Vol 18 No 9, September 2009
1674-1056/2009/18(09)/3736-06

Chinese Physics B

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Control of soliton characteristics of the condensate by an arbitrary x-dependent external potential*

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(Received 29 March 2009; revised manuscript received 5 April 2009)

This paper presents a family of soliton solutions of the one-dimensional nonlinear Schrödinger equation which describes the dynamics of the dark solitons in Bose–Einstein condensates with an arbitrary x-dependent external potential. The obtained results show that the external potential has an important effect on the dark soliton dynamical characteristics of the condensates. The amplitude, width, and velocity of the output soliton are relative to the source position of the external potential. The smaller the amplitude of the soliton is, the narrower its width is, and the slower the soliton propagates. The collision of two dark solitons is nearly elastic.

Keywords: Bose–Einstein condensates, dark solitons, x-dependent external potential
PACC: 0530J, 0290, 1110L

1. Introduction

Recently, the physics of dilute Bose–Einstein condensates (BECs) has gained much interest.[1−4] Several experiments on soliton of the BECs in harmonic[5,6] or optical lattice[7,8] potentials have been accomplished, and different techniques of creating different external potentials have been proposed. In the realistic experimental setting, an external electromagnetic field is used to produce, trap, and manipulate the BEC. For example, a harmonic potential can be created by a pair of external magnetic fields.[9] An optical lattice potential may be formed by counter-propagating laser beams and with beams enclosing an angle.[10] Especially, random potentials can also be created by laser speckles[11,12] or by randomly varying magnetic fields in the close proximity of a current-carrying wire.[13] Quasirandom potentials may be formed by using two-color quasiperiodic noncommensurate optical lattices.[14]

Theoretically, the physics of BECs can be described by the Gross–Pitaevskii (GP) equation, whose mathematical form is equal to the nonlinear Schrödinger equation.[1,2] To our knowledge, only two parameters in the GP equation can be changed. One is the external potential, and the other is the inter-atomic interaction strength. It is possible that the technologies of controlling dynamical characteristics are varying the trapped external potential by virtue of magnetic fields or (and) adjusting the interatomic interaction strength via the process of Feshbach resonance.[15−17] To date, the effect of the interatomic interaction on the nonlinear dynamics of the BECs has been intensively investigated.[18−30] Meanwhile, physicists have also investigated some types of external potentials influencing the number of BECs properties, including static and dynamical properties.[31−43] For example, for the BECs confined in the constant potential, there appears an alternative evolvement phenomenon between the bright and dark solitons.[40] When the BECs are trapped in the harmonic potential,[3,5] there exists dark (bright) solitons for the repulsive (attractive) interaction. While the condensates trapped in the tanh-shaped external potential,[41] the BECs exhibit bright (dark) solitons for the repulsive (attractive) interaction. For an expulsive parabolic potential, the bright soliton can be compressed into very high local matter density by increasing the absolute value of the atomic scattering length.[42] Cozzini *et al*[43] investigated a quadratic plus quartic trap influencing the properties of the vortex lattice.

Since the external potential heavily influences the series of properties of the BECs, we here study the soli-

*Project supported by the Natural Science Foundation of Hunan Province of China (Grant No 07JJ3002), and the Fund of the 11th Five-year Plan for Key Construction Academic Subject (Optics) of Hunan Province of China.
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ton characteristics of the condensates trapped in an arbitrary x-dependent external potential. It is shown that the amplitude, width, and velocity of the output soliton are relative to the source position of the external potential. The smaller the amplitude of the soliton is, the narrower its width is, and the slower the soliton propagates.

