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Generation of a χ-type four-atom entangled state*

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This paper proposes a scheme to generate a new χ-type four-atom entangled state for the first time by using linear optics elements, four one-sided cavities (one three-level atom) and a conventional photon detector. The linear optical elements and conventional photon detector are simple and accessible in experiments, which makes the scheme more feasible with current technology. In addition, the state $|\chi^{00}\rangle_{3214}$ with probability 1 can be generated as long as there is no photon loss.

Keywords: quantum entanglement, linear optics, χ-type entangled state
PACC: 0367

1. Introduction

Entanglement, a remarkable feature in the field of quantum physics, has played a key role as a valuable resource for quantum communication, quantum information processing, quantum teleportation and quantum dense coding.[1−7] Therefore, the exploration and preparation of various entangled states has become a highlight in quantum information processing. Bipartite entanglement is well understood,[8] whereas multipartite entanglement is still under extensive exploration.[9] Multiple particle entanglement has many interesting properties, for example, when any one of the three qubits of W state is traced out, entanglement yet remains between the other two particles. It has been shown that multi-particle entangled states have more advantages than the two-particle entangled states in their applications in quantum cloning, teleportation and dense coding. So the preparation and manipulation of multi-particle entangled states have attracted much attention. Many people have proposed theoretic schemes to prepare Greeberger–Horne–Zeilinger (GHZ) states, cluster states and W states.[10−20] Recently, in order to teleport an arbitrary two-qubit state, Yeo and Chua proposed a genuine four-qubit entangled state $|\chi^{00}\rangle_{3214}$,[21] namely the χ-type entangled state,

$$|\chi^{00}\rangle_{3214} = \frac{\sqrt{2}}{4}[|0000\rangle - |0011\rangle - |0101\rangle + |0110\rangle + |1001\rangle + |1010\rangle + |1100\rangle + |1111\rangle]_{3214}. \quad (1)$$

Lately, Wu *et al* investigated quantum nonlocality of the χ-type entangled state $|\chi^{00}\rangle_{3214}$.[22] It has been shown that the state violates optimally a new Bell inequality. The well-known three types of multipartite entangled state i.e., GHZ, cluster and W state, however, do not have such entanglement properties. Another important property is that it has the maximum entanglement between qubits $(3,2)$ and $(1,4)$, and between qubits $(3,1)$ and $(2,4)$,[23] that is, they are difficult to destroy by local operations. More importantly, it has many applications in quantum information processing (QIP) and fundamental tests of quantum physics, such as teleportation and dense coding.[21] Very recently, Wang and Yang proposed a scheme to generate a $|\chi^{00}\rangle_{3214}$-like state in an ion-trap system.[23] But the state they produced has less entanglement with von Neumann measure between ions $(1,2)$ and $(3,4)$ than that of the χ-type entangled state $|\chi^{00}\rangle_{3214}$.[21] In this paper, inspired by the protocol of Song *et al*,[14] we propose a simple scheme for generating the state $|\chi^{00}\rangle_{3214}$ with linear optics elements

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and a conventional photon detector. The scheme is easy to realize due to the fact that the quantum state of light is robust against decoherence and photons are ideal carriers for transmitting quantum information over a long distance. In addition, we only need a conventional photon detector to distinguish between the vacuum and nonvacuum Fock number states. This advantage also makes the scheme more feasible in experiments.

2. Generation of χ-type four-atom entangled state

Now let us describe our scheme in detail. The basic building model[24,25] involved in our scheme is shown in Fig.1. A single-photon pulse with horizontal (H) polarization enters a one-sided cavity in which a Λ-type three-level atom is confined. Levels $|0\rangle$ and $|e\rangle$ are resonantly coupled with the H component of the input photon. Suppose the input photon is of H polarization; it will have a resonant interaction with the cavity if the atom is in state $|1\rangle$. When $\kappa T \gg 1$ is satisfied (where T is the duration of the input photon pulse and κ is the cavity decay rate), the pulse will be reflected by the cavity with its pulse shape almost unchanged but its phase added by π. In contrast, when the atom is in state $|0\rangle$, the pulse will be reflected by the cavity with both its shape and phase unchanged. If the pulse is of vertical (V) polarization, it will be reflected by the mirror without any change. Therefore, the controlled phase flip (CPF) gate between the atom and the photon can be described by the unitary operator $U = \mathrm{e}^{(\mathrm{i}\,\pi|1\rangle\langle 1|\otimes|H\rangle\langle H|)}$.

