Vol 18 No 9, September 2009
1674-1056/2009/18(09)/3670-07

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

# Non-commutative Fock–Darwin system and its magnetism properties*

Yu Xiao-Min(余晓敏)[a)†] and Li Kang(李 康)[b)‡]

[a)] *Dean Office, Hangzhou Dianzi University, Hangzhou* 310018, *China*
[b)] *Department of Physics, Hangzhou Normal University, Hangzhou* 310036, *China*

(Received 29 December 2008; revised manuscript received 11 May 2009)

The Fock–Darwin system is studied in noncommutative quantum mechanics. We not only obtain its energy eigenvalues and eigenstates in noncommutative phase space, but also give an electron orbit description as well as the general expressions of the magnetization and the susceptibility in a noncommutative situation. Further, we discuss two particular cases of temperature and present some interesting results different from those obtained from usual quantum mechanics such as the susceptibility dependent on a magnetic field at high temperatures, the occurrence of the magnetization in a zero magnetic field and zero temperature limit, and so on.

**Keywords:** Landau diamagnetism, space–space non-commutativity, momentum–momentum non-commutativity

**PACC:** 0365, 1110Q

## 1. Introduction

In recent years, intensive research activities have been devoted to the study of physics effects on non-commutative (NC) space[1−9] where the coordinate $\hat{x}_i$ and momentum $\hat{p}_i$ satisfy

$$[\hat{x}_i, \hat{x}_j] = \mathrm{i}\Theta_{ij}, \quad [\hat{p}_i, \hat{p}_j] = 0, \quad [\hat{x}_i, \hat{p}_j] = \mathrm{i}\hbar\delta_{ij}. \tag{1}$$

Here $\{\Theta_{ij}\}$ is an antisymmetric matrix with very small elements representing the noncommutative property of the space. In the low energy effective theory of D-brane with $B$ field background and an extreme situation such as in the string scale or at very high energy levels, not only may the space non-commutativity appear, but also the effects of momentum non-commutativity can be significant, which is called NC phase space. In this space the momentum operators satisfy the commutation relation

$$[\hat{p}_i, \hat{p}_j] = \mathrm{i}\bar{\Theta}_{ij}, \tag{2}$$

where $\{\bar{\Theta}_{ij}\}$ is another antisymmetric matrix representing the non-commutativity of momentum. There are many papers about the study of various aspects of noncommutative problems, such as the energy levels in hydrogen atoms,[10,11] the Aharonov–Bohm (AB) effect,[12] the Aharonov–Casher (AC) effect[13,14] and the He–Mckellar–Wilkens effect.[15] As is well known, a system of spinless electrons moving in a plane, subjected to a perpendicular magnetic field $\boldsymbol{B}$, plays a very important role in modern physics, and it was first investigated by Landau.[16] Its interesting behaviours such as Landau diamagnetism and quantum Hall effect have been discussed in the framework of NC space recently.[17,18] If we neglect Coulomb interactions and restrict the system to a harmonic confining potential, it will become the Fock–Darwin system.[19] In the present paper, we focus our attention on NC problems of this system and make use of a standard coherent state method to further study its magnetization and magnetism in NC space and NC phase space.

The remainder of the present paper is organized as follows. In Section 2, by using a generalized Bopps shift formulation, instead of a star product method, we obtain the levels and the eigenstates of this system in the corresponding NC situation. In Section 3, by using a coherent state method, we study its magnetism in a NC situation and give the general expressions of the magnetization and the susceptibility. In Section 4, we

*Project supported by the National Natural Science Foundation of China (Grant Nos 10575026 and 10875035) and the Natural Science Foundation of Zhejiang Province, China (Grant No Y607437).
†E-mail: yuxiaomin010101@sohu.com
‡Corresponding author. E-mail: kangli@hznu.edu.cn

discuss two special cases of temperatures, present the more brief formulas of the magnetization and the susceptibility in the two kinds of conditions and exhibit some interesting results which do not exist in commutative space. We expect these discussions to be able to provide some clues to verifying the presence of NC space and NC phase space in the future. Conclusions and some remarks are given in the last section.

## 2. Electrons on NC phase space

In usual quantum mechanics, a planar spinless electron confined by a harmonic potential $\frac{1}{2}m{\omega_0}^2$ and moving in an external uniform magnetic field $\boldsymbol{B} = \nabla \times \boldsymbol{A}$, perpendicular to the plane, is described by the Fock–Darwin Hamiltonian

$$H = \frac{1}{2m}\left(P + \frac{e}{c}A\right)^2 + \frac{1}{2}mw_0^2 r^2, \tag{3}$$

where $\boldsymbol{P} = (p_x, p_y)$ is the canonical momentum and $r^2 = x^2 + y^2$. The Bose–Einstein statistics in noncommutative quantum mechanics require both space–space and momentum–momentum not to be commutative. Thus we need to consider physical problems in NC phase space. The static Schrödinger equation of this system in NC phase space can be expressed as

$$\left[\frac{1}{2m}\left(P + \frac{e}{c}A\right)^2 + \frac{1}{2}mw_0^2 r^2\right] * \psi = E\psi, \tag{4}$$

where the star product $*$ is the Moyal–Weyl product defined in Ref.[20]. By replacing the star product with a general ordinary product shift, i.e. shifting coordinates $x, y$ and momentums $p_x, p_y$ with