2. Hydro-dynamical model and asymptotic expansion

As is well known, an effective one dimensional (1D) GP equation in the dimensionless variables is[1,2,44]

$$\mathrm{i}\frac{\partial\psi}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+V(x)\psi+g_{1\mathrm{D}}|\psi|^2\psi, \tag{1}$$

where $V(x)$ is an arbitrary x-dependent external potential and $g_{1\mathrm{D}}=2|a_s|/a_0$. Here a_s is the s-wave scattering length and $a_0=\sqrt{\hbar/m\omega_\perp}$ (transverse harmonic-oscillator length) is determined the atomic mass m and the transverse oscillator frequency $\omega_\perp$. Expressing the wave function in terms of modulus and phase, we set $\psi(x,t)=A(x,t)\exp[-\mathrm{i}\,\mu t+\mathrm{i}\,\varphi(x,t)]$. Substituting it into Eq.(1) and then separating the real and imaginary parts, we obtain the following two hydro-dynamical equations

$$\frac{\partial A}{\partial t}+\frac{\partial A}{\partial x}\frac{\partial\varphi}{\partial x}+\frac{1}{2}A\frac{\partial^2\varphi}{\partial x^2}=0, \tag{2}$$

$$-\frac{1}{2}\frac{\partial^2 A}{\partial x^2}+\left[\frac{1}{2}\left(\frac{\partial\varphi}{\partial x}\right)^2-\mu+\frac{\partial\varphi}{\partial t}+V(x)\right]A+g_{1\mathrm{D}}A^3=0. \tag{3}$$

Note that Eqs.(2) and (3) are still non-integrable, the exact analytical solutions of the solitary excitation of this system are not available. By virtue of the approximation approach described in Refs.[45]–[48], we introduce an external potential $V(x)$ which is a function of fast variable $x_0=x$ representing the direction of soliton propagation. Slow variables $\xi=\varepsilon(x-\lambda t)$ and $\tau=\varepsilon^3 t$ characterize the slow variation of soliton dynamics. The λ is a parameter to be determined later. Without loss of generality, we assume that the condensate background u_0 is a positive constant. Use the asymptotic expansion, $A=u_0+\varepsilon^2\left[a^{(0)}(\xi,\tau)+\varepsilon^2 a^{(1)}(\xi,\tau)+\ldots\right]$, and $\varphi=\varepsilon\left[\varphi^{(0)}(\xi,\tau)+\varepsilon^2\varphi^{(1)}(\xi,\tau)+\ldots\right]$, where ε is a small parameter characterizing the relative amplitude of the excitation. Inserting them into Eqs.(2) and (3), and then separating each order in terms of the power of ε, respectively, from Eq.(2), we obtain

$$\lambda\frac{\partial a^{(0)}}{\partial\xi}-\frac{1}{2}u_0\frac{\partial^2\varphi^{(0)}}{\partial\xi^2}=0, \tag{4}$$

$$\lambda\frac{\partial a^{(1)}}{\partial\xi}-\frac{1}{2}u_0\frac{\partial^2\varphi^{(1)}}{\partial\xi^2}=\frac{\partial a^{(0)}}{\partial\xi}\frac{\partial\varphi^{(0)}}{\partial\xi}+\frac{1}{2}a^{(0)}\frac{\partial^2\varphi^{(0)}}{\partial\xi^2}+\frac{\partial a^{(0)}}{\partial\tau}, \tag{5}$$

and from Eq.(3), we have

$$-\mu u_0+V(x_0)u_0+g_{1\mathrm{D}}u_0^3=0, \tag{6}$$

$$-\mu a^{(0)}+V(x_0)a^{(0)}+3g_{1\mathrm{D}}u_0^2a^{(0)}=\lambda u_0\frac{\partial\varphi^{(0)}}{\partial\xi}, \tag{7}$$

$$\begin{aligned}&-\mu a^{(1)}+V(x_0)a^{(1)}+3g_{1\mathrm{D}}u_0^2a^{(1)}\\&=\frac{1}{2}\frac{\partial^2 a^{(0)}}{\partial\xi^2}-\frac{1}{2}u_0\left(\frac{\partial\varphi^{(0)}}{\partial\xi}\right)^2-3g_{1\mathrm{D}}u_0\left(a^{(0)}\right)^2\\&\quad+\lambda u_0\frac{\partial\varphi^{(1)}}{\partial\xi}+\lambda a^{(0)}\frac{\partial\varphi^{(0)}}{\partial\xi}-u_0\frac{\partial\varphi^{(0)}}{\partial\tau}.\end{aligned} \tag{8}$$