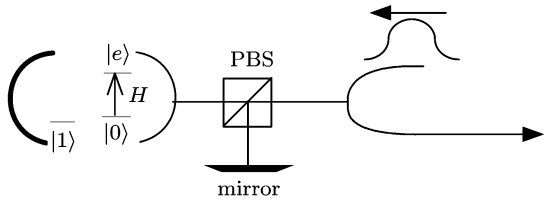


Fig.1. Schematic setup to implement the controlled phase flip gate with atom and single-photon pulse.

We assume that the input photon is initially in the state $|H\rangle$ and the four atoms trapped in different cavities are in the state $|+\rangle$, where$(|\pm\rangle = \frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. The setup is schematically shown in Fig.2. We will see that the photon in $|V\rangle$ will be reflected by the polarizing beam splitter (PBS), while the photon in $|H\rangle$ will pass through the PBS. After the input photon passes through $H_{(1,2)}$ and $\mathrm{PBS}_{(1,2)}$, the state of the whole system is

$$\begin{aligned}|\chi_1\rangle \rightarrow & \frac{1}{\sqrt{2}}|H\rangle|+++\rangle_{1234} \\ & -\frac{1}{2}|V\rangle|+++\rangle_{1234} \\ & +\frac{1}{2}|H\rangle_{\mathrm{DL}}|+++\rangle_{1234}, \end{aligned} \tag{2}$$

where the subscript DL (delay line) indicates the item which will take a longer time to reach the next optics elements than the others. In the same way, after the photon passes through C_1, and Cavity_1, the state of the whole system is transformed into

$$\begin{aligned}|\chi_2\rangle \rightarrow & \frac{1}{\sqrt{2}}|H\rangle|-++\rangle_{1234} \\ & -\frac{1}{2}|V\rangle|+++\rangle_{1234} \\ & +\frac{1}{2}|H\rangle_{\mathrm{DL}}|+++\rangle_{1234}. \end{aligned} \tag{3}$$

From Fig.2, we have the following processes:

$$\begin{aligned}\xrightarrow{H_{(3,4)},\mathrm{PBS}_{(3,4)},\mathrm{HWP}_{(1,2)}} & \frac{2+\sqrt{2}}{4}|H\rangle|-++\rangle_{1234} \\ & -\frac{1}{2}|V\rangle|+++\rangle_{1234} \\ & +\frac{1}{2}|H\rangle_{\mathrm{DL}}|+++\rangle_{1234} \\ & +\frac{\sqrt{2}}{4}|H\rangle_{\mathrm{DL}}|-++\rangle_{1234}, \end{aligned} \tag{4}$$

$$\begin{aligned}\xrightarrow{C_2,\mathrm{Cavity_2}} & \frac{1}{2}(|H\rangle|--+\rangle_{1234} \\ & -|V\rangle|+++\rangle_{1234}) \\ & +\frac{1}{2}|H\rangle_{\mathrm{DL}}|-++\rangle_{1234} \\ & +\frac{\sqrt{2}}{4}|H\rangle_{\mathrm{DL}}|-++\rangle_{1234} \\ & +\frac{\sqrt{2}}{4}|H\rangle|-++\rangle_{1234}. \end{aligned} \tag{5}$$

When the photon in state $|H\rangle_{\mathrm{DL}}|+-++\rangle_{1234}$ passes through H_5, it will be transformed into $\frac{1}{\sqrt{2}}(|H\rangle_{\mathrm{DL}}|+-++\rangle_{1234} + |V\rangle_{\mathrm{DL}}|+-++\rangle_{1234})$. We will see that the three parts of vertical polarization components $|V\rangle|+++\rangle_{1234}$, $|V\rangle|--+\rangle_{1234}$ and $|V\rangle_{\mathrm{DL}}|+-+\rangle_{1234}$ are reflected by PBS_5, and go through switch K_1 at different times. When the first two parts $|V\rangle|+++\rangle_{1234}$ and $|V\rangle|--+\rangle_{1234}$ arrive at the switch K_1, the working state of K_1 is transmitting. After the first two parts have gone through K_1, we change the transmitting state of K_1 into the reflecting state, so the last one $|V\rangle_{\mathrm{DL}}|+-+\rangle_{1234}$ will be reflected by K_1. After the photon passes through H_5, PBS_5, half-wave plate $\mathrm{HWP}_{(3,4)}$, H_{p} and K_1, the state of the whole system is transformed into