$$\begin{aligned} x &\longrightarrow \hat{x} = \alpha x - \frac{1}{2\hbar\alpha}\theta p_y, \\ y &\longrightarrow \hat{y} = \alpha y + \frac{1}{2\hbar\alpha}\theta p_x, \\ p_x &\longrightarrow \hat{p}_x = \alpha p_x + \frac{1}{2\hbar\alpha}\bar{\theta} y, \\ p_y &\longrightarrow \hat{p}_y = \alpha p_y - \frac{1}{2\hbar\alpha}\bar{\theta} x, \end{aligned} \tag{5}$$

where $\hat{x}, \hat{y}$ and $\hat{p}_x, \hat{p}_y$ are the coordinates and momentums in NC phase space respectively, $\Theta_{xy} = \theta$ and $\bar{\Theta}_{xy} = \bar{\theta}$. The noncommutative version of the Hamiltonian (3) can be written as

$$\begin{aligned} H &= \frac{1}{2m}\left(\hat{P} + \frac{e}{c}\hat{A}\right)^2 + \frac{1}{2}m{\omega_0}^2(\hat{x}^2 + \hat{y}^2) \\ &= \frac{1}{2m}\left[\left(\left(\alpha p_x + \frac{1}{2\hbar\alpha}\bar{\theta} y\right) - \frac{eB}{2c}\left(\alpha y + \frac{1}{2\hbar\alpha}\theta p_x\right)\right)^2 + \left(\left(\alpha p_y - \frac{1}{2\hbar\alpha}\bar{\theta} x\right) + \frac{eB}{2c}\left(\alpha x - \frac{1}{2\hbar\alpha}\theta p_y\right)\right)^2\right] \\ &\quad + \frac{1}{2}m{\omega_0}^2\left(\alpha x - \frac{1}{2\hbar\alpha}\theta p_y\right)^2 + \left(\alpha y + \frac{1}{2\hbar\alpha}\theta p_x\right)^2. \end{aligned} \tag{6}$$

Here the symmetric gauge, $\hat{A} = \left(-\frac{B}{2}\hat{y}, \frac{B}{2}\hat{x}, 0\right)$, has been used. If we set

$$\begin{aligned} \bar{\omega} &= \sqrt{\left[\left(\frac{\bar{\theta}}{\hbar\alpha m} - \alpha\omega_{\rm c}\right)^2 + 4\alpha^2{\omega_0}^2\right]\left[\left(\alpha - \frac{eB\theta}{4\hbar\alpha c}\right)^2 + \frac{m^2{\omega_0}^2\theta^2}{4\hbar^2\alpha^2}\right]}, \\ M &= \left[\frac{1}{m}\left(\alpha - \frac{eB\theta}{4\hbar\alpha c}\right)^2 + \frac{m\omega_0^2\theta^2}{4\hbar^2\alpha^2}\right]^{-1}, \\ \frac{\bar{\omega}_{\rm c}}{2} &= \frac{1}{m}\left(\alpha - \frac{eB\theta}{4\hbar\alpha c}\right)\left(-\frac{\bar{\theta}}{2\hbar\alpha} + \frac{eB\alpha}{2c}\right) - \frac{\frac{1}{2}m\omega_0^2\theta}{\hbar}, \end{aligned} \tag{7}$$

where $w_{\rm c} = \dfrac{eB}{mc}$ is the cyclotron frequency in commutative space, the Hamiltonian has the form

$$\begin{aligned} H &= \frac{1}{2M}({p_x}^2 + {p_y}^2) + \frac{1}{8}M\bar{\omega}^2(x^2 + y^2) \\ &\quad + \frac{\bar{\omega}_{\rm c}}{2}(xp_y - yp_x). \end{aligned} \tag{8}$$

Introducing the creation and annihilation operators as follows:

$$a_1 = \sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(x + \frac{2{\rm i}p_x}{M\bar{\omega}}\right) - {\rm i}\sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(y + \frac{2{\rm i}p_y}{M\bar{\omega}}\right),$$

$$a_1^+ = \sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(x - \frac{2\mathrm{i}p_x}{M\bar{\omega}}\right) + \mathrm{i}\sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(y - \frac{2\mathrm{i}p_y}{M\bar{\omega}}\right),$$
$$a_2 = \mathrm{i}\sqrt{\frac{M\bar{\omega}}{\hbar}}\left(x + \frac{2\mathrm{i}p_x}{M\bar{\omega}}\right) - \sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(y + \frac{2\mathrm{i}p_y}{M\bar{\omega}}\right),$$
$$a_2^+ = -\mathrm{i}\sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(x - \frac{2\mathrm{i}p_x}{M\bar{\omega}}\right) - \sqrt{\frac{M\bar{\omega}}{8\hbar}}\left(y - \frac{2\mathrm{i}p_y}{M\bar{\omega}}\right), \tag{9}$$

the Hamiltonian $H$ for the motion in the plane in NC phase space can be written as

$$H = \frac{\hbar\bar{\omega}}{2}(a_1^+a_1 + a_2^+a_2 + 1) + \frac{\bar{\omega}_\mathrm{c}}{2}(a_1^+a_1 - a_2^+a_2). \tag{10}$$