From Eq.(6), we obtain that the chemical potential of the system is $\mu=V(x_0)+g_{1\mathrm{D}}u_0^2$. Comparing Eq.(4) with Eq.(7), we have $\lambda=\pm\sqrt{g_{1\mathrm{D}}}u_0$. If $V(x)$ in Eq.(1) is an external harmonic potential, the group velocity of BEC with repulsive interatomic interaction is $V_\mathrm{g}=\pm\sqrt{g_{1\mathrm{D}}}u_0$, where the positive (negative) sign corresponds to the wave propagating to the right (left).[20,21,49] Thereby, the parameter λ is group velocity of the system. From Eqs.(5) and (8), we obtain

$$\frac{\partial\varphi^{(0)}}{\partial\tau}+\frac{3}{4}\left(\frac{\partial\varphi^{(0)}}{\partial\xi}\right)^2-\frac{1}{8V_\mathrm{g}}\frac{\partial^3\varphi^{(0)}}{\partial\xi^3}=0. \tag{9}$$

Equation (9) can also be called the Korteweg–de Vries (KdV) equation. Making transformation $\eta=\varepsilon\varphi^{(0)}$, $\xi=\varepsilon X=\varepsilon(x-V_\mathrm{g}t)$, and $\tau=\varepsilon^3 t$, from Eq.(9) we have

$$\frac{\partial\eta}{\partial t}+\frac{3}{4}\left(\frac{\partial\eta}{\partial X}\right)^2-\frac{1}{8V_\mathrm{g}}\frac{\partial^3\eta}{\partial X^3}=0. \tag{10}$$

Subsequently, by means of the generalized Hopf–Cole transformation, we set $\eta=-\frac{1}{V_\mathrm{g}}\frac{\partial\ln v}{\partial X}$. The single soliton solution of Eq.(10) can be given by

$$\eta=-\frac{k}{V_\mathrm{g}}\exp(kX-\omega t+\delta_0)\,\mathrm{sech}(kX-\omega t+\delta_0), \tag{11}$$

where k and δ_0 are arbitrary constants. Here $\omega=-k^3/(2V_\mathrm{g})$. Setting $k^2=2V_\mathrm{g}^2A_0$ (A_0 is a positive integrating constant) and using $\xi=\varepsilon X=\varepsilon(x-V_\mathrm{g}t)$, we get the following single soliton solution of Eq.(1):

$$\psi = u_0 \left(1 - A_0 \operatorname{sech}^2 \theta_0\right) \exp\left[-\mathrm{i}\,\mu t - \sqrt{2A_0}\,\mathrm{i} \exp\left(\theta_0\right) \operatorname{sech}\left(\theta_0\right)\right], \tag{12}$$

where $\theta_0 = V_\mathrm{g}\sqrt{2A_0}\,[x + A_0 V_\mathrm{g} t - V_\mathrm{g} t - x_{00}]$, in which $x_{00} = -V_\mathrm{g}\sqrt{2A_0}\delta_0$ is another constant denoting the initial position of soliton on the pedestal background.

Similarly, we obtain the following double solitons solution of Eq.(1):

$$\psi = u_0 \left\{1 - V_\mathrm{g}\left(A_2 - A_1\right) \frac{A_2 \mathrm{csch}^2 \theta_2 + A_1 \operatorname{sech}^2 \theta_1}{\left(\sqrt{A_2}\coth\theta_2 - \sqrt{A_1}\tanh\theta_1\right)^2}\right\} \times \exp\left\{-\mathrm{i}\,\mu t - \mathrm{i}\sqrt{2}\left[\left(\sqrt{A_1} + \sqrt{A_2}\right) + \frac{A_2 - A_1}{\sqrt{A_2}\coth\theta_2 - \sqrt{A_1}\tanh\theta_1}\right]\right\}, \tag{13}$$

where $\theta_1 = V_\mathrm{g}\sqrt{2A_1}\,[x - V_\mathrm{g} t + A_1 V_\mathrm{g} t - x_{01}]$, $\theta_2 = V_\mathrm{g}\sqrt{2A_2}\,[x - V_\mathrm{g} t + A_2 V_\mathrm{g} t - x_{02}]$, in which A_1 and A_2 are positive integrating constants, x_{01} and x_{02} are constants denoting the initial position of soliton on the pedestal background.

