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Abundant exact solutions for a strong dispersion-managed system equation*

Li Hua-Mei(李画眉)[a)†], Li Yi-Shen(李翊神)[b)], and Lin Ji(林 机)[a)]

[a)] *Department of Physics, Zhejiang Normal University, Jinhua* 321004, *China*

[b)] *Department of Mathematics, University of Science and Technology of China, Hefei* 230026, *China*

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The generalized nonlinear Schrödinger equation (NLSE), which governs the dynamics of dispersion-managed (DM) solitons, is considered. A novel transformation is constructed such that the DM fibre system equation with optical loss (gain) is transformed to the standard NLSE under a restricted condition. Abundant new soliton and periodic wave solutions are obtained by using the transformation and the solutions of standard NLSE. Further, we discuss their main properties and the interaction scenario between two neighbouring solitons by using direct computer simulation.

Keywords: dispersion-managed soliton, transformation, exact solution
PACC: 0340K, 4265S, 0290

1. Introduction

Optical transmission with using a short optical pulse train is fundamental technology for achieving a high speed and long distance global network. Among many optical transmission formats, an optical soliton, which is based on the exact balance between the group velocity dispersion and the self-phase modulation, offers great potential to realize an advanced optical transmission system since the soliton pulse can maintain its waveform over a long distance.[1] An ideal soliton can exist only in a lossless fibre with constant dispersion. However, a real fibre has loss and varying dispersion, which severely degrades the quality of the soliton pulse. Recent studies have shown that a periodic dispersion compensation seems to be the most effective way of improving the optical transmission system.[2−16] Basically, in dispersion-management (DM) techniques, a transmission line with a periodic dispersion map is used, such that each period is composed of two types of fibres, generally with different lengths and opposing group velocity dispersion (GVD).[2−4] Because of the periodic splicing of anomalous and normal dispersion fibres, there is an abrupt discontinuity in the GVD of the DM fibre system. It is very difficult to obtain exact analytical solutions for DM solitons.[17]

Based on the general particle-like nature of solitons, Tchofo Dinda *et al*[18] presented a complete collective variable theory for DM solitons to gain more insight into their dynamical behaviour. Hasegawa *et al*[19] considered the lossless DM fibre system, and after removing the fast varying chirp part of the DM soliton they derived the averaged DM fibre system equation and obtained a chirped stationary solution. Nakkeeran[20] presented the Lax pair and derived exact soliton solutions by using the Bäcklund transformation for the averaged DM soliton equation with optical losses. Very recently Xu *et al*[21] used the Hasegawa approach to derive the averaged DM soliton equation for a DM fibre system with loss or gain. For a special case of such a system equation, they have derived exact soliton solutions by using the Darboux transformation (DT). In the present paper, we consider directly the DM fibre system equation with optical loss (gain). The rest of this paper is organized as follows. In Section 2, we present a novel transformation such that the DM fibre system equation with optical loss (gain) is transformed into the standard NLSE under a restricted condition. Therefore, once one solution of the NLSE is provided, we can immediately obtain one solution for the DM fibre system equation. By using the transformation and the solutions of the NLSE, we can obtain abundant soliton

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†Corresponding author. E-mail: lihuamei@zjnu.cn

and periodic wave solutions. As examples, soliton and Jacobian elliptic function solutions in explicit forms are generated. Then we further investigate their main properties and the interaction scenario between two neighbouring solitons by using direct computer simulation. The last section is devoted to a short summary and discussion.

2. Transformation and exact solutions of the DM fibre system equation

2.1. Transformation

The NLSE which governs the dynamics of the DM fibre system is given by

$$\mathrm{i}\frac{\partial u}{\partial z}+\frac{d(z)}{2}\frac{\partial^2 u}{\partial T^2}+|u|^2u+\mathrm{i}\,\Gamma(z)u=0, \tag{1}$$

where $u=u(z,T)$ is the envelope of the axial electric field, $d(z)$ is the periodically varying GVD parameter representing anomalous and normal dispersions, and $\Gamma(z)$ is the loss (gain) coefficient.