Therefore, the energy eigenvalues and the corresponding eigenstates respectively are

$$E_{n_1,n_2} = \frac{\hbar\bar{\omega}}{2}(n_1 + n_2 + 1) + \frac{\hbar\bar{\omega}_\mathrm{c}}{2}(n_1 - n_2),$$
$$n_1,\ n_2 = 0, 1, 2\ldots \tag{11}$$

and

$$|n_1, n_2\rangle = \frac{1}{\sqrt{n_1!n_1!}}(a_1^+)^{n_1}(a_2^+)^{n_2}|0,0\rangle, \tag{12}$$

where $|0,0\rangle$ is the vacuum of $H$. In terms of the relation $\theta\bar{\theta} = 4\alpha^2(1-\alpha^2)$ in Ref.[20], we know that when $\alpha = 1$, which leaves $\bar{\theta} = 0$, then the space is restricted to NC space where only momentum–momentum is commutative. So the energy levels turn into the case in NC space, i.e.

$$E_{n_1,n_2} = \left(\frac{\hbar\sqrt{4\omega_0^2 + \omega_\mathrm{c}^2}}{2} \frac{-Be\theta}{8c\hbar}\sqrt{4\omega_0^2 + \omega_\mathrm{c}^2}\right)(n_1 + n_2 + 1) + \left(\frac{\hbar\omega_\mathrm{c}}{2} - \frac{\omega_\mathrm{c}^2 m\theta}{8\hbar} - \frac{\omega_0^2 m\theta}{2\hbar}\right)(n_1 - n_2). \tag{13}$$

When $\theta = 0$, the result returns to the case of general quantum mechanics, i.e. $E_{n_1,n_2} = \left(\frac{\hbar\sqrt{4\omega_0^2 + \omega_\mathrm{c}^2}}{2}\right)(n_1 + n_2 + 1) + \frac{\hbar\omega_\mathrm{c}}{2}(n_1 - n_2)$.

## 3. Non-commutative magnetization and magnetism

A coherent state is one of the most important states, since it can simulate a classical field. As the average number of photons increases, the uncertainty becomes negligible and the field is more and more like a classical one. By adopting the coherent state method, electron orbits can be described in classical language and the expectation value of any observable in the system could be computed.[21] Here we study the magnetization and magnetism of the system in NC space and NC phase space. Therefore, we introduce the normalized coherent state as

$$|z_1, z_2\rangle = \exp\left[-\frac{1}{2}(|z_1|^2 + |z_2|^2)\right] \times \exp[z_1a_1^+ + z_2a_2^+]|0,0\rangle \tag{14}$$

where $z_x$ and $z_y$ are complex parameters. We still use the standard form of the partition function in NC phase space, i.e.

$$Z = \mathrm{Tr}\mathrm{e}^{-\frac{H}{kT}} = \frac{1}{\pi^2}\int \mathrm{d}^2z_1\mathrm{d}^2z_2\langle z_1, z_2|\mathrm{e}^{-\frac{1}{kT}[\frac{\hbar\bar{\omega}}{2}(a_1^+a_1 + a_2^+a_2 + 1) + \frac{\bar{\omega}_\mathrm{c}}{2}(a_1^+a_1 - a_2^+a_2)]}|z_1, z_2\rangle. \tag{15}$$

To approximately calculate this partition function, we need to know the expectation values of the coordinates under the corresponding coherent state $|z_1, z_2\rangle$ in the Heisenberg picture and the results are given by

$$\bar{\hat{x}} = \mathrm{Re}\left[\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} - \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)z_1(t) + (\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} + \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}})(-\mathrm{i}z_2(t))\right],$$
$$\bar{\hat{y}} = \mathrm{Im}\left[-\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} - \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)z_1(t) + \left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} + \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)(-\mathrm{i}z_2(t))\right], \tag{16}$$

where $z_1(t) = z_1\mathrm{e}^{-\mathrm{i}\frac{(\bar{\omega}+\bar{\omega}_\mathrm{c})t}{2}}$ and $z_2(t) = z_2\mathrm{e}^{-\mathrm{i}\frac{(\bar{\omega}-\bar{\omega}_\mathrm{c})t}{2}}$. Thus the wave packet centroid of the coherent state $|z_1, z_2\rangle$ executes the motion as shown in Fig.1: when the point $A$ $\left(\mathrm{Re}\left[\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} + \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)(-\mathrm{i}z_2(t)\right)\right]\right.$, $\left.\mathrm{Im}\left[\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} + \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)(-\mathrm{i}z_2)(t)\right]\right)$ makes a circular motion with radius $R_1 = \left|\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}} + \frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)\right.$

