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Faithful quantum entanglement sharing based on linear optics with additional qubits*

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This paper presents a scheme for faithfully distributing a pure entanglement between two parties over an arbitrary collective-noise channel with linear optics. The transmission is assisted by an additional qubit against collective noise. The receiver can take advantage of the time discrimination and the measurement results of the assistant qubit to reconstruct a pure entanglement with the sender. Although the scheme succeeds probabilistically, the resource used to get a pure entanglement state is finite, and so is easier to establish entanglement in practice than quantum entanglement purification.

Keywords: entanglement sharing, quantum communication, linear optics
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Quantum entanglement is one of the most emblematical characters of quantum systems. The nonlocality and veracious relation between different parts of an entangled quantum system make it have various applications in several fields of quantum information, such as quantum teleportation,[1] entanglement swapping,[2] quantum cryptography,[3−8] quantum computation,[9] and so on. Using entanglement to carry out quantum communication, first we establish the entanglement between remote parties.[9] Generally, entangled states are produced locally by parameter down conversion in experiment. Then particles such as photons will be transmitted through quantum channels to every party participating in the communication later. However, in practice, due to the imperfection of the channel (such as thermal fluctuation and vibration of the medium), the entangled states transmitted usually turn into mixed ones, which will decrease the fidelity and the success probability of quantum communication. Several methods are presented to reduce the influence of the channel noise, such as quantum error correction,[9,10] quantum error rejection,[11] a decoherence-free (DF) subsystem,[12] entanglement purification,[13−17] and so on.

Usually, no matter what error model is chosen, the noise in a channel is often referred to as a collective one. In other words, we suppose that the fluctuation of the channel is so slow in time that the alteration is considered to be the same over a sequence of several photons. Quantum error correction[9,10] and quantum entanglement purification[13−17] have the same essence to confront the noise, i.e., sacrificing several samples to get a useful one. In detail, parts of samples are measured for indicating errors and then the errors in the remaining photons are corrected with some unitary operations. These methods can solve the problem theoretically. In nature, quantum error correction uses some physical qubits to encode one logic bit against a collective noise.[8,9] However, in entanglement purification protocols,[13−16] the steps to get a perfect entangled state, whose fidelity is 100% with a mixed ensemble, are infinite in principle, which means that we cannot obtain an absolutely pure entangled state in practice. The DF subspaces method[12] is encoding

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a quantum state into a DF subspace, which is invariant under a collective noise. However, this method requires the physical qubits to be entangled.

Recently, Kalamidas[11] proposed an error rejection scheme by using a time-bin code, which can obtain the original state after its transmission through the noisy quantum channel with unit probability. The setup is simple except for the fast polarization modulator (Pockels cells), whose synchronization makes it hard to implement with current techniques.[18] Later, de Brito and Ramos[19] presented a different setup based on the similar idea without Pockels cells, in which coherent states were used, instead of single-photon pulses. However, the efficiency is low due to the loss on the way. Han *et al* [20] proved that the original scheme is also suitable for a mixed state, which will have some important applications in quantum networks. These protocols split a single photon into two time-bins and the two parts counteract some errors, and then an uncorrupted state comes out in a definite time. In 2005, Yamamoto *et al* [21] proposed a qubit distribution scheme against an arbitrary collective noise by using an additional qubit to transmit a single photon. Although the maximal success probability is about 1/8 if it uses two Mach–Zehnder (MZ) interferometers to decode the state, the scheme is simple and flexible.

In this paper, we present a scheme for distributing a pure entangled two-photon quantum system over an arbitrary collective-noise channel between two parties, following some ideas in Ref.[12]. The setup involves a beam splitter (BS), a half-wave plate (HWP) and three polarizing beam splitters (PBSs), which are simple and flexible in the laboratory. The reference photon is set in a fixed direction and there is no entanglement between the signal and the reference photons. An uncorrupted state will arrive at a definite time. In other words, the fidelity is retained by sacrificing some success probability.

