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Global impulsive exponential synchronization of stochastic perturbed chaotic delayed neural networks*

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In this paper, the global impulsive exponential synchronization problem of a class of chaotic delayed neural networks (DNNs) with stochastic perturbation is studied. Based on the Lyapunov stability theory, stochastic analysis approach and an efficient impulsive delay differential inequality, some new exponential synchronization criteria expressed in the form of the linear matrix inequality (LMI) are derived. The designed impulsive controller not only can globally exponentially stabilize the error dynamics in mean square, but also can control the exponential synchronization rate. Furthermore, to estimate the stable region of the synchronization error dynamics, a novel optimization control algorithm is proposed, which can deal with the minimum problem with two nonlinear terms coexisting in LMIs effectively. Simulation results finally demonstrate the effectiveness of the proposed method.

Keywords: exponential synchronization, chaotic delayed neural networks, impulsive control, stochastic perturbation

PACC: 0540, 0545

1. Introduction

During the past two decades, chaos synchronization has attracted considerable attention due to its great potential applications in secure communication, chemical reactions and biological systems.[1,2] After the pioneering work of Ott,[3] Pecora and Carroll,[4,5] many different methods have been used theoretically and experimentally to synchronize the chaotic systems, such as linear and nonlinear feedback control,[6−8] backstepping control,[9,10] variable structure control,[11,12] adaptive control,[13−15]impulsive control,[16−19] fuzzy model-based control,[20−22] active control,[23] etc. Among these methods, impulsive control is an efficient method to deal with the dynamical systems which cannot be controlled by continuous control methods. In addition, in the synchronization process the response system receives the information from the drive system only at discrete time instants, which dramatically reduces the amount of synchronization information transmitted from the drive system to the response system and makes the method more efficient in a large number of real-life applications.[24]

Recently, it has been revealed that some types of delayed neural networks (DNNs) can exhibit some complicated dynamics and even chaotic behaviour if the parameters and time delays are appropriately chosen.[25,26] Thus, the synchronization problem of chaotic DNNs has received extensive consideration.[15,27−29]

When chaos synchronization is applied to engineering, such as secure communication, these chaos-based synchronization schemes are rather sensitive to external perturbations which in many cases are of great uncertainties and hence may be treated as random, as fluctuations from the release of neurotransmitters, and other probabilistic causes. By virtue of the stability theory for stochastic differential equations, there has been an increasing interest in the study of exponential synchronization of chaotic delayed systems with stochastic perturbations.[28−30] In the work mentioned above was used just a continuous

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feedback control method, which greatly increases the possibility of code breaking in the process of signal transmission. By comparison, the impulsive control method to be used in this paper is just composed of linear feedback of the error signal and only the state information at discrete time instants is needed, which is more efficient, secure and thus useful in a great number of applications, especially for secure communication. Therefore, it is necessary to study the exponential synchronization of chaotic delayed systems with stochastic perturbation via impulsive control. However, due to the difficulty in investigating the stability of impulsive stochastic delayed differential equations (ISDDEs), to the best of our knowledge, few results about this problem are available.

Motivated by the aforementioned discussion, in this paper we address a practical issue of using the impulsive control method to exponentially synchronize a class of chaotic DNNs with stochastic perturbation. Based on the Lyapunov stability theory, stochastic analysis approach and the efficient impulsive differential inequality, some new sufficient conditions in the form of a linear matrix inequality (LMI) are derived, which can be conducted readily using Matlab LMI Toolbox.[31] Specially, to estimate the stable region of synchronization error dynamics (i.e. the maximum upper bound of impulsive distance under a given control gain), a novel and practical optimization control algorithm is proposed, which is first presented and can deal with the minimum problem with two nonlinear terms coexisting in LMIs effectively.

Throughout the paper, the notations R, R^n and $R^{n\times m}$ denote the real number, n-dimensional Euclidean space and the set of all real matrices, respectively. $R^+ := [0,\infty)$. $N := \{1,2,\ldots\}$. For real symmetric matrices $\boldsymbol{X}$ and $\boldsymbol{Y}$, the notation $\boldsymbol{X} > \boldsymbol{Y}$ ($\boldsymbol{X} \geq \boldsymbol{Y}$, $\boldsymbol{X} < \boldsymbol{Y}$, $\boldsymbol{X} \leq \boldsymbol{Y}$) means that $\boldsymbol{X}$–$\boldsymbol{Y}$ is positive definite (positive semi-definite, negative definite, negative semi-definite). $\boldsymbol{I}$ denotes the identity matrix with appropriate dimensions. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. $\lambda_{\max}(\boldsymbol{S})$ ($\lambda_{\min}(\boldsymbol{S})$) denotes the maximal (minimal) eigenvalue of matrix $\boldsymbol{S}$. For $\bar{\tau} > 0$, $\mathcal{C}([t_0-\tau,t_0];R^n)$ denotes the family of continuous function ϕ from $[t_0-\bar{\tau},t_0]$ to R^n. Let $(\Omega,\mathcal{F},\mathcal{F}_{t\geq 0},\mathcal{P})$ be a complete probability space with a filtration $\mathcal{F}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets). $L^p_{\mathcal{F}_0}([t_0-\bar{\tau},t_0];R^n)$ denotes the family of all $\mathcal{F}_0$-measurable $\mathcal{C}([t_0-\bar{\tau},t_0];R^n)$-valued random variables $\varphi = \varphi(s) : t_0-\bar{\tau} \leq s \leq t_0$ such that $\sup_{t_0-\bar{\tau}\leq s\leq t_0} E(\|\phi(s)\|^p) < \infty$, where $p > 0$. $\mathrm{E}\{\cdot\}$, $\mathrm{Tr}\{\cdot\}$ and $\mathrm{D}^+\{\cdot\}$ represent the mathematical expectation operator, the trace of a matrix and the upper and right (Dini) derivative, respectively. $\mathrm{PC}(l) := \{\phi : [-\bar{\tau},\infty) \to R^l, l \in N$, $\phi(t)$ is continuous everywhere except for the finite number of points t_k at which $\phi(t_k^+) = \phi(t_k)$ and $\phi(t_k^-)$ exist$\}$. $\mathcal{C}^{1,2}(R^+ \times R^n; R^+)$ denotes the family of all nonnegative functions $V(t,x)$ on $R^+ \times R^n$ which are continuously twice differentiable with respect to x and differentiable with respect to t. The symbol "$*$" within a matrix represents the symmetric term of the matrix.