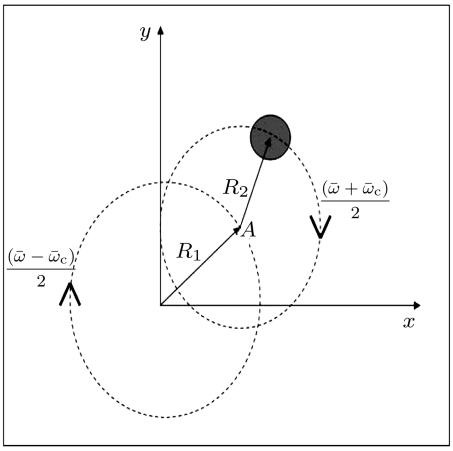



**Fig.1.** Motion of the coherent state $|z_1, z_2\rangle$. The shaded region represents the Gaussian packet. When the point $A$ $\left(\mathrm{Re}\left[\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}}+\frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)(-\mathrm{i}z_2(t))\right],\right.$ $\left.\mathrm{Im}\left[\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}}+\frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)(-\mathrm{i}z_2)(t)\right]\right)$ makes a circular motion with radius $R_1 = \left|\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}}+\frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)(-\mathrm{i}z_2)\right|$ at cyclotron frequency $\frac{\bar{\omega}-\bar{\omega}_c}{2}$, the wave packet centroid of the coherent state follows the motion with radius $R_2 = \left|\left(\alpha\sqrt{\frac{2\hbar}{M\bar{\omega}}}-\frac{\theta}{2\alpha}\sqrt{\frac{M\bar{\omega}}{2\hbar}}\right)z_1\right|$ at cyclotron frequency $\frac{\bar{\omega}+\bar{\omega}_c}{2}$ around the point $A$ at the same time.

$\times(-\mathrm{i}z_2)\Big|$ at cyclotron frequency $\dfrac{\bar{\omega}-\bar{\omega}_c}{2}$, the wave packet centroid of the coherent state follows the motion with radius $R_2 = \left|\left(\alpha\sqrt{\dfrac{2\hbar}{M\bar{\omega}}}-\dfrac{\theta}{2\alpha}\sqrt{\dfrac{M\bar{\omega}}{2\hbar}}\right)z_1\right|$ at cyclotron frequency $\dfrac{\bar{\omega}+\bar{\omega}_c}{2}$ around the point $A$ at the same time. For the system in a plane with radius $R$, the partition function can be approximately written as

$$
\begin{aligned}
&Z\\
&= \frac{4}{{l_1}^2{l_2}^2}\mathrm{e}^{-\frac{1}{2kT}\hbar\bar{\omega}}\int_0^\infty |z_2'|\exp\left[\left|\frac{z_2'}{l_2}\right|^2\left(\mathrm{e}^{-\frac{\hbar(\bar{\omega}-\bar{\omega}_c)}{2kT}}-1\right)\right]\\
&\quad \mathrm{d}|z_2'|\int_0^\infty |z_1'|\exp\left[\left|\frac{z_1'}{l_1}\right|^2\left(\mathrm{e}^{-\frac{\hbar(\bar{\omega}+\bar{\omega}_c)}{2kT}}-1\right)\right]\mathrm{d}|z_1'|, \quad (17)
\end{aligned}
$$

where $l_1 = \alpha\sqrt{\dfrac{2\hbar}{M\bar{\omega}}}-\dfrac{\theta}{2\alpha}\sqrt{\dfrac{M\bar{\omega}}{2\hbar}}$ and $l_2 = \alpha\sqrt{\dfrac{2\hbar}{M\bar{\omega}}}+\dfrac{\theta}{2\alpha}\sqrt{\dfrac{M\bar{\omega}}{2\hbar}}$. The result (17) is attributed to the following two considerations. 1) The coherent states with the centroid outside the plate are all excluded. The limitation to the sum over the other coherent states is equivalent to the cutoff of energy eigenstates used by Landau. 2) Because the radius $R \gg l_1, l_2$ and the exponential $\exp\left[\left|\dfrac{z_1'}{l_1}\right|^2\left(\mathrm{e}^{-\frac{\hbar(\bar{\omega}+\bar{\omega}_c)}{2kT}}-1\right)\right]$ and $\exp\left[\left|\dfrac{z_2'}{l_2}\right|^2\left(\mathrm{e}^{-\frac{\hbar(\bar{\omega}-\bar{\omega}_c)}{2kT}}-1\right)\right]$ fall off rapidly with $|z_1'|$ and $|z_2'|$ respectively, the integration region of $|z_1'|$ and $|z_2'|$ can be safely chosen from zero to infinity. So from the perspective of the partition function, the system with radius $R$ is approximately equivalent to the one with infinite side. Calculating Eq.(17), yields

$$
Z = \frac{1}{4\sinh\left[\dfrac{\hbar(\bar{\omega}+\bar{\omega}_c)}{4kT}\right]\sinh\left[\dfrac{\hbar(\bar{\omega}-\bar{\omega}_c)}{4kT}\right]}. \quad (18)
$$

