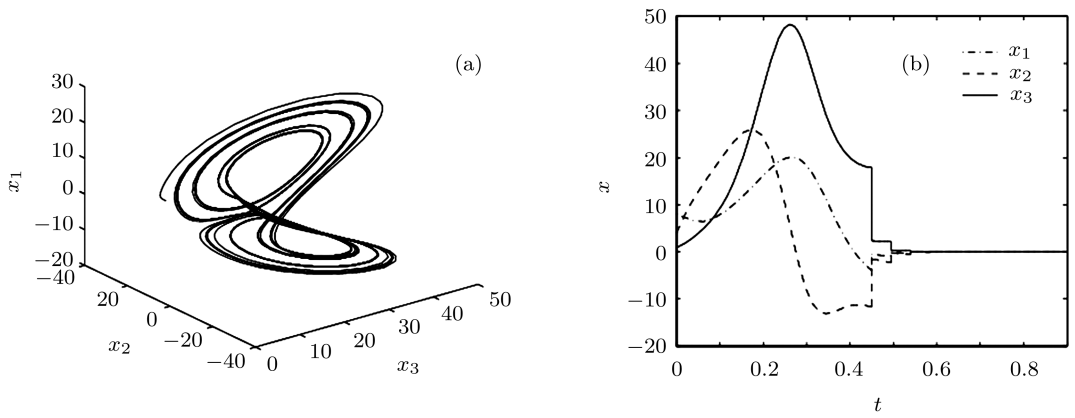


Fig.1. Chaotic behaviour and impulsive fuzzy control of Lorenz system with delay $\tau = 0.044$. (a) Chaotic attractor of Lorenz's system with $\tau = 0.044$, (b) Impulsive fuzzy control begins at $t = 0.45$ with $\tau = 0.044, \alpha = 0.045$.

Remark 4.1. If Theorem 3.2 is used for system (25), computation yields $\lambda_{\mathrm{A}} = 18.4109$ and $b = 10$. But for $q \ge \max\{\mathrm{e}^{\alpha c}, \mathrm{e}^{2\lambda\alpha}\} \ge \mathrm{e}^{\alpha c}$, if $\alpha \ge 0.0283404$, then the equation $c = \lambda_{\mathrm{A}} + 2b\sqrt{q}$ has no real root. So we may not use Theorem 3.2 to obtain the control matrix $\boldsymbol{D}_i$ when $\tau \ge 0.0283404$. We can use Theorem 3.1 to design the control matrix $\boldsymbol{D}_i$ for $0 < \tau \le 0.0467203$. However, when $\tau \le 0.0283403$, we find that the result in Theorem 3.2 is less conservative than that in Theorem 3.1 through careful calculation. For some examples, setting $\tau = 0.0283403$, and $\alpha = t_k - t_{k-1} = 0.028340301$, we obtain $||\boldsymbol{I} + \boldsymbol{D}_i|| < 0.2835$ by using Theorem 3.2 and $||\boldsymbol{I} + \boldsymbol{D}_i|| < 0.2445$ by using Theorem 3.1. Moreover, for $\tau = 0.014$ and $\alpha = t_k - t_{k-1} = 0.015$, we obtain $||\boldsymbol{I} + \boldsymbol{D}_i|| < 0.7038$ by using Theorem 3.2 and $||\boldsymbol{I} + \boldsymbol{D}_i|| < 0.3418$ by using Theorem 3.1. For $\tau_0(t) = 0.014$, $\boldsymbol{\phi}(s) = (8, 4, 1)^{\mathrm{T}}$, and $-0.044 \le s \le 0$, the chaotic behaviour is shown in Fig.2(a). Choosing $\boldsymbol{D}_1 = \boldsymbol{D}_2 = \mathrm{diag}(-0.2965, -0.2965, -0.2965)$ and $||\boldsymbol{I} + \boldsymbol{D}_i|| = 0.7035$, the simulation of impulsive fuzzy control is given in Fig.2(b).

Example 4.2. The Chen system with delay is given as

$$\begin{aligned} x_1'(t) &= \beta(x_2(t) - x_1(t)), \\ x_2'(t) &= (\eta - \beta)x_1(t) + \eta x_2(t) - x_1(t)x_3(t - \tau_0(t)), \\ x_3'(t) &= x_1(t)x_2(t - \tau_0(t)) - \gamma x_3(t), \end{aligned} \tag{27}$$

where β, γ and η are constants. Assume that $x_1(t) \in [-\zeta, \zeta]$ and $\zeta > 0$. Let $\beta = 35, \gamma = 3, \eta = 28$, and $\zeta = 40$, In a way similar to the discussion of Example 4.1, we can give the fuzzy model of system (27)