$$|\chi_3\rangle \rightarrow \frac{\sqrt{2}}{4}|H\rangle(|---+\rangle_{1234}+|-+++\rangle_{1234}-|++++\rangle_{1234})-\frac{\sqrt{2}}{4}|H\rangle_{\widetilde{\mathrm{DL}}}|+-++\rangle_{1234}$$
$$+\frac{\sqrt{2}}{4}|H\rangle_{\mathrm{DL}}(|+-++\rangle_{1234}+|-+++\rangle_{1234}+|++++\rangle_{1234}+|--++\rangle_{1234}). \quad (6)$$

Here, DL and $\widetilde{\mathrm{DL}}$ are used for distinguishing different pathes. Then the photon goes through C_3, Cavity_3 and K_2, the corresponding state is

$$|\chi_4\rangle \rightarrow \frac{\sqrt{2}}{4}|H\rangle(|----+\rangle_{1234}+|-+++\rangle_{1234}-|+++-+\rangle_{1234})-\frac{\sqrt{2}}{4}|H\rangle_{\widetilde{\mathrm{DL}}}|+-++\rangle_{1234}$$
$$+\frac{\sqrt{2}}{4}|H\rangle_{\mathrm{DL}}(|+--+\rangle_{1234}+|-+-+\rangle_{1234}+|++++\rangle_{1234}+|--++\rangle_{1234}). \quad (7)$$