3. Numerical results and discussions

To obtain the effect of the external potentials on the solitary excitation, we further give a numerical calculation of the dynamical properties of the soliton of the BECs trapped in different external potentials.

First, we probe the stability problem of the soliton. As a typical example, Fig.1 shows space-time evolution of the density of the BECs trapped in a constant external potential $V_1(x) = 1.0$. We see that there exists a sinus in the amplitude function of the density distribution at $x = 0$ at initial time, which can be associated with the dark soliton. With the time going on, the dark soliton propagates rightward without attenuation and changes in shape (including its height and width). It illustrates that the dark soliton is a stable propagating solitary wave, which is the result from the interplay between the nonlinearity and atomic dispersion.

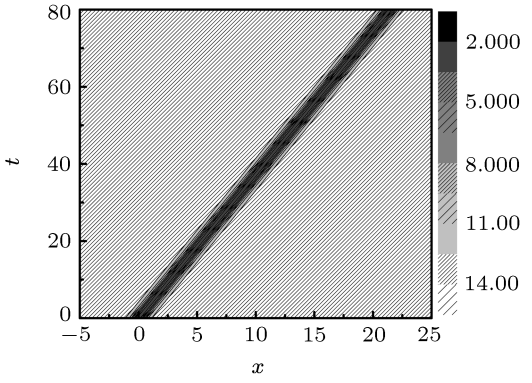


Fig.1. The space-time evolution of the density of the condensate trapped in a constant external potential $V_1(x) = 1.0$. Other parameters are $g_{1\mathrm{D}} = 0.1$, $A_0 = 0.8$, $\mu = 2.8$, and $x_{00} = 0$.

Subsequently, we study the effect of the different external potentials on the dynamical characteristics of the soliton. Here, we choose some representative external potentials, such as a constant, a harmonic, an expulsive parabolic, and a harmonic plus quartic anharmonic potential. Figure 2 plots the density distribution of the source position $x_0 = 8.0$ of the output soliton of the BECs trapped in the different external potentials. One sees that when the condensates are confined whether in a constant potential or in others (a harmonic, an expulsive parabolic, or a harmonic plus quartic anharmonic potential) a dark soliton is still exhibited. It is mainly due to the fact that the repulsive interaction among atoms is exhibited. However, when the BECs are trapped in different external potentials, the amplitude of the solitons occurs differently. This is due to the external potential. From Fig.3, which shows the width of the soliton of the BECs trapped in the different external potentials, we find that the external potentials have a modulating effect on the width of the soliton. For a constant potential, the width of the soliton stays unchanged (see solid line *a* in Fig.3). While the BECs are trapped in a harmonic potential, the width of the soliton firstly increases then decreases, and a maximum exists at the center of the harmonic potential (see dashed line *b* in Fig.3). When the condensates are confined in an expulsive parabolic potential, the width of the soliton firstly decreases then increases, and a minimum occurs at the center of the potential (see dashed line *c* in Fig.3). For a harmonic plus quartic anharmonic

potential, the curve of the width of the soliton is similar with those of a harmonic potential. The former is steeper than the latter, whose main effect is expected to come from the contribution of the quartic anharmonic external potential. Meanwhile, we depict the velocity of the soliton of the BECs in different potentials in Fig.4. One may see that the curves of the velocity of the soliton of the BECs in the different potentials are in qualitative agreement with those of the corresponding width. From the above discussion, we find that the characteristics (the amplitude, width, and velocity) of the soliton can be controlled by an arbitrary x-dependent external potential. Furthermore, from Fig.2 and the position $x = 8.0$ of both Figs.3 and 4, we can conclude that the smaller the amplitude of the soliton is, the narrower its width is, and the slower the soliton propagates.