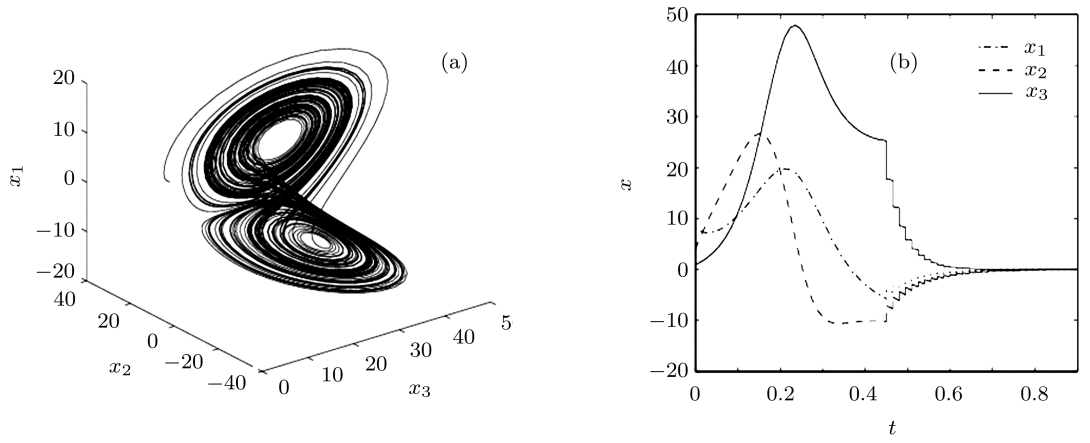


Fig.2. Chaotic behaviour and impulsive fuzzy control of Lorenz system with delay $\tau = 0.014$, (a) Chaotic attractor of Lorenz's system with $\tau = 0.014$, (b) Impulsive fuzzy control begins at $t = 0.45$ with $\tau = 0.014, \alpha = 0.015$.

like that of system (25), where $\boldsymbol{A}_1 = \boldsymbol{A}_2 = \begin{bmatrix} -\beta & \beta & 0 \\ \eta-\beta & \eta & 0 \\ 0 & 0 & -\gamma \end{bmatrix}$, $\boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -\zeta \\ 0 & \zeta & 0 \end{bmatrix}$, $\boldsymbol{B}_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \zeta \\ 0 & -\zeta & 0 \end{bmatrix}$, $M_1(x_1(t)) = \frac{1}{2}\left(1 + \frac{x_1(t)}{\zeta}\right)$, and $M_2(x_1(t)) = \frac{1}{2}\left(1 - \frac{x_1(t)}{\zeta}\right)$.

Using Theorem 3.1, we can give the impulsive fuzzy control matrix $\boldsymbol{D}_i$ for $0 < \alpha \leq 0.0029661804$. When Theorem 3.2 is used for system (27), computation yields $\lambda_{\mathrm{A}} = 61.9420$ and $b = 40$. By solving the equation $c = \lambda_{\mathrm{A}} + 2b\sqrt{q}$, where $q = \mathrm{e}^{\alpha c}$, we can give the control matrix $\boldsymbol{D}_i$ for $0 < \alpha \leq 0.00732930036$. Especially, for $\tau = 0.0064$ and $\alpha = t_k - t_{k-1} = 0.0065$, according to Theorem 3.2, we obtain $c = 231.9571$ and $||\boldsymbol{I} + \boldsymbol{D}_i|| < 0.4705$. Choosing $\boldsymbol{D}_1 = \boldsymbol{D}_2 = \mathrm{diag}(-0.53, -0.53, -0.53)$, $||\boldsymbol{I} + \boldsymbol{D}_i|| = 0.47$, and $0 < \lambda < 0.1788$, we easily find all of the conditions to be satisfied. So the trivial solution of the impulsive fuzzy control system is exponentially stable with the convergence rate $\lambda/2$. The simulations of the chaotic behaviour and impulsive fuzzy control are given in Fig.3.

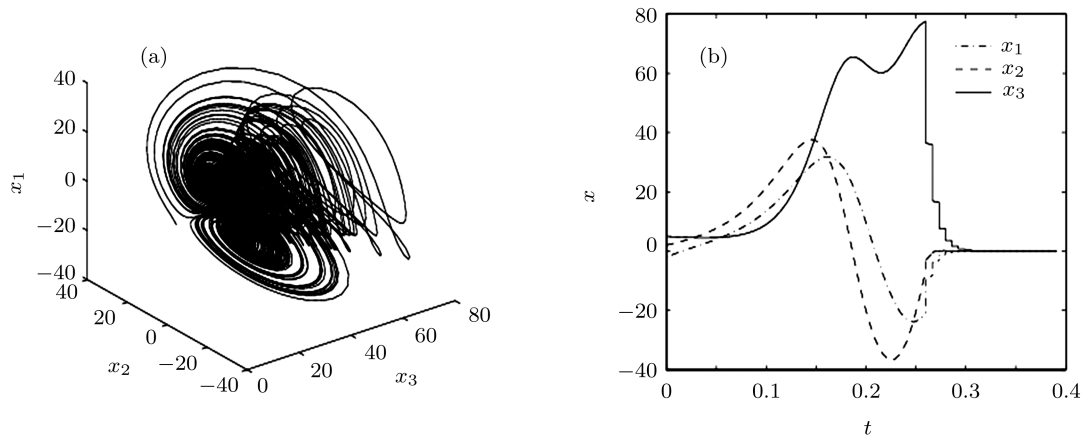


Fig.3. Chaotic behaviour and impulsive fuzzy control of the Chen system with delay $\tau = 0.0064$. (a) Chaotic attractor of Chen's system with $\tau = 0.0064$, (b) Impulsive fuzzy control begins at $t = 0.26$ with $\tau = 0.0064$, $\alpha = 0.0065$.

5. Conclusion

In this paper, an impulsive TS fuzzy framework for controlling TS fuzzy chaotic systems is proposed through combining impulsive control technique with intelligent fuzzy logic methodology. The proposed control method has many advantages, such as, small impulse control, fuzzy intelligence and unified control method. Some criteria have been derived based on the Lyapunov–Razumikhin method to guarantee the global exponential stability of the impulsive fuzzy model. The design procedures for estimating the bound of control matrices proposed in this paper have the virtues of simplicity and computational attractiveness. The effectiveness of our method has been demonstrated through numerical simulations of chaotic systems with time-delay including the Lorenz system and Chen system.

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