In order to obtain some solutions of Eq.(1), firstly we present a novel transformation

$$u(z,T)=\frac{r_0}{\sqrt{d(z)}}q(\xi,\tau)\exp\left(-2\int\Gamma(z)\mathrm{d}z+\mathrm{i}\,\Theta(z,T)\right),$$

$$\begin{aligned}\xi=&\frac{r_0}{d(z)}\exp\left(-2\int\Gamma(z)\mathrm{d}z\right)T\\&-r_0b_{10}\int\frac{\exp(-4\int\Gamma(z)\mathrm{d}z)}{d(z)}\mathrm{d}z+a_{20},\end{aligned}$$

$$\tau=r_0^2\int\frac{\exp(-4\int\Gamma(z)\mathrm{d}z)}{d(z)}\mathrm{d}z+G_0,$$

$$\begin{aligned}\Theta(z,T)=&\frac{\frac{\mathrm{d}}{\mathrm{d}z}d(z)+2d(z)\Gamma(z)}{2d(z)^2}T^2\\&+\frac{b_{10}\exp(-2\int\Gamma(z)\mathrm{d}z)}{d(z)}T\\&-\frac{b_{10}^2}{2}\int\frac{\exp(-4\int\Gamma(z)\mathrm{d}z)}{d(z)}\mathrm{d}z+b_0.\end{aligned} \tag{2}$$

$q(\xi,\tau)$ is a solution of the standard NLSE

$$\mathrm{i}\,q_\tau+\frac{1}{2}q_{\xi\xi}+|q|^2q=0, \tag{3}$$

where $q=q(\xi,\tau)$, r_0, G_0, a_{20}, b_{10} and b_0 are arbitrary constants, $d(z)$ and $\Gamma(z)$ must satisfy the condition

$$\begin{aligned}&2d(z)^2\frac{\mathrm{d}}{\mathrm{d}z}\Gamma(z)+4d(z)^2\Gamma(z)^2+2\left[\frac{\mathrm{d}}{\mathrm{d}z}d(z)\right]d(z)\Gamma(z)\\&+\left[\frac{\mathrm{d}^2}{\mathrm{d}z^2}d(z)\right]d(z)-[\frac{\mathrm{d}}{\mathrm{d}z}d(z)]^2=0.\end{aligned} \tag{4}$$

When $d(z)$ is an arbitrary constant, $\Gamma(z)=0$. If $d(z)$ is considered to be an arbitrary function, $\Gamma(z)$ is given by

$$\Gamma(z)=\frac{1}{2}\left[\frac{d(z)}{C_0+\int d(z)\mathrm{d}z}-\frac{\frac{\mathrm{d}}{\mathrm{d}z}d(z)}{d(z)}\right], \tag{5}$$

where C_0 is an arbitrary constant. Then the transformation is rewritten as follows:

$$u(z,T)=\frac{r_0}{\sqrt{C_0+\int d(z)\mathrm{d}z}}q(\xi,\tau)\exp\left[\mathrm{i}\,\Theta(z,T)+\frac{1}{2}\left(\frac{\frac{\mathrm{d}}{\mathrm{d}z}d(z)}{d(z)}-\frac{d(z)}{C_0+\int d(z)\mathrm{d}z}\right)\right],$$

$$\xi=\frac{1}{C_0+\int d(z)\mathrm{d}z}\left[r_0T+a_{20}\int d(z)\mathrm{d}z+a_{20}C_0+r_0b_{10}\right],$$

$$\tau=\frac{G_0\int d(z)\mathrm{d}z+G_0C_0-r_0^2}{C_0+\int d(z)\mathrm{d}z},$$

$$\begin{aligned}\Theta(z,T)=&-\frac{T^2}{2d(z)^2}\left[\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{d(z)}{C_0+\int d(z)\mathrm{d}z}\right)\left(C_0+\int d(z)\mathrm{d}z\right)-\frac{\mathrm{d}}{\mathrm{d}z}d(z)\right]\\&+\frac{b_{10}}{C_0+\int d(z)\mathrm{d}z}T+\frac{2b_0\int d(z)\mathrm{d}z+2b_0C_0+b_{10}^2}{2[C_0+\int d(z)\mathrm{d}z]}.\end{aligned} \tag{6}$$

Using the transformation (6) and the solutions of the standard NLSE (3), we can obtain soliton and periodic wave solutions.