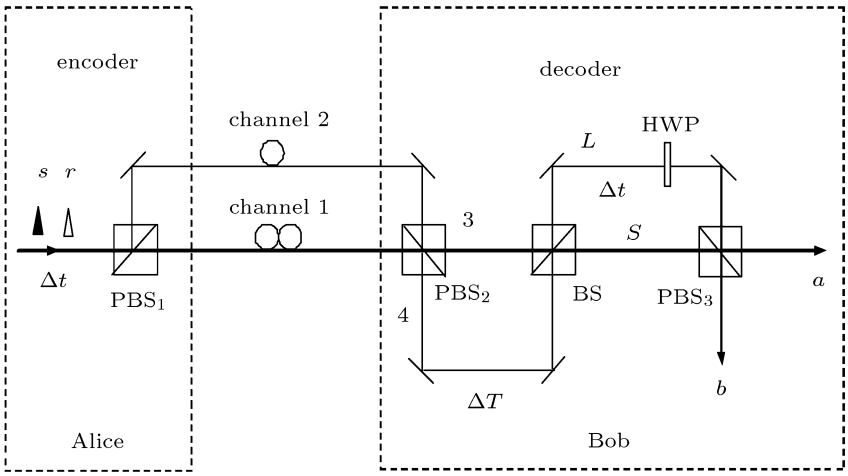


Fig.1. Schematic diagram of the proposed faithful entanglement transmission scheme, which is similar to Ref.[21]. The sender Alice separates the reference photon r and the signal photon s by Δt temporally. Each of the two photons are split into two spatial parts by the PBS. Photons transmitted through the quantum channels 1 and 2 are received by Bob. Bob's unbalanced interferometer (a Mach–Zehnder interferometer) is adjusted to counteract the temporally different Δt. ΔT ($\geq 3\Delta t$) is the time delay between port 3 and port 4. Bob extracts the original signal state by measuring one of the two photons in the X basis at a special time.

Now let us consider a simple case. Suppose there are two parties, the sender Alice and the receiver Bob, who want to share an unknown two-qubit entangled state $|\psi\rangle_{\rm AB}$. In general, an unknown entangled state of two photons can be written as

$$|\psi\rangle_{\rm AB} = \alpha|ab\rangle_{\rm AB} + \beta|\bar{a}\bar{b}\rangle_{\rm AB}, \tag{1}$$

$$|\alpha|^2 + |\beta|^2 = 1, \tag{2}$$

where $a, b, \bar{a}, \bar{b} \in \{H, V\}$ and $\langle \bar{a}|a\rangle = 0, \langle \bar{b}|b\rangle = 0$. H and V represent horizontal and vertical polarizations, respectively, i.e., the two eigenvectors of the measuring basis Z. Alice sends the particle B through a noisy channel to Bob, with the help of an additional photon r which is prepared in the state $\frac{1}{\sqrt{2}}(|H\rangle_{\rm r} + |V\rangle_{\rm r})$. Here, we just investigate the share of the following state $|\phi\rangle_{\rm AB}$ for demonstrating the principle of this faithful entanglement sharing scheme; the other cases are the same as this one with or without some unitary operations.

$$|\phi\rangle_{\rm AB} = \alpha|HH\rangle_{\rm AB} + \beta|VV\rangle_{\rm AB}. \tag{3}$$

The principle of this scheme is shown in Fig.1. The letter s in the diagram, which indicates the signal photon transmitted, represents the photon B in this situation. The signal photon B and the reference photon r have a time interval about Δt. Either the reference photon r or the signal one s going ahead is feasible. We describe the situation that r is going ahead. The initial state of these three photons is

$$\begin{aligned} |\Psi\rangle_{\rm ABr} &= (\alpha|HH\rangle + \beta|VV\rangle)_{\rm AB} \\ &\otimes \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)_{\rm r}. \end{aligned} \tag{4}$$

The two photons transmitted are split by the PBS into two spatial modes, H and V. The PBS transmits H states and reflects V states. That is, the H and V components go through channels 1 and 2, respectively. The effect of the quantum noise in the channel can be expressed by unitary operations

$$\begin{aligned} &|H\rangle_1 \to \delta_1|H\rangle_1 + \eta_1|V\rangle_1, \\ &|V\rangle_2 \to \delta_2|H\rangle_2 + \eta_2|V\rangle_2, \\ &|\delta_i|^2 + |\eta_i|^2 = 1, \quad (i = 1, 2). \end{aligned} \tag{5}$$

The subscript 1 and 2 indicates the two outputs of the first PBS, and also the two quantum channels. It is obvious that this noise is an arbitrary collective noise, neither just a collective dephasing noise nor a collective depolarizing one. The two channels are modulated to have the same length. That is, the two parts of each photon arrive at Bob's PBS at the same time. The two photons maintain the temporal delay Δt in Bob's laboratory. The state at the output of Bob's PBS ($\rm PBS_2$) is

$$\begin{aligned} |\Psi\rangle_{\rm ABr} &\xrightarrow{\rm noise} [\alpha(\delta_1|HH\rangle_{\rm AB1} + \eta_1|HV\rangle_{\rm AB1}) \\ &+ \beta(\delta_2|VH\rangle_{\rm AB2} + \eta_2|VV\rangle_{\rm AB2})] \\ &\otimes \frac{1}{\sqrt{2}}[(\delta_1|H\rangle_{r1} + \eta_1|V\rangle_{r1}) \\ &+ \delta_2|H\rangle_{r2} + \eta_2|V\rangle_{r2})] \\ &\xrightarrow{\rm PBS_2} [\alpha(\delta_1|HH\rangle_{\rm AB3} + \eta_1|HV\rangle_{\rm AB4}) \\ &+ \beta(\delta_2|VH\rangle_{\rm AB4} + \eta_2|VV\rangle_{\rm AB3})] \\ &\otimes \frac{1}{\sqrt{2}}(\delta_1|H\rangle_{r3} + \eta_1|V\rangle_{r4} \\ &+ \delta_2|H\rangle_{r4} + \eta_2|V\rangle_{r3}). \end{aligned} \tag{6}$$