In terms of the standard formulas for the free energy and the magnetization, $F = -nkT\ \ln Z$ and $M = -\dfrac{\partial F}{\partial B}$, the magnetization is given by

$$
\begin{aligned}
M = &-\frac{n\hbar}{2}\left(\frac{\alpha^2 e}{2cm}-\frac{Be^2\theta}{4c^2m\hbar}+\frac{e\theta\bar{\theta}}{8cm\alpha^2\hbar^2}+\frac{1}{\bar{\omega}}\left(\frac{B\alpha^4e^2}{2c^2m^2}-\frac{\alpha^2e\bar{\theta}}{2c\hbar m^2}-\frac{3B^2\alpha^2e^3\theta}{8c^3hm^2}+\frac{Be^2\theta\bar{\theta}}{2c^2\hbar^2m^2}-\frac{e\theta\bar{\theta}^2}{8c\hbar^3m^2\alpha^2}\right.\right.\\
&\left.\left.+\frac{B^3e^4\theta^2}{16c^4m^2\hbar^2}-\frac{3B^2e^3\theta^2\bar{\theta}}{32c^3\hbar^3m^2\alpha^2}+\frac{Be^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}-\frac{\alpha^2e\theta\omega_0^2}{2c\hbar}+\frac{Be^2\theta^2\omega_0^2}{4c^2\hbar^2}-\frac{e\omega_0^2\theta^2\bar{\theta}}{8c\hbar^3\alpha^2}\right)\right)\coth\left[\frac{1}{2kT}\left(\frac{\hbar\bar{\omega}}{2}+\frac{\hbar\bar{\omega}_c}{2}\right)\right]\\
&-\frac{n\hbar}{2}\left(-\frac{\alpha^2e}{2cm}+\frac{Be^2\theta}{4c^2m\hbar}-\frac{e\theta\bar{\theta}}{8cm\alpha^2\hbar^2}+\frac{1}{\bar{\omega}}\left(\frac{B\alpha^4e^2}{2c^2m^2}-\frac{\alpha^2e\bar{\theta}}{2c\hbar m^2}-\frac{3B^2\alpha^2e^3\theta}{8c^3hm^2}+\frac{Be^2\theta\bar{\theta}}{2c^2\hbar^2m^2}-\frac{e\theta\bar{\theta}^2}{8c\hbar^3m^2\alpha^2}\right.\right.\\
&\left.\left.+\frac{B^3e^4\theta^2}{16c^4m^2\hbar^2}-\frac{3B^2e^3\theta^2\bar{\theta}}{32c^3\hbar^3m^2\alpha^2}+\frac{Be^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}-\frac{\alpha^2e\theta\omega_0^2}{2c\hbar}+\frac{Be^2\theta^2\omega_0^2}{4c^2\hbar^2}-\frac{e\omega_0^2\theta^2\bar{\theta}}{8c\hbar^3\alpha^2}\right)\right)\\
&\times\coth\left[\frac{1}{2kT}\left(\frac{\hbar\bar{\omega}}{2}-\frac{\hbar\bar{\omega}_c}{2}\right)\right], \quad (19)
\end{aligned}
$$

and the susceptibility is

$$
\chi = -\frac{\hbar}{2}\left(-\frac{e^2\theta}{4c^2\hbar m}+\frac{1}{\bar{\omega}}\left(\frac{\alpha^4e^2}{2c^2m^2}-\frac{3B\alpha^2e^3\theta}{4C^3m^2\hbar}+\frac{e^2\theta\bar{\theta}}{2C^2m^2\hbar^2}+\frac{3B^2e^4\theta^2}{16C^4m^2\hbar^2}-\frac{3Be^3\theta^2\bar{\theta}}{16c^3\hbar^3m^2\alpha^2}+\frac{e^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}\right.\right.
$$