2. Problem formulation and preliminaries

A class of chaotic DNNs to be studied in this paper can be described as follows:

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}f(\boldsymbol{x}(t)) + \boldsymbol{B}f(\boldsymbol{x}_\tau(t)) + \boldsymbol{U}, \quad (1)$$

where $\boldsymbol{x}(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$ is the neuron state vector, $\boldsymbol{C} = \mathrm{diag}(c_1, c_2, \ldots, c_n)^{\mathrm{T}}$ is a diagonal matrix with $c_i > 0$. The activation function, $f(\boldsymbol{x}(t)) = [f_1(x_1), f_2(x_2), \ldots, f_n(x_n)]^{\mathrm{T}}$, describes the behaviour in which the neurons respond to each other. $\boldsymbol{U} = [U_1, U_2, \ldots, U_n]^{\mathrm{T}}$ denotes the input vector. $\boldsymbol{A} = (a_{ij})_{n\times n}$ and $\boldsymbol{B} = (b_{ij})_{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. The transmission delay may be unknown but it is bounded, i.e. $0 \leq \tau(t) \leq \bar{\tau}$. $\boldsymbol{x}_\tau(t) := \boldsymbol{x}(t-\tau(t))$.

With regard to the function $f(\boldsymbol{x}(t))$, the following assumption is given.

Assumption 1 Each function $f_i : R \to R$ with $f_i(0) = 0$ is bounded and satisfies the Lipschitz condition with a Lipschitz constant $\sigma_i > 0$, i.e. $\left|f_i(\widehat{v}) - f_i(\breve{v})\right| \leq \sigma_i \left|\widehat{v} - \breve{v}\right|$ for all $\widehat{v}$, $\breve{v} \in R$, $i = 1, 2, \ldots, n$.

Remark 1 Formula (1) unifies several well-known chaotic DNNs. In particular, if the activation function $f_i(x_i)$ is sigmoid, then expression (1) describes the dynamics of chaotic delayed Hopfield neural networks (DHNNs). Similarly, if the activation function $f_i(x_i) = (|x_i+1| - |x_i-1|)/2$, then expression (1) describes the dynamics of chaotic delayed cellular neural networks (DCNNs).

We take system (1) as the drive system and the response system is given by

$$\mathrm{d}\tilde{\boldsymbol{x}}(t) = (-\boldsymbol{C}\tilde{\boldsymbol{x}}(t) + \boldsymbol{A}f(\tilde{\boldsymbol{x}}(t)) + \boldsymbol{B}f(\tilde{\boldsymbol{x}}_\tau(t)) + \boldsymbol{U})\,\mathrm{d}t$$

$$+\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\mathrm{d}\omega(t), \tag{2}$$

where $\tilde{\boldsymbol{x}}(t)$ is the state vector of the response system, $\boldsymbol{e}(t)=\tilde{\boldsymbol{x}}(t)-\boldsymbol{x}(t)=[\tilde{x}_1(t)-x_1(t),\tilde{x}_2(t)-x_2(t),\ldots,\tilde{x}_n(t)-x_n(t)]^{\mathrm{T}}$ is the error vector, $\tilde{\boldsymbol{x}}_\tau(t):=\tilde{\boldsymbol{x}}(t-\tau(t))$, $\boldsymbol{e}_\tau(t):=\boldsymbol{e}(t-\tau(t))$, $\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_n(t)]^{\mathrm{T}}$ is an n-dimensional Brown motion defined on $(\Omega,\mathcal{F},\mathcal{F}_{t\geq 0},\mathcal{P})$. $\vartheta: R^+\times R^n\times R^n\to R^{n\times n}$ is the diffusion coefficient matrix and satisfies the following assumption.

Assumption 2 $\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))$ is locally Lipschitz continuous and satisfies the linear growth condition. $\vartheta(t,0,0)=0$. Moreover, there exist constant matrices of appropriate dimensions $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ such that

$$\begin{aligned}&\mathrm{Tr}[\vartheta^{\mathrm{T}}(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))]\\ &\leq\|\boldsymbol{\Sigma}_1\boldsymbol{e}(t)\|^2+\|\Sigma_2 e_\tau(t)\|^2.\end{aligned}$$

Note that the stochastic perturbation term $\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\mathrm{d}\omega(t)$ in expression (2) can be regarded as a result from the stochastic uncertainties of the DNNs and the occurrence of the internal error when the simulation circuits are constructed. Therefore, the investigation of the stochastic perturbation effect on chaos synchronization is more meaningful and practical.

At discrete time t_k, the state variables of the drive system are transmitted to the response system as the control input such that the state variables of the response system are suddenly changed at these instants. Therefore, the impulsive controlled response system can be written in the following form:

$$\begin{aligned}\mathrm{d}\tilde{x}(t)&=(-\boldsymbol{C}\tilde{\boldsymbol{x}}(t)+\boldsymbol{A}f(\tilde{\boldsymbol{x}}(t))+\boldsymbol{B}f(\tilde{\boldsymbol{x}}_\tau(t))+\boldsymbol{U})\,\mathrm{d}t\\&\quad+\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\mathrm{d}\omega(t),\ t\geq t_0=0,\ t\neq t_k,\\ \tilde{\boldsymbol{x}}(t_k)&=\tilde{\boldsymbol{x}}(t_k^-)+(\boldsymbol{W}_k-\boldsymbol{I})\boldsymbol{e}(t_k^-),\ t=t_k,\ k\in N,\end{aligned}\tag{3}$$

where $\boldsymbol{W}_k\in R^{n\times n}$ is the control gain matrix. Suppose that discrete instant set $\{t_k\}$ satisfies $0<t_1<t_2<\ldots<t_k<t_{k+1}<\ldots$, $\lim\limits_{k\to\infty}t_k=\infty$. $\lim\limits_{h\to 0^+}\tilde{\boldsymbol{x}}(t_k-h)=\tilde{\boldsymbol{x}}(t_k^-)$, $\lim\limits_{h\to 0^+}\boldsymbol{x}(t_k+h)=\tilde{\boldsymbol{x}}(t_k^+)=\tilde{\boldsymbol{x}}(t_k)$ implies that $\tilde{\boldsymbol{x}}(t)$ is right-continuous at $t=t_k$. Thus, from expressions (1) and (3), and from $\boldsymbol{x}(t_k)=\boldsymbol{x}(t_k^-)$ as well, the following error system can be obtained:

$$\begin{aligned}\mathrm{d}\boldsymbol{e}(t)&=(-\boldsymbol{C}\boldsymbol{e}(t)+\boldsymbol{A}h(\boldsymbol{e}(t))+\boldsymbol{B}h(e_\tau(t)))\mathrm{d}t\\&\quad+\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\mathrm{d}\omega(t),\ t\geq 0,\ t\neq t_k,\\ \boldsymbol{e}(t_k)&=\boldsymbol{W}_k\boldsymbol{e}(t_k^-),\ t=t_k,\ k\in N,\\ \boldsymbol{e}(t)&=\varphi(t),\ -\bar{\tau}\leq t\leq 0,\end{aligned}\tag{4}$$

where $h(\boldsymbol{e}(t)):=f(\boldsymbol{e}(t)+\boldsymbol{x}(t))-f(\boldsymbol{x}(t))=f(\tilde{\boldsymbol{x}}(t))-f(\boldsymbol{x}(t))$.