2.2. Single-soliton solution of DM fibre system equation (1)

By using DT,[22] a single-soliton solution q_1 of Eq.(3) is given by[23]

$$q_1 = \frac{2k_1[\cos(2\beta_1) - \mathrm{i}\sin(2\beta_1)]}{\cosh(2\alpha_1)} \tag{7}$$

with $\alpha_1 = k_1(\xi+2v_1\tau)+\alpha_{10}, \beta_1 = v_1\xi+(v_1^2-k_1^2)\tau$ and here $q = q(\xi,\tau)$. If we give $d(z) = \cos(\omega z)$, then we obtain a single-soliton solution to Eq.(1) as follows:

$$u_{11} = 2k_1 r_0 \cos(\omega z)\left(\frac{\omega}{\omega C_0 + \sin(\omega z)}\right)^{3/2} \times \frac{\cos(2\beta_1) - \mathrm{i}\sin(2\beta_1)}{\cosh(2\alpha_1)} \exp(\mathrm{i}\,\Theta(z,T)) \tag{8}$$

with

$$\alpha_1 = \frac{k_1}{\omega C_0 + \sin(\omega z)}[\omega r_0 T + a_{20}\sin(\omega z) + \omega a_{20} C_0 + \omega r_0 b_{10} + 2v_1(G_0\sin(\omega z) + \omega G_0 C_0 - \omega r_0^2)] + \alpha_{10},$$

$$\beta_1 = \frac{1}{\omega C_0 + \sin(\omega z)}[v_1(\omega r_0 T + a_{20}\sin(\omega z) + \omega a_{20} C_0 + \omega r_0 b_{10}) + (v_1^2 - k_1^2)(G_0\sin(\omega z) + \omega G_0 C_0 - \omega r_0^2)],$$

$$\Theta(z,T) = \frac{1}{2(\omega C_0 + \sin(\omega z))}(\omega T^2 + 2\omega b_{10} T + 2b_0\sin(\omega z) + 2\omega b_0 C_0 + \omega b_{10}^2).$$

To our knowledge, this kind of solution has not been reported yet. In order to understand its main property, we plot the soliton solution to Eq.(8) in Fig.1.

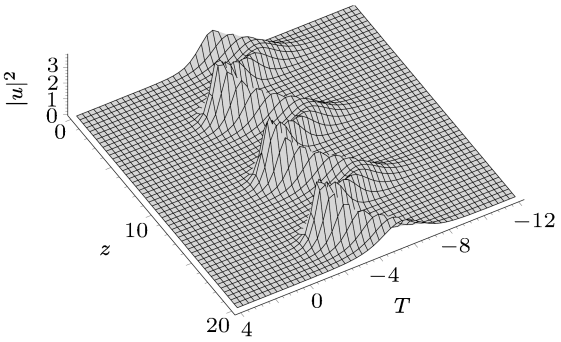


Fig.1. Evolution of a bright soliton expressed by $|u_{11}|^2$ with parameters $\omega = 1$, $r_0 = 1$, $G_0 = 1$, $C_0 = 2$, $b_{10} = 1$, $a_{20} = 1$, $k_1 = 1$, $v_1 = 0.4$, and $\alpha_{10} = 0$.

We find that the soliton amplitude, width and centre all vary periodically with z value. Those behaviours are clearly seen in Figs.2(a)–2(c).