$$
\begin{aligned}
&+\frac{e^2\theta^2\omega_0^2}{4c^2\hbar^2}\Big)-\frac{2}{\bar{\omega}^3}\left(\frac{B\alpha^4e^2}{2c^2m^2}-\frac{\alpha^2e\bar{\theta}}{2c\hbar m^2}-\frac{3B^2\alpha^2e^3\theta}{8c^3hm^2}+\frac{Be^2\theta\bar{\theta}}{2c^2\hbar^2m^2}-\frac{e\theta\bar{\theta}^2}{8c\hbar^3m^2\alpha^2}+\frac{B^3e^4\theta^2}{16c^4m^2\hbar^2}-\frac{3B^2e^3\theta^2\bar{\theta}}{32c^3\hbar^3m^2\alpha^2}\right.\\
&\left.+\frac{Be^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}-\frac{\alpha^2e\theta\omega_0^2}{2c\hbar}+\frac{Be^2\theta^2\omega_0^2}{4c^2\hbar^2}-\frac{e\omega_0^2\theta^2\bar{\theta}}{8c\hbar^3\alpha^2}\right)^2\Big)\coth\left[\frac{1}{2kT}\left(\frac{\hbar\bar{\omega}}{2}+\frac{\hbar\bar{\omega}_{\rm c}}{2}\right)\right]\\
&-\frac{\hbar}{2}\left(\frac{e^2\theta}{4c^2\hbar m}+\frac{1}{\bar{\omega}}\left(\frac{\alpha^4e^2}{2c^2m^2}-\frac{3B\alpha^2e^3\theta}{4C^3m^2\hbar}+\frac{e^2\theta\bar{\theta}}{2C^2m^2\hbar^2}+\frac{3B^2e^4\theta^2}{16C^4m^2\hbar^2}-\frac{3Be^3\theta^2\bar{\theta}}{16c^3\hbar^3m^2\alpha^2}+\frac{e^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}+\frac{e^2\theta^2\omega_0^2}{4c^2\hbar^2}\right)\right.\\
&-\frac{2}{\bar{\omega}^3}\left(\frac{B\alpha^4e^2}{2c^2m^2}-\frac{\alpha^2e\bar{\theta}}{2c\hbar m^2}-\frac{3B^2\alpha^2e^3\theta}{8c^3hm^2}+\frac{Be^2\theta\bar{\theta}}{2c^2\hbar^2m^2}-\frac{e\theta\bar{\theta}^2}{8c\hbar^3m^2\alpha^2}+\frac{B^3e^4\theta^2}{16c^4m^2\hbar^2}-\frac{3B^2e^3\theta^2\bar{\theta}}{32c^3\hbar^3m^2\alpha^2}\right.\\
&\left.\left.+\frac{Be^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}-\frac{\alpha^2e\theta\omega_0^2}{2c\hbar}+\frac{Be^2\theta^2\omega_0^2}{4c^2\hbar^2}-\frac{e\omega_0^2\theta^2\bar{\theta}}{8c\hbar^3\alpha^2}\right)^2\right)\coth\left[\frac{1}{2kT}\left(\frac{\hbar\bar{\omega}}{2}-\frac{\hbar\bar{\omega}_{\rm c}}{2}\right)\right]\\
&+\frac{\hbar^2}{4kT}\left(\frac{\alpha^2e}{2cm}-\frac{Be^2\theta}{4c^2m\hbar}+\frac{e\theta\bar{\theta}}{8cm\alpha^2\hbar^2}+\frac{1}{\bar{\omega}}\left(\frac{B\alpha^4e^2}{2c^2m^2}-\frac{\alpha^2e\bar{\theta}}{2c\hbar m^2}-\frac{3B^2\alpha^2e^3\theta}{8c^3hm^2}+\frac{Be^2\theta\bar{\theta}}{2c^2\hbar^2m^2}-\frac{e\theta\bar{\theta}^2}{8c\hbar^3m^2\alpha^2}\right.\right.\\
&\left.\left.+\frac{B^3e^4\theta^2}{16c^4m^2\hbar^2}-\frac{3B^2e^3\theta^2\bar{\theta}}{32c^3\hbar^3m^2\alpha^2}+\frac{Be^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}-\frac{\alpha^2e\theta\omega_0^2}{2c\hbar}+\frac{Be^2\theta^2\omega_0^2}{4c^2\hbar^2}-\frac{e\omega_0^2\theta^2\bar{\theta}}{8c\hbar^3\alpha^2}\right)\right)^2\\
&\times\operatorname{csch}\left[\frac{1}{2kT}\left(\frac{\hbar\bar{\omega}}{2}+\frac{\hbar\bar{\omega}_{\rm c}}{2}\right)\right]^2+\frac{\hbar^2}{4kT}\left(-\frac{\alpha^2e}{2cm}+\frac{Be^2\theta}{4c^2m\hbar}-\frac{e\theta\bar{\theta}}{8cm\alpha^2\hbar^2}+\frac{1}{\bar{\omega}}\left(\frac{B\alpha^4e^2}{2c^2m^2}-\frac{\alpha^2e\bar{\theta}}{2c\hbar m^2}\right.\right.\\
&-\frac{3B^2\alpha^2e^3\theta}{8c^3hm^2}+\frac{Be^2\theta\bar{\theta}}{2c^2\hbar^2m^2}-\frac{e\theta\bar{\theta}^2}{8c\hbar^3m^2\alpha^2}+\frac{B^3e^4\theta^2}{16c^4m^2\hbar^2}-\frac{3B^2e^3\theta^2\bar{\theta}}{32c^3\hbar^3m^2\alpha^2}+\frac{Be^2\theta^2\bar{\theta}^2}{32c^2\hbar^4m^2\alpha^4}-\frac{\alpha^2e\theta\omega_0^2}{2c\hbar}\\
&\left.\left.+\frac{Be^2\theta^2\omega_0^2}{4c^2\hbar^2}-\frac{e\omega_0^2\theta^2\bar{\theta}}{8c\hbar^3\alpha^2}\right)\right)^2\operatorname{csch}\left[\frac{1}{2kT}\left(\frac{\hbar\bar{\omega}}{2}-\frac{\hbar\bar{\omega}_{\rm c}}{2}\right)\right]^2. \qquad (20)
\end{aligned}
$$

By setting $\alpha=1$ and $\bar{\theta}=0$, the magnetization and the susceptibility in NC space can be obtained quickly from expressions (19) and (20). Further, if we require $\theta=0$ in addition, the results (19) and (20) can be converted into expressions in commutative space.

# 4. Temperature limit

So far we have presented the general expressions of the magnetization $M$ and the susceptibility $\chi$ in NC phase space which are somewhat complicated. Here we consider two interesting cases, namely high temperature and zero temperature limits, and the results are more simple.