Definition 1 The global exponential synchronization of (1) and (3) is said to be achieved if, for a suitably designed impulsive controller, the trivial solution $\boldsymbol{e}(t)=0$ of the error system (4) is globally exponentially stable in mean square, i.e. there exist constants $\mu\geq 1$ and $\lambda>0$, such that

$$E(\|\boldsymbol{e}(t)\|^2)\leq\mu\mathrm{e}^{-\lambda(t-t_0)}\sup_{t_0-\bar{\tau}\leq s\leq t_0}E(\|\varphi(s)\|^2)\tag{5}$$

for any initial condition $\boldsymbol{e}(s)=\varphi(s)$ on $t_0-\bar{\tau}\leq s\leq t_0$ in $L^2_{\mathcal{F}_0}([t_0-\bar{\tau},t_0];R^n)$, and the constant $\lambda>0$ is defined as the exponential synchronization rate.

Our goal is to set the control gain $\boldsymbol{W}_k$ and the impulsive distance $\delta_k=t_k-t_{k-1}$ $(k\in N)$ such that the error dynamics (4) is globally exponentially stable in mean square, which implies that the impulsive controlled response system (3) exponentially synchronizes with the drive system (1) for an arbitrary initial condition. For the sake of analytical simplification, we can choose constant control gain and identical impulsive distance, which does not cause any loss of generality in the sense of synchronization analysis, i.e.

$$\boldsymbol{W}_k=w\boldsymbol{I},\ \delta_k=\delta.\tag{6}$$

Next, we introduce several lemmas to be used in the following.

Lemma 1[32] Given arbitrary real matrices $\boldsymbol{X}_1\in R^{n\times k}$, $\boldsymbol{X}_2\in R^{n\times k}$, $\boldsymbol{\Lambda}\in R^{n\times n}$, and $\boldsymbol{\Lambda}=\boldsymbol{\Lambda}^{\mathrm{T}}>0$, the following inequality holds:

$$\boldsymbol{X}_1^{\mathrm{T}}\boldsymbol{X}_2+\boldsymbol{X}_2^{\mathrm{T}}\boldsymbol{X}_1\leq\boldsymbol{X}_1^{\mathrm{T}}\boldsymbol{\Lambda}\boldsymbol{X}_1+\boldsymbol{X}_2^{\mathrm{T}}\boldsymbol{\Lambda}^{-1}\boldsymbol{X}_2.\tag{7}$$

Lemma 2[24] (Impulsive delay differential inequality) Let $0\leq\tau(t)\leq\bar{\tau}$, $F(t,u,u(t-\tau(t))):R^+\times R\times R\to R$ be nondecreasing in $u(t-\tau(t))$, and $I_k(u):R\to R$ be nondecreasing in u. Suppose that $u(t)$, $v(t)\in PC(1)$ satisfy

$$\begin{aligned}&D^+u(t)\leq F(t,u(t),u(t-\tau(t))),\ t\geq 0,\\ &u(t_k)\leq I_k(u(t_k^-)),\ k\in N,\\ &D^+v(t)>F(t,v(t),v(t-\tau(t))),\ t\geq 0,\\ &v(t_k)\geq I_k(v(t_k^-)),\ k\in N.\end{aligned}$$

Then $u(t)\leq v(t)$ for $-\bar{\tau}\leq t\leq 0$ implies $u(t)\leq v(t)$ for $t\geq 0$.

3. Main results

3.1. Global exponential synchronization via impulsive control

Before starting our main results, to facilitate the understanding of the following contents, we need the following related notations.

For a general stochastic system $\mathrm{d}\boldsymbol{x}(t) = g_1(t,\boldsymbol{x})\mathrm{d}t + g_2(t,\boldsymbol{x})\mathrm{d}\omega(t)$ on $t \geq 0$ with initial value $\boldsymbol{x}_0 \in R^n$, $g_1 : R^+ \times R^n \to R^n$ and $g_2 : R^+ \times R^n \to R^{n\times n}$, for each $V(t,\boldsymbol{x}) \in \mathcal{C}^{1,2}(R^+ \times R^n; R^+)$, an operator $\mathcal{L}$ is defined from $R^+ \times R^n$ to R by

$$\begin{aligned}\mathcal{L}V(t,\boldsymbol{x}) =\ & V_t(t,\boldsymbol{x}) + V_x(t,\boldsymbol{x})g_1(t,\boldsymbol{x}) \\ & +\frac{1}{2}\mathrm{Tr}[g_2^{\mathrm{T}}(t,\boldsymbol{x})V_{xx}(t,\boldsymbol{x})g_2(t,\boldsymbol{x})],\end{aligned}$$

where

$$V_t(t,\boldsymbol{x}) = \frac{\partial V(t,\boldsymbol{x})}{\partial t},$$

$$V_x(t,\boldsymbol{x}) = \left(\frac{\partial V(t,\boldsymbol{x})}{\partial x_1},\ldots,\frac{\partial V(t,\boldsymbol{x})}{\partial x_n}\right),$$

$$V_{xx}(t,\boldsymbol{x}) = \left(\frac{\partial^2 V(t,\boldsymbol{x})}{\partial x_i \partial x_j}\right)_{n\times n}.$$