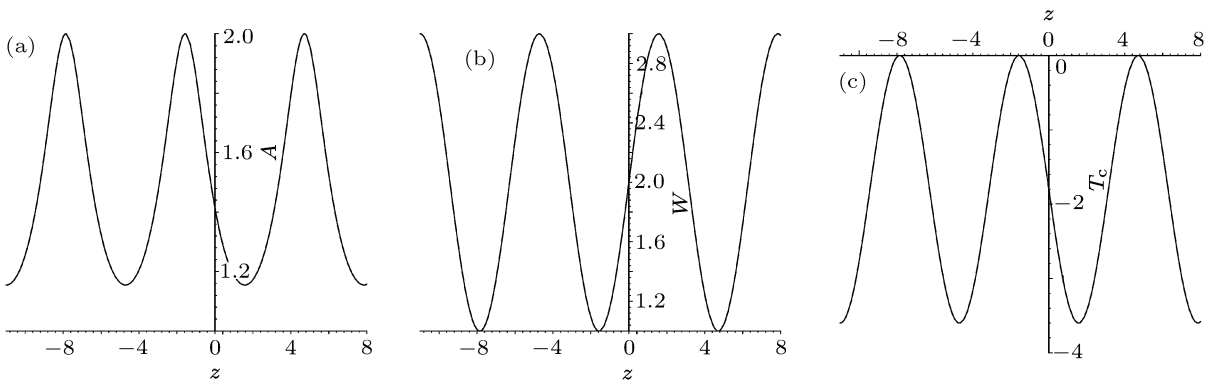


Fig.2. Dynamics of the bright soliton amplitude with parameters $\omega = 1$, $r_0 = 1$, $C_0 = 2$, and $k_1 = 1$ (a); dynamics of the bright soliton width with parameters $\omega = 1$, $r_0 = 1$, $C_0 = 2$, and $k_1 = 1$ (b); dynamics of the bright soliton centre with parameters $\omega = 1$, $r_0 = 1$, $G_0 = 1$, $C_0 = 2$, $b_{10} = 1$, $a_{20} = 1$, $k_1 = 1$, and $v_1 = 0.4$ (c).

If we give $d(z) = z$, we can obtain another single-soliton solution to Eq.(1) as follows:

$$u_{12} = 2k_1 r_0 z\left(\frac{2}{2C_0 + z^2}\right)^{3/2} \frac{\cos(2\beta_1) - \mathrm{i}\sin(2\beta_1)}{\cosh(2\alpha_1)} \exp(\mathrm{i}\,\Theta(z,T)) \tag{9}$$

with

$$\alpha_1 = \frac{k_1}{2C_0 + z^2}[2r_0T + a_{20}z^2 + 2a_{20}C_0 + 2r_0b_{10} + 2v_1(G_0z^2 + 2G_0C_0 - 2r_0^2)] + \alpha_{10},$$

$$\beta_1 = \frac{1}{2C_0 + z^2}[v_1(2r_0T + a_{20}z^2 + 2a_{20}C_0 + 2r_0b_{10}) + (v_1^2 - k_1^2)(G_0z^2 + 2G_0C_0 - 2r_0^2)],$$

$$\Theta(z,T) = \frac{1}{2C_0 + z^2}(T^2 + 2b_{10}T + b_0z^2 + 2b_0C_0 + b_{10}^2).$$

The solution to Eq.(9) is plotted in Fig.3. The properties of this soliton amplitude, width and centre in Fig.3 are different from those in Fig.1. Similarly, we can obtain many exact solutions of Eq.(1) since $d(z)$ is an arbitrary function of z.

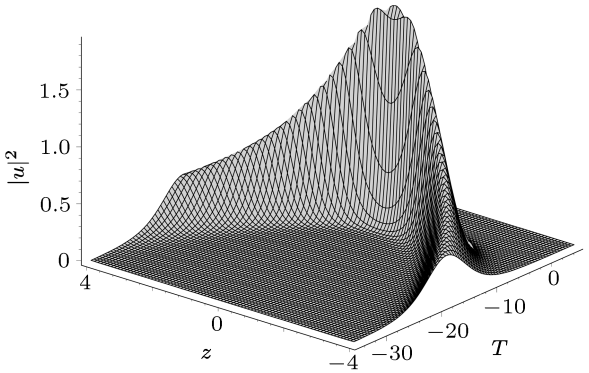


Fig.3. Evolution of a bright soliton expressed by $|u_{12}|^2$ with parameters $\omega = 1$, $r_0 = 1$, $G_0 = 1$, $C_0 = 2$, $b_{10} = 1$, $a_{20} = 1$, $k_1 = 1$, $v_1 = 0.4$, and $\alpha_{10} = 0$.