## 4.1. High temperature approximation

In this case the magnetization $M$ (19) can be approximated by

$$
\begin{aligned}
&M_{\rm NC\ phase\ space}\\
&=-\frac{B\hbar^2n\alpha^4e^2}{12c^2m^2kT}+\frac{B^2\hbar n\alpha^2e^3\theta}{16c^3m^2kT}\\
&\quad+\frac{ne\hbar\alpha^2\bar{\theta}}{12ckm^2T}+\frac{\hbar n\alpha^2e\theta\omega_0^2}{12ckT}\\
&=-\frac{B\hbar^2n\alpha^4e^2}{12c^2m^2kT}+\frac{B^2\hbar n\alpha^2e^3\theta}{16c^3m^2kT}\\
&\quad+\frac{ne\hbar\alpha^4(1-\alpha^2)}{3ckm^2T\theta}+\frac{\hbar n\alpha^2e\theta\omega_0^2}{12ckT}, \qquad (21)
\end{aligned}
$$

where we have only considered the first order modification from space–space non-commutativity and momentum–momentum non-commutativity and used the relation $\theta\bar{\theta}=4\alpha^2(1-\alpha^2)$. Thus, the magnetizations in NC space and commutative (C) space are respectively

$$
M_{\rm NC\ space}=-\frac{B\hbar^2ne^2}{12c^2m^2kT}+\frac{B^2\hbar ne^3\theta}{16c^3m^2kT}+\frac{\hbar ne\theta\omega_0^2}{12ckT},
$$

and

$$
M_{\rm C\ space}=-\frac{B\hbar^2ne^2}{12c^2m^2kT}. \qquad (22)
$$

According to expressions (21) and (22), we find that although the magnetizations in the three spaces are all proportional to the inverse of temperature $T$, magnetizations $M_{\rm NC\ space}$ and $M_{\rm NC\ phase\ space}$ depend on the frequency of the harmonic potential $\omega_0$ that does not appear in commutative space.

In terms of the formula $\chi=\frac{1}{n}\frac{\partial M}{\partial B}$, the susceptibility in commutative space is given by

$$
\chi_{\rm C\ space}=-\frac{\hbar^2e^2}{12c^2m^2kT}, \qquad (23)
$$

which is obviously applicable to the case of low frequency $\omega_0$ and consistent with the result in Ref.[21]. The susceptibilties in NC space and NC phase space

are respectively

$$\chi_{\text{NC space}} = -\frac{\hbar^2 e^2}{12c^2m^2kT} + \frac{B\hbar e^3\theta}{8c^3m^2kT},$$

and

$$\chi_{\text{NC phase space}} = -\frac{\hbar^2\alpha^4 e^2}{12c^2m^2kT} + \frac{B\hbar e^3\alpha^2\theta}{8c^3m^2kT}. \quad (24)$$

Although the systems in the three spaces all exhibit Landau diamagnetism at high temperatures, the dependence of susceptibility on the parameter of the system in the NC situation is different from that in the case of usual quantum mechanics and they are both related to magnetic field B.

## 4.2. Zero temperature limit

For the other interesting case, $T \to 0$, the magnetizations in commutative space, NC space and NC phase space can be expressed respectively as

$$M_{\text{C space}} \to -\frac{B\hbar ne^2}{2cm\sqrt{B^2e^2 + 4c^2m^2\omega_0^2}},$$

$$M_{\text{NC space}} \to \frac{-2Bc\hbar ne^2 + B^2ne^3\theta + 2c^2m^2ne\theta\omega_0^2}{4c^2m\sqrt{B^2e^2 + 4c^2m^2\omega_0^2}},$$

$$M_{\text{NC phase space}} \to \frac{-2Bc\hbar ne^2 + B^2ne^3\theta + 2c^2m^2ne\theta\omega_0^2}{4c^2m\sqrt{B^2e^2 + 4c^2m^2\omega_0^2}} + \frac{2c^2mne\bar{\theta}\omega_0^2}{\left(B^2e^2 + 4c^2m^2\omega_0^2\right)^{3/2}}$$

$$= \frac{-2Bc\hbar ne^2 + B^2ne^3\theta + 2c^2m^2ne\theta\omega_0^2}{4c^2m\sqrt{B^2e^2 + 4c^2m^2\omega_0^2}} + \frac{8c^2mne\alpha^2(1-\alpha^2)\omega_0^2}{\theta\left(B^2e^2 + 4c^2m^2\omega_0^2\right)^{3/2}}, \quad (25)$$

which retain only the first order modification from space–space non-commutativity and momentum–momentum non-commutativity. Notice that expression (25) is not applicable to the case of the frequency $\omega_0 \to 0$. For a normal but not special system, we find that in the magnetic field $B \to 0$, $M_{\text{C space}}$ tends to be zero and the system exhibits no magnetization characteristic in commutative space, but a similar phenomenon does not occur in the NC situation where the magnetizations are given by

$$M_{\text{NC space}} \to \frac{ne\theta\omega_0}{4c},$$

$$M_{\text{NC phase space}} \to \frac{ne\theta\omega_0}{4c} + \frac{ne\bar{\theta}}{4cm^2\omega_0} = \frac{ne\theta\omega_0}{4c} + \frac{ne\alpha^2(1-\alpha^2)}{\theta cm^2\omega_0}. \quad (26)$$