Theorem 1 If the control gain and impulsive distance are chosen as expression (6) and there exist a positive matrix $\boldsymbol{P} > 0$, positive diagonal matrices $\boldsymbol{Q} > 0$, $\boldsymbol{S} > 0$, a real matrix $\boldsymbol{Z}$ with appropriate dimensions and scalars r, $\tilde{r} > 0$, $\rho > 0$ such that the following conditions hold:

$$0 < |w| < 1, \tag{8}$$

$$\begin{bmatrix} \boldsymbol{\Xi} & \boldsymbol{PA} & \boldsymbol{PB} \\ * & -\boldsymbol{Q} & 0 \\ * & * & -\boldsymbol{S} \end{bmatrix} < 0, \tag{9}$$

$$\boldsymbol{P} < \rho\boldsymbol{I}, \tag{10}$$

$$\boldsymbol{Z} < r\boldsymbol{P}, \tag{11}$$

$$\boldsymbol{\Sigma}_\sigma S \boldsymbol{\Sigma}_\sigma + \rho\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{\Sigma}_2 < \tilde{r}\boldsymbol{P}, \tag{12}$$

$$\frac{2\ln|w|}{\delta} + r + \frac{\tilde{r}}{w^2} < 0, \tag{13}$$

where $\boldsymbol{\Xi} = -2\boldsymbol{PC} + \boldsymbol{\Sigma}_\sigma\boldsymbol{Q}\boldsymbol{\Sigma}_\sigma + \rho\boldsymbol{\Sigma}_1^{\mathrm{T}}\boldsymbol{\Sigma}_1 - \boldsymbol{Z}$, $\boldsymbol{\Sigma}_\sigma = \mathrm{diag}\{\sigma_1,\sigma_2,\ldots,\sigma_n\}$, then the error system (4) is globally exponentially stable in mean square, which implies that the response system (3) globally exponentially synchronizes with the drive system (1), and the exponential synchronization rate $\lambda > 0$ is a unique solution of

$$\lambda - a + b\mathrm{e}^{\lambda\bar{\tau}} = 0, \tag{14}$$

in which $a = -2\ln|w|/\delta - r$, $b = \tilde{r}/w^2$.

Proof Denote $y(\lambda) = \lambda - a + b\mathrm{e}^{\lambda\bar{\tau}}$. From inequality (13), we have $a - b > 0$, thus $y(0) < 0$, $y(+\infty) > 0$ and $y'(\lambda) > 0$. Using the continuity and the monotonicity of $y(\lambda)$, Eq.(14) has a unique positive solution, i.e. $\lambda > 0$.

Now we construct a Lyapunov-like function in the form of

$$V(t,\boldsymbol{e}(t)) = \boldsymbol{e}^{\mathrm{T}}(t)P\boldsymbol{e}(t). \tag{15}$$

By Itô's differential formula,[28] the stochastic derivative of $V(t,\boldsymbol{e}(t))$ along the trajectory of system (4) is

$$\begin{aligned}&\mathrm{d}V(t,\boldsymbol{e}(t)) \\ =\ & \mathcal{L}V(t,\boldsymbol{e}(t))\mathrm{d}(t) \\ & +V_e(t,\boldsymbol{e}(t))\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\mathrm{d}\omega(t).\end{aligned} \tag{16}$$

For $t \in [t_{k-1},t_k)$, $k \in N$, taking the mathematical expectation on both sides of Eq.(16), we can obtain the following Dini derivative

$$D^+E(V(t,\boldsymbol{e}(t))) = E(\mathcal{L}V(t,\boldsymbol{e}(t))). \tag{17}$$

From expressions (10)–(12) and Lemma 1, for $t \in [t_{k-1},t_k)$, $k \in N$, the Dini derivative of $V(t,\boldsymbol{e}(t))$ along the trajectory of system (4) is obtained as follows:

$$\begin{aligned}&\mathcal{L}V(t,\boldsymbol{e}(t))\mathrm{d}(t) \\ =\ & V_t(t,\boldsymbol{e}(t)) + V_e(t,\boldsymbol{e}(t))(-C\boldsymbol{e}(t) + \boldsymbol{A}h(\boldsymbol{e}(t)) + \boldsymbol{B}h(\boldsymbol{e}_\tau(t))) \\ & +\frac{1}{2}\mathrm{Tr}\left(\vartheta^{\mathrm{T}}(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))V_{ee}(t,\boldsymbol{e}(t))\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\right) \\ \leq\ & 2e^{\mathrm{T}}(t)\boldsymbol{P}\left(-\boldsymbol{C}\boldsymbol{e}(t) + \boldsymbol{A}h(\boldsymbol{e}(t)) + \boldsymbol{B}h(\boldsymbol{e}_\tau(t))\right) + \rho\mathrm{Tr}\left(\vartheta^{\mathrm{T}}(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\vartheta(t,\boldsymbol{e}(t),\boldsymbol{e}_\tau(t))\right) \\ \leq\ & -2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{PC}\boldsymbol{e}(t) + 2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{PA}h(\boldsymbol{e}(t)) + 2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{PB}h(\boldsymbol{e}_\tau(t)) + \rho\left(\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{\Sigma}_1^{\mathrm{T}}\boldsymbol{\Sigma}_1\boldsymbol{e}(t) + \boldsymbol{e}_\tau^{\mathrm{T}}(t)\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{\Sigma}_2\boldsymbol{e}_\tau(t)\right)\end{aligned}$$

$$
\begin{aligned}
&\leq -2\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{PC}\boldsymbol{e}(t)+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{PAQ}^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}(t)+h^{\mathrm{T}}(\boldsymbol{e}(t))\boldsymbol{Q}\boldsymbol{h}(\boldsymbol{e}(t))+\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{PBS}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{e}(t)\\
&\quad +h^{\mathrm{T}}(\boldsymbol{e}_\tau(t))\boldsymbol{S}h(\boldsymbol{e}_\tau(t))+\rho\left(\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{\Sigma}_1^{\mathrm{T}}\boldsymbol{\Sigma}_1\boldsymbol{e}(t)+\boldsymbol{e}_\tau^{\mathrm{T}}(t)\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{\Sigma}_2\boldsymbol{e}_\tau(t)\right)\\
&\leq \boldsymbol{e}^{\mathrm{T}}(t)(-2\boldsymbol{PC}+\boldsymbol{\Sigma}_\sigma\boldsymbol{Q}\boldsymbol{\Sigma}_\sigma+\rho\boldsymbol{\Sigma}_1^{\mathrm{T}}\boldsymbol{\Sigma}_1+\boldsymbol{PAQ}^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{PBS}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P})\boldsymbol{e}(t)+\boldsymbol{e}_\tau^{\mathrm{T}}(t)(\boldsymbol{\Sigma}_\sigma\boldsymbol{S}\boldsymbol{\Sigma}_\sigma+\rho\boldsymbol{\Sigma}_2^{\mathrm{T}}\boldsymbol{\Sigma}_2)\boldsymbol{e}_\tau(t)\\
&\leq \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{\Pi}\boldsymbol{e}(t)+r\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t)+\tilde{r}\boldsymbol{e}_\tau^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_\tau(t),
\end{aligned}
$$