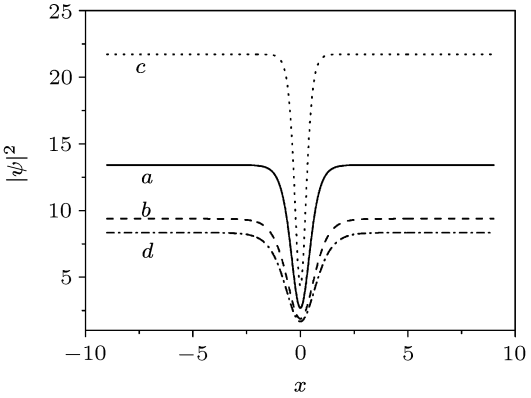


Fig.2. The density distribution of the condensate trapped in the different external potentials (a) a constant external potential $V_1(x) = 1.0$ (solid line), (b) a harmonic potential $V_2(x) = \frac{1}{2}\lambda x^2$ with $\lambda = 0.06$ (dashed line), (c) an expulsive parabolic potential $V_3(x) = -\frac{1}{2}\lambda x^2$ with $\lambda = 0.06$ (dotted line), and (d) a harmonic plus quartic anharmonic potential $V_4(x) = \frac{1}{2}\lambda x^2 + \frac{1}{4}\beta\lambda^2 x^4$ with $\lambda = 0.06$ and $\beta = 0.05$ (dash dotted line). The source position of the output soliton is chosen at $x_0 = 8.0$. Other parameters used are the same as those in Fig.1.

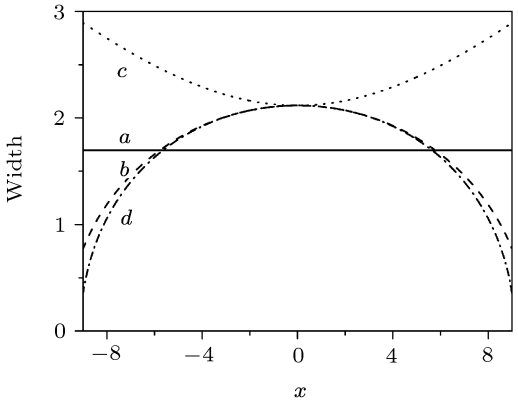


Fig.3. The width of the soliton of the condensates trapped in the different external potentials. The parameters used are the same as those in Fig.2.

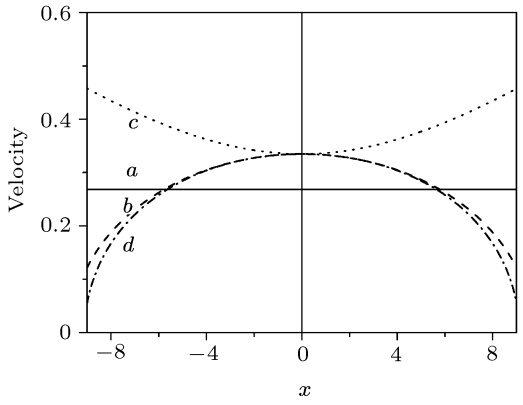


Fig.4. The velocity of the soliton of the condensates trapped in the different external potentials. The parameters used are the same as those in Fig.2.

The stability of the propagating soliton suggests a possibility to consider collision between two solitons. Finally, we discuss the effect of the different external potentials on the collision of two solitons. Figures 5(a)–5(d) represent the space-time distribution of two solitons of the condensates trapped in a constant, a harmonic, an expulsive parabolic, and a harmonic plus quartic anharmonic external potential, respectively. The source position of the output soliton is also chosen on the spot $x_0 = 8.0$. From Fig.5(a), we see that at the initial stage a smaller amplitude and narrower soliton is on the right-hand side (which is called a small soliton), while a larger amplitude and wider soliton (which may be called a large soliton) is on the left-hand side. Clearly, the two dark solitons propagate in a direct way. With the time going on, the large soliton chases after the small soliton. It illustrates that the velocity of the large soliton is larger than that of the small soliton. In the collision process, the amplitude of the large soliton decreases a little, while that of the small soliton increases a little. After the collision, the two solitons asymptotically separate away, but the propagating mode does not change. It remarkably indicates no energy exchange between the two dark solitons. So, we conclude that the collision of two dark solitons is nearly elastic. Similar conclusions of a large soliton chasing after a small soliton are also found for the condensates in other external potentials (see Figs.5(b)–5(d)). By comparing Figs.5(a)–5(d), we find that their amplitudes have slight variation. This suggests that the amplitude, width, and velocity of the solitons are relative to the source position of the different external potentials.