2.3. Double-soliton solution of DM fibre system equation (1)

Note that q_1 is generated by one-fold DT. It is natural to produce n-solition $q_n (n \geq 1$ is an integer) of the NLSE by n-fold DT, and further to provide u_n for Eq.(1) by using the transformation (6). To illustrate this more clearly, set $n = 2$; q_2 is given by[23]

$$q_2 = \frac{4[(w_1\cos(2\beta_1) + w_2\cos(2\beta_3)) - \mathrm{i}(w_1\sin(2\beta_1) + w_2\sin(2\beta_3))]}{-4k_1k_3\cos(\beta_1 - \beta_3) + w_3 + w_4} \tag{10}$$

of the NLSE with

$$\begin{aligned}
w_1 &= k_1[k_1^2 - k_3^2 + (v_1 - v_3)^2]\cosh(2\alpha_3) - 2\,\mathrm{i}\,k_1k_3(v_1 - v_3)\sinh(2\alpha_3),\\
w_2 &= k_3[-k_1^2 + k_3^2 + (v_1 - v_3)^2]\cosh(2\alpha_1) + 2\,\mathrm{i}\,k_1k_3(v_1 - v_3)\sinh(2\alpha_1),\\
w_3 &= [(k_1 - k_3)^2 + (v_1 - v_3)^2]\cosh[2(\alpha_1 + \alpha_3)],\\
w_4 &= [(k_1 + k_3)^2 + (v_1 - v_3)^2]\cosh[2(\alpha_1 - \alpha_3)],\\
\alpha_i &= k_i(\xi + 2v_i\tau_i) + \alpha_{i0},\\
\beta_i &= v_i\xi + (v_i^2 - k_i^2)\tau, \quad i = 1, 3.
\end{aligned}$$

Furthermore we achieve a new double-soliton solution with $d(z) = \cos(\omega z)$ for Eq.(1) as follows:

$$u_{21} = 4r_0\cos(\omega z)\left(\frac{\omega}{\omega C_0 + \sin(\omega z)}\right)^{3/2}\frac{[(w_1\cos(2\beta_1) + w_2\cos(2\beta_3)) - \mathrm{i}(w_1\sin(2\beta_1) + w_2\sin(2\beta_3))]}{-4k_1k_3\cos(\beta_1 - \beta_3) + w_3 + w_4}\exp(\mathrm{i}\,\Theta), \tag{11}$$

where w_1, w_2, w_3, and w_4 are obtained from expression (10) and

$$\alpha_i = \frac{k_i}{\omega C_0 + \sin(\omega z)}[\omega r_0T + a_{20}\sin(\omega z) + \omega a_{20}C_0 + \omega r_0b_{10} + 2v_i(G_0\sin(\omega z) + \omega G_0C_0 - \omega r_0^2)] + \alpha_{i0},$$

$$\beta_i = \frac{1}{\omega C_0 + \sin(\omega z)}[v_i(\omega r_0T + a_{20}\sin(\omega z) + \omega a_{20}C_0 + \omega r_0b_{10}) + (v_i^2 - k_i^2)(G_0\sin(\omega z) + \omega G_0C_0 - \omega r_0^2)], \quad i = 1, 3,$$

$$\Theta(z,T) = \frac{1}{2(\omega C_0 + \sin(\omega z))}(\omega T^2 + 2\omega b_{10}T + 2b_0\sin(\omega z) + 2\omega b_0C_0 + \omega b_{10}^2).$$

We plot the double-soliton solutions to Eq.(1) in Fig.4.