where $\boldsymbol{\Pi}=\boldsymbol{\Xi}+\boldsymbol{PAQ}^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{PBS}^{-1}\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}$. By Schur complement,[31] one can easily derive that inequality (9) is equivalent to $\boldsymbol{\Pi}<0$, which implies that

$$
\begin{aligned}
&\mathcal{L}V(t,\boldsymbol{e}(t))\\
&\leq \boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{\Pi}\boldsymbol{e}(t)+r\boldsymbol{e}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}(t)+\tilde{r}\boldsymbol{e}_\tau^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{e}_\tau(t)\\
&\leq rV(t,\boldsymbol{e}(t))+\tilde{r}V(t-\tau(t),\boldsymbol{e}_\tau(t)). \qquad (18)
\end{aligned}
$$

For simplicity, in the following, we denote $V(t,\boldsymbol{e}(t))$ and $V(t-\tau(t),\boldsymbol{e}_\tau(t))$ by $V(t)$ and $V(t-\tau(t))$, respectively. Take the expectation on both sides of inequality (18), and from Eq.(17) we have

$$
\begin{aligned}
D^+E(V(t)) &= E(\mathcal{L}V(t))\\
&\leq rE(V(t))+\tilde{r}E(V(t-\tau(t))). \qquad (19)
\end{aligned}
$$

On the other hand, for $t=t_k$, $k\in N$, we have

$$
\begin{aligned}
E(V(t_k)) &= E(\boldsymbol{e}^{\mathrm{T}}(t_k^-)\boldsymbol{W}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{W}_k\boldsymbol{e}(t_k^-))\\
&= w^2E(\boldsymbol{e}^{\mathrm{T}}(t_k^-)\boldsymbol{P}\boldsymbol{e}(t_k^-))\\
&= w^2E(V(t_k^-)). \qquad (20)
\end{aligned}
$$

For any $\varepsilon>0$, let $v(t)$ be a unique solution of the following impulsive delayed differential equation:

$$
\begin{aligned}
&\dot{v}(t)=rv(t)+\tilde{r}v(t-\tau(t))+\varepsilon,\ t\neq t_k,\ t\geq 0,\\
&v(t_k)=w^2v(t_k^-),\ k\in N,\\
&v(t)=\rho E(\|\varphi(t)\|^2),\ \ -\bar{\tau}\leq t\leq 0. \qquad (21)
\end{aligned}
$$

Denote $F(t,E(V(t)),E(V(t-\tau(t))))=rE(V(t))+\tilde{r}E(V(t-\tau(t)))$ and $I_k(E(V(t)))=w^2E(V(t))$, then the function F and I_k will satisfy the monotonicity given in Lemma 2.

Since $E(V(t))\leq\rho E(\|\varphi(t)\|^2)=v(t)$ for $-\bar{\tau}\leq t\leq 0$, it follows from Lemma 2 that

$$
E(V(t))\leq v(t),\ t\geq 0. \qquad (22)
$$

By the formula for the variation of parameters,[33] the solution $v(t)$ of Eq.(21) can be represented as

$$
\begin{aligned}
v(t) &= W(t,0)v(0)\\
&\quad +\int_0^t W(t,s)(\tilde{r}v(s-\tau(s))+\varepsilon)\mathrm{d}s, \qquad (23)
\end{aligned}
$$

where $W(t,s)$, $0\leq s\leq t$, is the Cauchy matrix of the following linear system:

$$
\begin{aligned}
&\dot{\eta}(t)=r\eta(t),\ t\neq t_k,\ t\geq 0,\\
&\eta(t_k)=w^2\eta(t_k^-),\ k\in N. \qquad (24)
\end{aligned}
$$

By the representation of the Cauchy matrix,[33] we have the following estimate by inequality (8) and $\delta=t_k-t_{k-1}$:

$$
\begin{aligned}
W(t,s) &= \mathrm{e}^{r(t-s)}\prod_{s\leq t_k\leq t}w^2\\
&\leq \mathrm{e}^{\left(-a-\frac{2\ln|w|}{\delta}\right)(t-s)}w^{2\left(\frac{t-s}{\delta}-1\right)}\\
&= w^{-2}\mathrm{e}^{-a(t-s)},\ 0\leq s\leq t. \qquad (25)
\end{aligned}
$$

Let $\gamma=w^{-2}\rho\sup_{-\bar{\tau}\leq s\leq 0}E(\|\varphi(s)\|^2)$. Accordingly, it follows from expressions (23) and (25) that

$$
\begin{aligned}
v(t) &\leq w^{-2}\rho\mathrm{e}^{-at}\|\varphi(0)\|^2\\
&\quad +\int_0^t w^{-2}\mathrm{e}^{-a(t-s)}(\tilde{r}v(s-\tau(s))+\varepsilon)\mathrm{d}s\\
&\leq \gamma\mathrm{e}^{-at}+\int_0^t \mathrm{e}^{-a(t-s)}\\
&\quad \times\left(bv(s-\tau(s))+\frac{\varepsilon}{w^2}\right)\mathrm{d}s,\ t\geq 0. \qquad (26)
\end{aligned}
$$

Since λ, ρ, ε, $a-b>0$ and $0<|w|<1$, it yields

$$
\begin{aligned}
v(t) &< \frac{\rho\|\varphi(t)\|^2}{w^2}<\gamma\mathrm{e}^{-\lambda t}+\frac{\varepsilon}{w^2(a-b)},\\
&\quad -\bar{\tau}\leq t\leq 0. \qquad (27)
\end{aligned}
$$

In the following, by reduction to absurdity, we will prove the following inequality holds

$$
v(t)<\gamma\mathrm{e}^{-\lambda t}+\frac{\varepsilon}{w^2(a-b)},\ t\geq 0. \qquad (28)
$$

If this not true, by the estimate of inequality (27) and $v(t)\in\mathrm{PC}(1)$, then there exists a $t^*>0$ such that

$$
v(t^*)\geq\gamma\mathrm{e}^{-\lambda t^*}+\frac{\varepsilon}{w^2(a-b)} \qquad (29)
$$

and

$$
v(t)<\gamma\mathrm{e}^{-\lambda t}+\frac{\varepsilon}{w^2(a-b)},\ t<t^*. \qquad (30)
$$