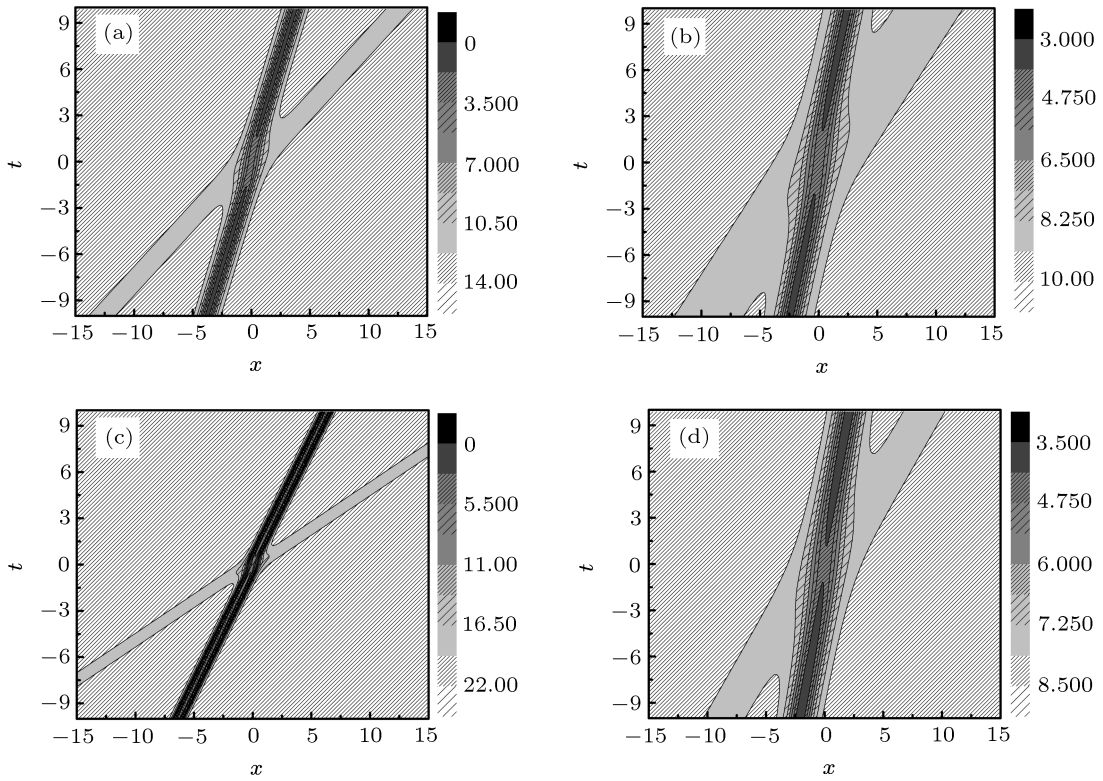


Fig.5. The space-time evolution of the density of two solitons of the condensates trapped in the different external potentials (a) $V_1(x) = 1.0$, (b) $V_2(x) = \frac{1}{2}\lambda x^2$, (c) $V_3(x) = -\frac{1}{2}\lambda x^2$, and (d) $V_4(x) = \frac{1}{2}\lambda x^2 + \frac{1}{4}\beta\lambda^2 x^4$. The parameters used are $A_1 = 0.1$, $A_2 = 0.7$, $x_{01} = 0$, and $x_{02} = 0$. The other parameters used are the same as those in Fig.2.

4. Conclusions

In summary, we develop a multiple-scale method combined with the generalized Hopf–Cole transformation to analytically study the dynamical characteristics of the soliton of BECs trapped in an arbitrary x-dependent external potential. We derive a family of soliton solutions of the BECs in x-dependent external potentials. Our numerical calculations show that the external potential has an important effect on the dark soliton dynamical characteristics. The amplitude, width, and velocity of the output soliton are relative to the source position of the external potential. That is to say, the soliton characteristics of the condensates can be controlled by an external potential. The smaller the amplitude of the soliton, the narrower its width is, and the slower the soliton propagates. For the case of the collision of a large soliton chasing after a small soliton, we find that the amplitude of the large soliton decreases a little, while that of the small soliton increases a little in the collision process. The collision of the two dark solitons is nearly elastic.

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