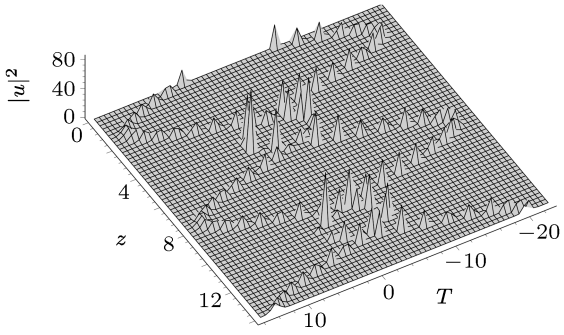


Fig.4. Interaction between two bright solitons expressed by $|u_{21}|^2$ with parameters. $\omega = 1$, $r_0 = 1$, $G_0 = 1$, $C_0 = 1.5$, $b_{10} = 1$, $a_{20} = 1$, $k_1 = 3$, $k_3 = 2$, $v_1 = 5$, $v_3 = -5$, $\alpha_{10} = 2$, and $\alpha_{30} = -2$.

If we give $d(z) = z$, we can obtain another double-soliton solution to Eq.(1) and plot it in Fig.5.

$$u_{22} = 4r_0 z\left(\frac{2}{2C_0 + z^2}\right)^{3/2} \frac{[(w_1\cos(2\beta_1) + w_2\cos(2\beta_3)) - \mathrm{i}(w_1\sin(2\beta_1) + w_2\sin(2\beta_3))]}{-4k_1k_3\cos(\beta_1 - \beta_3) + w_3 + w_4}\exp(\mathrm{i}\,\Theta), \tag{12}$$

where w_1, w_2, w_3, and w_4 are obtained from expression (10) and

$$\alpha_i = \frac{k_i}{2C_0 + z^2}[2r_0T + a_{20}z^2 + 2a_{20}C_0 + 2r_0b_{10} + 2v_i(G_0z^2 + 2G_0C_0 - 2r_0^2)] + \alpha_{i0},$$

$$\beta_i = \frac{1}{2C_0 + z^2}[v_i(2r_0T + a_{20}z^2 + 2a_{20}C_0 + 2r_0b_{10}) + (v_i^2 - k_i^2)(G_0z^2 + 2G_0C_0 - 2r_0^2)],\ i = 1, 3,$$

$$\Theta(z, T) = \frac{1}{2C_0 + z^2}(T^2 + 2b_{10}T + b_0z^2 + 2b_0C_0 + b_{10}^2).$$

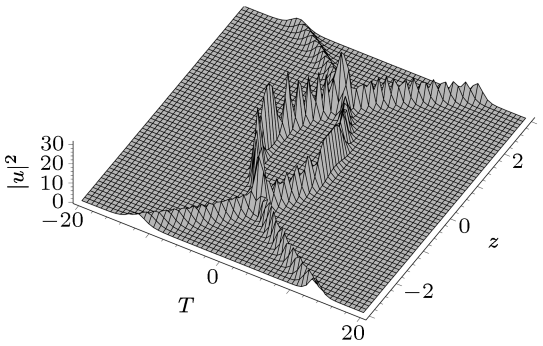


Fig.5. Interaction between two bright solitons expressed by $|u_{22}|^2$ with $\omega = 1$, $r_0 = 1$, $G_0 = 1$, $C_0 = 1.5$, $b_{10} = 1$, $a_{20} = 1$, $k_1 = 3$, $k_3 = 2$, $v_1 = 5$, $v_3 = -5$, $\alpha_{10} = 15$ and $\alpha_{30} = -15$.