By Eq.(14) and inequalities (26) and (30), we have

$$\begin{aligned} v(t^*) &\le \gamma \mathrm{e}^{-at^*} + \int_0^{t^*} \mathrm{e}^{-a(t^*-s)} \Big(bv(s-\tau(s)) + \frac{\varepsilon}{w^2} \Big) \mathrm{d}s \\ &< \mathrm{e}^{-at^*} \left\{ \gamma + \int_0^{t^*} \mathrm{e}^{as} \left[b\gamma \mathrm{e}^{-\lambda(s-\tau(s))} + \frac{b\varepsilon}{w^2(a-b)} + \frac{\varepsilon}{w^2} \right] \mathrm{d}s \right\} \\ &\le \mathrm{e}^{-at^*} \left\{ \gamma + b\gamma \mathrm{e}^{\lambda\bar{\tau}} \int_0^{t^*} \mathrm{e}^{(a-\lambda)s} \mathrm{d}s + \frac{a\varepsilon}{w^2(a-b)} \int_0^{t^*} \mathrm{e}^{as} \mathrm{d}s \right\} \\ &= \gamma \mathrm{e}^{-\lambda t^*} + \frac{\varepsilon}{w^2(a-b)} - \frac{\varepsilon \mathrm{e}^{-at^*}}{w^2(a-b)} \\ &< \gamma \mathrm{e}^{-\lambda t^*} + \frac{\varepsilon}{w^2(a-b)}. \end{aligned} \tag{31}$$

This contradicts inequality (29), and so the estimate of expression (28) holds. Letting $\varepsilon \to 0$, and from inequality (22) we have

$$E(V(t)) \le v(t) \le \gamma \mathrm{e}^{-\lambda t},\ t \ge 0, \tag{32}$$

i.e.,

$$E(\|\boldsymbol{e}(t)\|^2) \le \mu \mathrm{e}^{-\lambda t} \sup_{-\bar{\tau} \le s \le 0} E(\|\varphi(s)\|^2),\ t \ge 0, \tag{33}$$

where $\mu = \rho/(w^2 \lambda_{\min}(\boldsymbol{P})) \ge 1$.

From inequality (33), it is easy to conclude that the error system (4) is globally exponentially stable in mean square with exponential synchronization rate λ. This completes the proof.

3.2. Optimization control algorithm to estimate the stable region

From inequalities (8) and (13), if $r + w^{-2}\tilde{r} \le 0$, then we can take $\delta < +\infty$ (i.e. δ can be taken as any finite positive number). Otherwise, we can estimate the stable region of system (4) with the fixed control gain coefficient w, i.e. $\delta < -2\ln|w| / (r + w^{-2}\tilde{r})$, by solving the following optimization problem:

$$\begin{aligned} &\text{minimize } r + w^{-2}\tilde{r} > 0, \\ &\text{s.t. inequalities (8)} - \text{(12)}. \end{aligned} \tag{34}$$

For the above optimization problem (34), note that two nonlinear terms, i.e. $r\boldsymbol{P}$ and $\tilde{r}\boldsymbol{P}$, coexist in the LMIs (11) and (12), thus the conventional LMI solvers (feasp, mincx and gevp) are not applicable directly. In view of this fact, we present the following procedure (see Fig.1) to solve the optimization problem.

(1) Initialize the system parameters, including $\boldsymbol{C}, \boldsymbol{A}, \boldsymbol{B}, \boldsymbol{\Sigma}_\sigma, \boldsymbol{\Sigma}_1, \boldsymbol{\Sigma}_2, \bar{\tau}$, the control gain coefficient w that satisfies inequality (8), two given coefficients of accuracy $\varsigma > 0$ and $\zeta > 0$, two temporary flag numbers s_0 and s_1.

(2) To obtain the theoretic minimum value of r that satisfies the LMIs (8)–(12), we can solve the following generalized eigenvalue minimization problem (GEVP):[34]

$$\begin{aligned} &\text{minimize } r, \\ &\text{s.t. } \begin{cases} \text{inequalities (8)} - 10), \\ \boldsymbol{Z} < r\boldsymbol{P}. \end{cases} \end{aligned} \tag{35}$$

We denote the minimum value obtained by $\underline{\mu}$. Meanwhile, we choose a constant $\bar{\mu} > 0$ which satisfies $\bar{\mu} > \underline{\mu}$.

(3) Let $r = \bar{\mu}$ (i.e. choose $\bar{\mu}$ as the first value of r in the beginning of the new iterative operation).

(4) Fix r and use the following GEVP technique to obtain the minimum value of $\tilde{r}$:

$$\begin{aligned} &\text{minimize } \tilde{r} > 0, \\ &\text{s.t. } \begin{cases} \text{inequalities (8)} - \text{(11)}, \\ \boldsymbol{\Sigma}_\sigma \boldsymbol{S} \boldsymbol{\Sigma}_\sigma + \rho \boldsymbol{\Sigma}_2^{\mathrm{T}} \boldsymbol{\Sigma}_2 < \tilde{r}\boldsymbol{P}. \end{cases} \end{aligned} \tag{36}$$

(5) Check whether $r + w^{-2}\tilde{r} < s_0$, if yes, let $s_0 = r + w^{-2}\tilde{r}$, $s_1 = r$.

(6) Let $r = r - \varsigma$. Check whether $r < \underline{\mu}$, if so, go to Step (7). Otherwise go to Step (4).

(7) Check whether $s_0 \le \bar{\mu}$, if so, go to Step (8). Otherwise let $\bar{\mu} = \bar{\mu} + \zeta$ and then go to Step (3).

(8) Terminate the program and output the optimal value s_0, s_1 and $w^2(s_0 - s_1)$ (correspond to $\min\{r + w^{-2}\tilde{r}\}$, r and $\tilde{r}$, respectively).

Next, we give the following directions, which imply the rule for choosing $\underline{\mu}$ and the feasibility of the aforementioned algorithm.