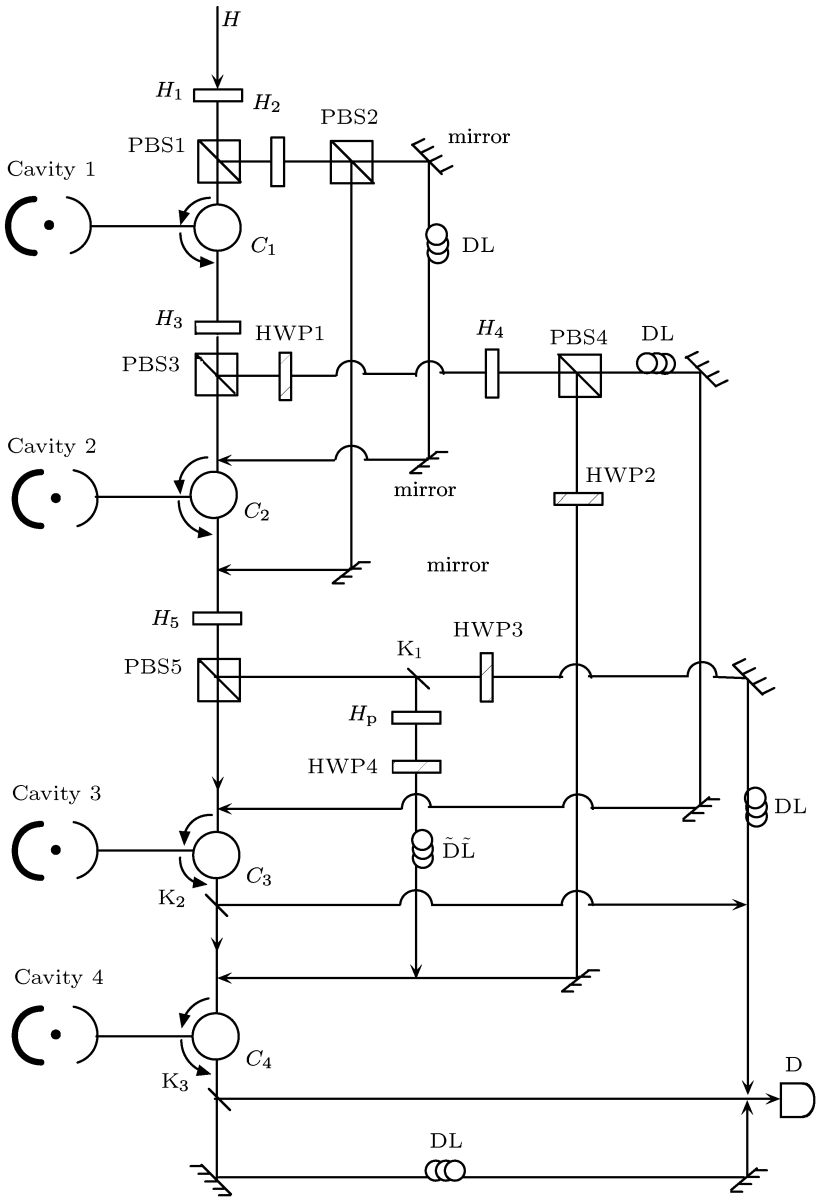


Fig.2. Schematic setup to generate the four-atom χ-type entangled state. The 45°-titled half-wave plate (HWP) rotates the photon polarization as $H \leftrightarrow V$, the half-wave plate H performs a unitary operation on the photon polarization state, $H|H\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)$, $H|V\rangle = \frac{1}{\sqrt{2}}(|H\rangle - |V\rangle)$ and the $\frac{\pi}{2}$–phase shifter P is used to change the sign of $|V\rangle$ ($H_{\mathrm{p}}|V\rangle = -|V\rangle$). The polarizing beam splitter (PBS) transmits the horizontal polarization mode $|H\rangle$ and reflects the vertical polarization mode $|V\rangle$. DL and D are delay line and conventional photon detector, respectively.

Similarly, the four parts ($|H\rangle|--++\rangle_{1234}$, $|H\rangle|+++\rangle_{1234}$, $|H\rangle_{\rm DL}|+-++\rangle_{1234}$ and $|H\rangle_{\rm DL}|-+++\rangle_{1234}$) would arrive at switch K_2 at different times. Once the first two parts $|H\rangle|--++\rangle_{1234}$ and $|H\rangle|++++\rangle_{1234}$ go through K_2, we change the transmitting state of K_2 into the reflecting state, so the last two parts $|H\rangle_{\rm DL}|+-++\rangle_{1234}$ and $|H\rangle_{\rm DL}|-+++\rangle_{1234}$ will be reflected by K_2.

Finally, the photon goes through C_4, Cavity_4 and K_3 (K_3 has the same effect as K_1 and K_2, and the components transmitted by K_3 are delayed); the photon–atom system is transformed into

$$\begin{aligned}|\chi_5\rangle \rightarrow & \frac{\sqrt{2}}{4}|H\rangle_{\rm DL}(|++++\rangle-|++--\rangle \\ & -|+-+-\rangle+|+--+\rangle \\ & +|---\,-\rangle+|--++\rangle \\ & +|-+-+\rangle+|-++\,-\rangle)_{1234}. \end{aligned} \quad (8)$$

We appropriately select the length of optical fibre to make sure that all the photons arrive at a detector at one time. If the detector makes a click, the atoms 1, 2, 3, 4 will collapse to the state

$$\begin{aligned}|\chi_6\rangle \rightarrow & \frac{\sqrt{2}}{4}(|++++\rangle-|++--\rangle \\ & -|+-+-\rangle+|+--+\rangle \\ & +|---\,-\rangle+|--++\rangle \\ & +|-+-+\rangle+|-++\,-\rangle)_{1234}. \end{aligned} \quad (9)$$

Thus, we have generated a χ-type four-atom entangled state in a simple way.

3. Discussion and summary

In conclusion, we have proposed a scheme to prepare the χ-type entangled state $|\chi^{00}\rangle_{3214}$ which has many interesting properties and optimally violates a new Bell inequality. In Ref.[23], Wang *et al* have prepared a $|\chi^{00}\rangle_{3214}$-like state in an ion-trap system. Although the state has the maximum entanglement between ions (3,2) and (1,4) and between ions (3,1) and (2,4) as the χ-type entangled state $|\chi^{00}\rangle_{3214}$, it has less entanglement between ions (1,2) and (3,4) than that of $|\chi^{00}\rangle_{3214}$. However, we have conveniently generated the χ-type entangled state $|\chi^{00}\rangle_{3214}$ by using linear optical elements and a conventional photon detector and appropriately selecting the length of optical fibre. We will be able to get the state $|\chi^{00}\rangle_{3214}$ with probability 1 as long as there is no photon loss. The main elements used in our scheme, such as HWPs, PBSs, switches and conventional photon detectors, have been used widely in other QIPs. Our scheme is easy to realize with current technology due to the advantage that the photon has weak interaction with the environment during the course of transmission.

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