The subscripts 3 and 4 represent the two outputs of the second PBS ($\rm PBS_2$). Output 4 links to another decoder, the same as that of output 3 (or it links to the same decoder with a time delay $\Delta T \geq 3\Delta t$, shown in Fig.1). Bob's decoder comprises a BS, a HWP which rotates the polarization of the photons by 90°, a PBS and two photon detectors. The optical path difference between L and S in the MZ interferometer is adjusted to compensate the temporal interval Δt. Now we consider the event, which has the probability 25% on the total instances, that the two photons both come out from the same output of $\rm PBS_2$, say output 3. The state of the three photons becomes

$$\begin{aligned} |\psi\rangle_{33} = \frac{1}{\sqrt{2}}[&\alpha\delta_1^2|HHH\rangle_{\rm ABr} \\ &+ \delta_1\eta_2(\beta|VVH\rangle_{\rm ABr} \\ &+ \alpha|HHV\rangle_{\rm ABr}) \\ &+ \beta\eta_2^2|VVV\rangle_{\rm ABr}]. \end{aligned} \tag{7}$$

It is easy to see that the two terms in the parentheses $\delta_1\eta_2(\beta|VVH\rangle_{\rm ABr} + \alpha|HHV\rangle_{\rm ABr})$ are not affected by the channel noise. Bob can extract an uncorrupt state with his decoder shown in Fig.1. We consider the case that the reference photon r passes through the path L and the signal photon B through the path S. In this time, the temporal difference between these two photons is counteracted by the MZ interferometer. In the first and the fourth terms of Eq.(7), the two photons have the same polarizations. After the action of HWP, these two terms will result in a two-photon state or a vacuum one in the two detectors. In contrast, photons in the state $\alpha|HHV\rangle_{\rm ABr} + \beta|VVH\rangle_{\rm ABr}$ result in one photon in each of the two modes a and b. The photon detector is set to measure in the basis $X = \frac{1}{\sqrt{2}}(|H\rangle \pm |V\rangle)$. If Bob detects a photon in the

state $\frac{1}{\sqrt{2}}(|H\rangle+|V\rangle)$, the state of photons A and B will collapse into $\alpha|HH\rangle_{\rm AB}+\beta|VV\rangle_{\rm AB}$, which is just the uncorrupt state that the two parties want to share. If the measurement outcome is $\frac{1}{\sqrt{2}}(|H\rangle-|V\rangle)$, the receiver can obtain the originally entangled state by taking a $\sigma_z=|H\rangle\langle H|-|V\rangle\langle V|$ operator on photon B. In the case that the reference photon passes through the path S or (and) the signal photon through the path L, at least one photon will arrive at the wrong time. To eliminate the contribution of the states having the wrong path or coming to the same detectable mode, postselection can be used other than sophisticated photon-number analysers.

From Eq.(7) and the analysis above, the success probability for sharing an uncorrupt entangled state is $(|\delta_1\eta_2|^2/2)(d^2/4)$. Here d is the detector efficiency and 1/4 comes from the decoding loss (50% probability that photons go to the wrong paths and 50% probability that both photons appear from port 4). The average value of $|\delta_1\eta_2|^2/2$ is 1/8 by rotating the collective polarization randomly in each channel. The situation that photons both appear from the output 4 is similar to the discussion above and the average success probability is the same. That is, taking the case that both photons appear in the output 4 into account, the success probability is 1/8, which is same as that in Ref.[21].

Certainly, the case with 50% probability that one photon comes out from output 3 and the other from output 4 also exists. The uncorrupt state can be extracted by using deterministic two-qubit operations on r and B. This will increase the total success probability to $d^2/4$.

In summary, we have proposed a faithful quantum entanglement sharing scheme for distant parties to share entangled states over an arbitrary collective-noise channel with the help of an additional state. The setup is composed of some passive linear-optics elements. The reference photon is initially set to a fixed polarization and no entanglement is needed between the reference and the signal. Although the scheme succeeds probabilistically, a definite pure entangled state would be obtained once it succeeds. Compared with quantum entanglement purification protocols,[13−17] the resource needed in this protocol is greatly reduced.

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