• In Step (2), by solving the GEVP (35), we can obtain the minimum value r (which is denoted by $\underline{\mu}$)

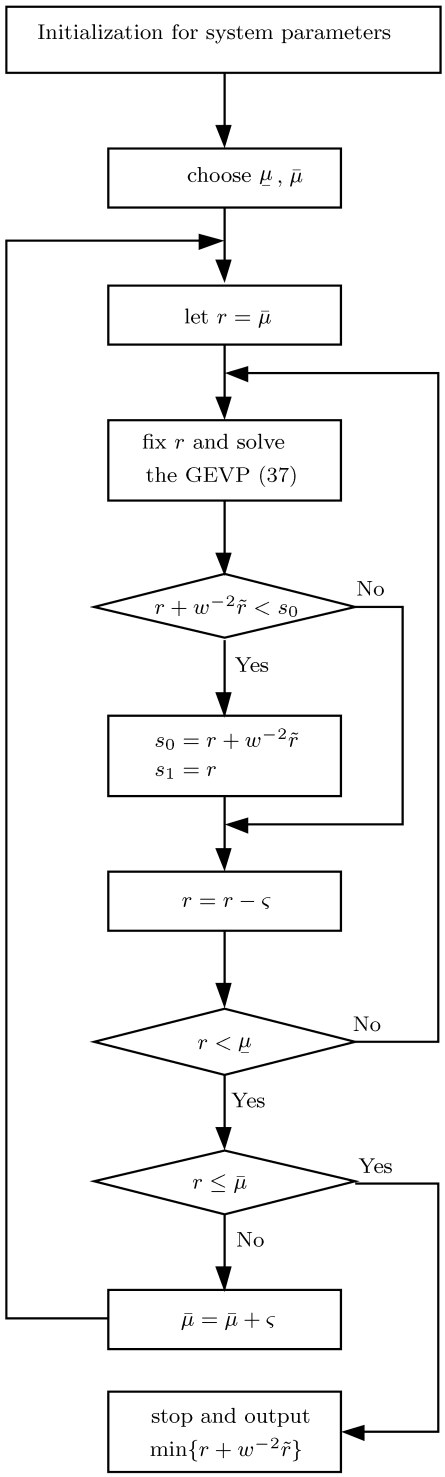


Fig.1. Flowchart for the computation of optimization problem (34).

that satisfies LMIs (8)–(11). If we add another LMI constraint, i.e. inequality (12), the derived r must be no less than $\underline{\mu}$. Thus, we can take $\underline{\mu}$ as the theoretic minimum value of r that satisfies LMIs (8)–(12).

• We choose $r = \bar{\mu}$ in Step (3), and for the first operation in Step (4), by the GEVP (36), we obtain a minimum value of $\tilde{r}$, which is denoted by $\bar{v}$. Let $\bar{s}_0 = \bar{\mu} + w^{-2}\bar{v}$. Obviously, $\bar{\mu} < \bar{s}_0$. If the constant $\bar{\mu}$ satisfies $s_0 \le \bar{\mu}$, where s_0 is the final output in Step (8) of the algorithm (34), then it implies $s_0 < \bar{s}_0$ and in this case s_0 can be regarded as the optimal value (i.e., $\min\{r + w^{-2}\tilde{r}\}$). Accordingly, constant $\bar{\mu}$ can be regarded as the theoretic maximum value of r. Otherwise, we choose another larger $\bar{\mu}$ (i.e., $\bar{\mu} + \zeta$) to repeat the above procedure until $\bar{\mu}$ satisfies $s_0 \le \bar{\mu}$.

• The interval $[\underline{\mu}, \bar{\mu}]$ contains the whole numerical range where r should be, which guarantees the reasonable estimation of the minimum s_0. Furthermore, the optimal value obtained in Step (8) becomes more accurate as ς decreases.

4. Illustrative example

Consider the following chaotic DNNs:[25]

$$\dot{\boldsymbol{x}}(t) = -\boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{A}f(\boldsymbol{x}(t)) + \boldsymbol{B}f(\boldsymbol{x}_\tau(t)) + \boldsymbol{U}, \quad (37)$$

where $\boldsymbol{C} = \boldsymbol{I}$, $\boldsymbol{U} = 0$, $f_i(x_i) = (|x_i + 1| - |x_i - 1|)/2$, $i = 1, 2$, $\tau(t) \equiv \bar{\tau} = 1$, the feedback matrix and the delay feedback matrix, which are specified respectively as

$$\boldsymbol{A} = \begin{bmatrix} 1 + \pi/4 & 20 \\ 0.1 & 1 + \pi/4 \end{bmatrix},$$

$$\boldsymbol{B} = \begin{bmatrix} -1.3\sqrt{2}\pi/4 & 0.1 \\ 0.1 & -1.3\sqrt{2}\pi/4 \end{bmatrix}.$$

As shown in Fig.2, system (37) possesses a chaotic behaviour.

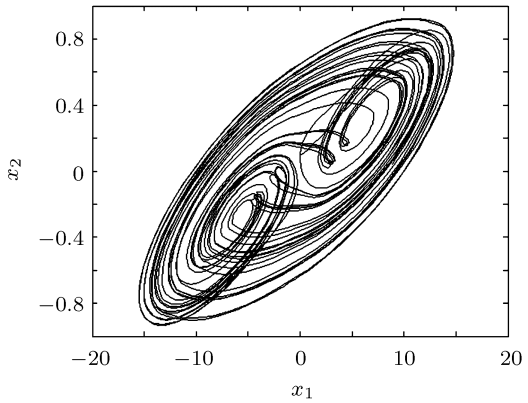


Fig.2. Chaotic behaviour of the chaotic delayed cellular neural networks (37).

From Assumption 1, it is easy to verify that $\boldsymbol{\Sigma}_\sigma = \boldsymbol{I}$. Let $\vartheta(t, \boldsymbol{e}(t), \boldsymbol{e}_\tau(t)) = \mathrm{diag}\left\{\frac{1}{\sqrt{2}}(e_1(t) + e_{1\tau}(t)),\ \frac{1}{\sqrt{2}}(e_2(t) + e_{2\tau}(t))\right\}$, hence we can easily obtain $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2 = \boldsymbol{I}$. We choose $w = 0.5$, and from Theorem 1 and the algorithm described in Fig.1, we have the global optimal value $s_0 = \min\{r + w^{-2}\tilde{r}\} = 42.1700 + 0.5^{-2} \times 1.7186 = 49.0444$, and a set of feasible solutions as follows:

$$\boldsymbol{P} = \begin{bmatrix} 15.2200 & -0.0533 \\ -0.0533 & 15.8625 \end{bmatrix},$$

$$\boldsymbol{Q} = \begin{bmatrix} 307.1211 & 0 \\ 0 & 307.1211 \end{bmatrix},$$

$$\boldsymbol{S} = \begin{bmatrix} 19.8621 & 0 \\ 0 & 19.8621 \end{bmatrix},$$

$$\boldsymbol{Z} = \begin{bmatrix} 635.9571 & 10.8228 \\ 10.8228 & 497.0093 \end{bmatrix},$$

$$\rho = 24.8996.$$

Therefore, from inequality (13), the estimate of the stable region of system (4) is given by

$$\delta < -\frac{2}{s_0}\ln|w| = 0.0283.$$

Figure 3 shows the time response curves of system (4) with $w = 0.5$ and $\delta = 0.028$, and the exponential synchronization rate can be obtained as $\lambda = 0.0577$ from Eq.(14) accordingly. The initial conditions of the drive system and the response system are taken as $\boldsymbol{x}(s) = [0.1, 0.1]^{\mathrm{T}}$ and $\tilde{\boldsymbol{x}}(s) = [0.2, 0.2]^{\mathrm{T}}$ for $-1 \leq s \leq 0$, respectively. Throughout the simulations, we use the fourth Runge-Kutta approach in time steps of 0.0001 s.