Further, we can obtain the susceptilities in the three spaces,

$$\chi_{\text{C space}} \to -\frac{2B^2\hbar e^4\omega_0^2}{c^4m^4\omega^5} - \frac{8\hbar e^2\omega_0^4}{c^2m^2\omega^5},$$

$$\chi_{\text{NC space}} \to -\frac{2B^2\hbar e^4\omega_0^2}{c^4m^4\omega^5} - \frac{8\hbar e^2\omega_0^4}{c^2m^2\omega^5} + \frac{B^5e^7\theta}{4c^7m^6\omega^5} + \frac{5B^3e^5\theta\omega_0^2}{2c^5m^4\omega^5} + \frac{6Be^3\theta\omega_0^4}{c^3m^2\omega^5},$$

$$\chi_{\text{NC phase space}} \to -\frac{2B^2\alpha^2\hbar e^4\omega_0^2}{c^4m^4\omega^5} - \frac{8\hbar\alpha^2e^2\omega_0^4}{c^2m^2\omega^5} + \frac{B^5e^7\theta}{4c^7m^6\omega^5} + \frac{5B^3e^5\theta\omega_0^2}{2c^5m^4\omega^5} + \frac{6Be^3\theta\omega_0^4}{c^3m^2\omega^5} - \frac{6Be^3\bar{\theta}\omega_0^2}{c^3m^4\omega^5}$$

$$= -\frac{2B^2\alpha^2\hbar e^4\omega_0^2}{c^4m^4\omega^5} - \frac{8\hbar\alpha^2e^2\omega_0^4}{c^2m^2\omega^5} + \frac{B^5e^7\theta}{4c^7m^6\omega^5} + \frac{5B^3e^5\theta\omega_0^2}{2c^5m^4\omega^5} + \frac{6Be^3\theta\omega_0^4}{c^3m^2\omega^5} - \frac{24Be^3\alpha^2(1-\alpha^2)\omega_0^2}{c^3m^4\theta\omega^5}, \quad (27)$$

where $\omega = \sqrt{\omega_c^2 + 4\omega_0^2}$. For the case of the magnetic field $B \to 0$, the susceptibilities of the normal system in the three spaces are $\chi_{\text{C space}} = \chi_{\text{NC space}} \to -\frac{\hbar e^2}{4c^2m^2\omega_0}$ and $\chi_{\text{NC phase space}} \to -\frac{\hbar e^2\alpha^2}{4c^2m^2\omega_0}$, separately. Thus the system in these spaces may all be inclined to display diamagnetism.

# 5. Conclusion and remarks

In this article, by use of the 'shift' method instead of doing tedious star product calculation, we obtain the energy eigenvalues and eigenstates of the Fock–Darwin System in NC space and NC phase space. Furthermore, utilizing the coherent state technique, we present the general expressions of the magnetization and the susceptibility in the NC situation as well as their corrections to those in the commutative

space as the consequence of the effects of space–space non-commutativity and momentum–momentum non-commutativity, and some interesting results which do not exist in commutative space, such as the magnetization depending on the frequency of the harmonic potential and the susceptibility related to the magnetic field at high temperatures, the emergence of the magnetization in zero magnetic field and zero temperatures limit, and so on. All of these results may help us to better verify the existence of NC space and NC phase space.

## References


[1] Seiberg N and Witten E 1999 *J. High Energy Phys.* **32** 9909

[2] Connes A, Douglas M R and Schwarz A 1998 *J. High Energy Phys.* **3** 9802

[3] Ardalan F, Arfaei H and Sheikh-Jabbari M M 1999 *J. High Energy Phys.* **16** 9902

[4] Curtright T, Fairlie D and Zachos C 1998 *Phys. Rev.* D **58** 25002
Mezincescu L 2000 hep-th/0007046.

[5] Chu S C and Ho P M 1999 *Nucl. Phys.* B **550** 151
Chu S C and Ho P M 2000 *Nucl. Phys.* B **568** 447

[6] Schomerus V 1999 *J. High Energy Phys.* **30** 9906

[7] Filk T 1996 *Phys. Lett.* B **376** 53

[8] Castellani Leonardo 2000 *Class. Quant. Grav.* **17** 3377

[9] Konechny A and Schwarz A 2002 *Phys. Rept.* **360** 353

[10] Nair V P 2001 *Phys. Lett.* B **505** 249
Nair V P and Polychronakos A P 2001 *Phys. Lett.* B **505** 267

[11] Chaichian M, Sheikh–Jabbari M M and Tureanu A 2001 *Phys. Rev. Lett.* **86** 2716

[12] Chaichian M, Presnajder P, Sheikh-Jabbari M M and Tureanu A 2002 *Phys. Lett.* B **527** 149

[13] Li K and Wang J H 2007 *Eur. Phys. J.* C **50** 1007

[14] Mirza B and Zarei M 2004 *Eur. Phys. J.* C **32** 583

[15] Li K, Sayipjamal D and Wang J H 2008 *Chin. Phys.* B **17** 1716

[16] Landau L D 1930 *Z. Phys.* **64** 629

[17] Lee B H, Moon K and Rim C 2001 *Phys. Rev.* D **64** 085014

[18] Basu B and Ghosh S 2005 *Phys. Lett.* A **346** 133
DayiÖ F and Jellal A 2001 *Phys. Lett.* A **287** 349
Li K, Cao X H and Wang D Y 2006 *Chin. Phys.* **15** 2236

[19] Fock V 1928 *Z. Phys.* **47** 446
Darwin C G 1930 *Proc. Cambridge Phil. Soc.* **27** 86

[20] Li K, Wang J H and Chen Chiyi 2005 *Mod. Phys. Lett.* A **20** 2165

[21] Feldman A and Kahn A H 1970 *Phys. Rev.* B **1** 4584