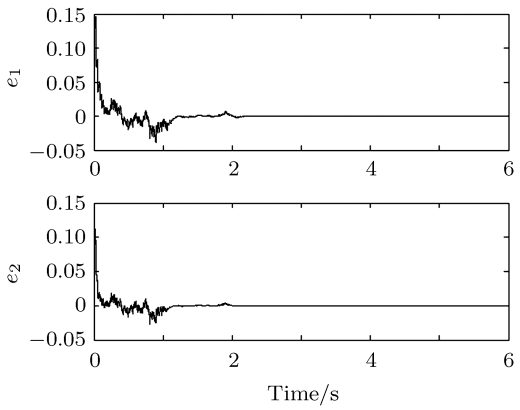


Fig.3. Time response curves of system (4) with $w = 0.5$ and $\delta = 0.028$.

5. Conclusions

In this paper, we have investigated the issue of global impulsive exponential synchronization for a class of chaotic DNNs with stochastic perturbation. Based on the efficient impulsive delay differential inequality and stochastic analysis approach, some new sufficient conditions expressed by LMIs are established. In addition, by using a novel and effective optimization control algorithm, the estimation of the maximum impulsive distance (i.e. the stable region of synchronization error dynamics) has also been given. Finally, some numerical simulations are given to demonstrate the effectiveness of the method. It should be pointed out that the method of impulsive synchronization between two different chaotic DNNs with stochastic perturbation is far from being straightforward due to their different structures and full parameter mismatch. Furthermore, the conditions we give for impulsive synchronization are only sufficient ones, less conservative conditions still need to be developed. These issues are interesting and important, and their further study is desirable.

References

[1] Boccaletti S, Kurths J, Osipov G, Valladares D L and Zhou C S 2002 *Phys. Rep.* **366** 1
[2] Chen G R and Dong X N 1998 *From Chaos to Order: Methodologies, Perspectives and Applications* (Singapore: World Scientific) p12
[3] Ott E, Grebogi C and Yorke J A 1990 *Phys. Rev. Lett.* **64** 1196
[4] Pecora L M and Carroll T L 1990 *Phys. Rev. Lett.* **64** 821
[5] Carroll T L and Pecora L M 1991 *IEEE Trans. Circuits Syst.* **38** 453
[6] Wang F Q and Liu C X 2006 *Phys. Lett.* A **360** 274
[7] Lü L, Zhang Q L and Guo Z A 2007 *Chin. Phys.* **16** 1603
[8] Huang H, Li H X and Zhong J 2006 *Int. J. Bifurc. and Chaos* **16** 281
[9] Lü L, Zhang Q L and Guo Z A 2008 *Chin. Phys.* B **17** 498
[10] Yassen M T 2007 *Phys. Lett.* A **360** 582
[11] Zhang H, Ma X K and Liu W Z 2004 *Chaos, Solitons and Fractals* **21** 1249
[12] Lou X Y and Cui B T 2008 *Chin. Phys.* B **17** 520
[13] Gao T G, Chen Z Q, Yuan Z Z and Yu D C 2007 *Chaos, Solitons and Fractals* **33** 922

[14] Zhang H G, Huang W, Wang Z L and Chai T Y 2006 *Phys. Lett.* A **350** 363
[15] Zhang H G, Xie Y H, Wang Z L and Zheng C D 2007 *IEEE Trans. Neural Netw.* **18** 1841
[16] Ma T D, Zhang H G and Wang Z L 2007 *Acta Phys. Sin.* **56** 3796 (in Chinese)
[17] Zhang H G, Huang W and Wang Z L 2005 *Dynamics of Continuous, Discrete and Impulsive Systems* Series B **12** 153
[18] Zhang R, Xu Z Y and He X M 2007 *Chin. Phys.* **16** 1912
[19] Zhang H G, Ma T D, Yu W and Fu J 2008 *Chin. Phys.* B **17** 3616
[20] Zhang H G, Zhao Y, Yu W and Yang D S 2008 *Chin. Phys.* B **17** 4056
[21] Yang D S, Zhang H G, Li A P and Meng Z Y 2007 *Acta Phys. Sin.* **56** 3121 (in Chinese)
[22] Zhao Y, Zhang H G and Zheng C D 2008 *Chin. Phys.* B **17** 529
[23] Lei Y M, Xu W and Xie W X 2007 *Chaos, Solitons and Fractals* **32** 1823
[24] Yang Z C and Xu D Y 2007 *IEEE Trans. Autom. Contr.* **52** 1448
[25] Gilli M 1993 *IEEE Trans. Circuits Syst.* I *Reg. Papers* **40** 849
[26] Lu H T 2002 *Phys. Lett.* A **298** 109
[27] Cao J D, Chen G R and Li P 2008 *IEEE Trans. Syst. Man Cybern.* B **38** 488
[28] Tang Y, Qiu R H, Fang J A, Miao Q Y and Xia M 2008 *Phys. Lett.* A **372** 4425
[29] Sun Y H, Cao J D and Wang Z D 2007 *Neurocomputing* **70** 2477
[30] Li X L and Cao J D 2008 *Journal of the Franklin Institute* **345** 779
[31] Boyd S, Ghaoui L E, Feron E and Balakrishnan V 1994 *Linear Matrix Inequalities in System and Control Theory* (Philadelphia, PA: SIAM) p7
[32] Sanchez E N and Perez J P 1999 *IEEE Trans. Circuits Syst.* I *Reg. Papers* **46** 1395
[33] Lakshmikantham V, Bainov D D and Simeonov P S 1989 *Theory of Impulsive Differential Equations* (Singapore: World Scientific) p83
[34] Guan X P and Chen C L 2004 *IEEE Trans. Fuzzy Syst.* **12** 236