2.4. Periodic wave solution of DM fibre system equation (1)

A Jacobian elliptic cn-function solution q_3 of Eq.(3) is given by

$$q_3 = A\mathrm{cn}\left(\frac{A}{m}\xi - \frac{AA_2}{m}\tau + A_1, m\right) \times \exp\left[\mathrm{i}\left(A_2\xi - \frac{A_2^2}{2}\tau - \frac{A^2(2m^2 - 1)}{2m^2}\tau + A_3\right)\right], \tag{13}$$

where m is a modulus of the Jacobian elliptic function and A, A_1, A_2, and A_3 are arbitrary constants. If we give $d(z) = \cos(\omega z)$, then we obtain a Jacobian elliptic function solution to Eq.(1) as follows:

$$u_{31} = Ar_0\cos(\omega z)\left(\frac{\omega}{\omega C_0 + \sin(\omega z)}\right)^{3/2} \times \mathrm{cn}\left(\frac{A}{m}\xi - \frac{AA_2}{m}\tau + A_1, m\right) \times \exp\left[\mathrm{i}\left(A_2\xi - \frac{A_2^2}{2}\tau - \frac{A^2(2m^2 - 1)}{2m^2}\tau + A_3 + \Theta(z, T)\right)\right] \tag{14}$$

with

$$\xi = \frac{\omega r_0T + a_{20}\sin(\omega z) + \omega a_{20}C_0 + \omega r_0b_{10}}{\omega C_0 + \sin(\omega z)},$$

$$\tau = \frac{G_0\sin(\omega z) + \omega G_0C_0 - \omega r_0^2)}{\omega C_0 + \sin(\omega z)},$$

$$\Theta(z,T) = \frac{1}{2(\omega C_0 + \sin(\omega z))}(\omega T^2 + 2\omega b_{10} T + 2b_0 \sin(\omega z) + 2\omega b_0 C_0 + \omega b_{10}^2).$$

We plot the Jacobian elliptic function solution to Eq.(1) in Fig.6.

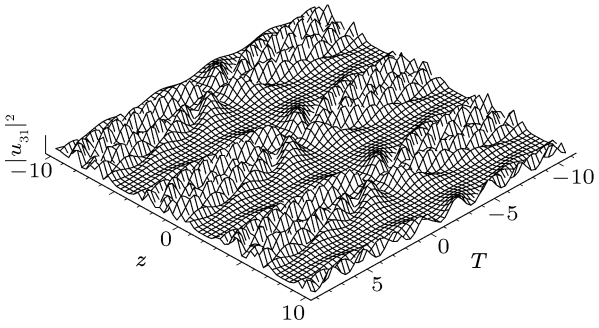


Fig.6. Evolution of a periodic wave solution expressed by $|u_{31}|^2$ with parameters $B_0 = 0.2$, $W_0 = 1$, and $X_0 = 2$.

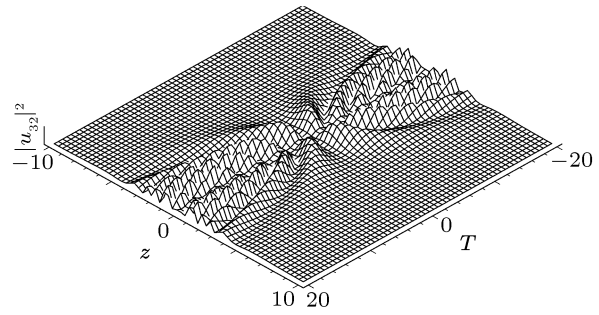


Fig.7. Evolution of a periodic wave solution expressed by $|u_{32}|^2$ with parameters $B_0 = 0.2$, $W_0 = 1$, and $X_0 = 2$.

If we give $d(z) = z$, we can obtain another Jacobian elliptic function solution to Eq.(1) and plot it in Fig.7.

3. Summary

We have considered directly the DM fibre system equation with optical loss (gain) instead of the averaged DM fibre system equation. We have presented a novel transformation such that the DM fibre system equation with optical loss (gain) is transformed into the standard nonlinear NLSE under a restricted condition. By using the transformation and the solutions of the NLSE, we have obtained abundant soliton and periodic wave solutions. For instance, one-soliton, two-soliton and Jacobian elliptic function solutions are generated. To the best of our knowledge, these solutions have not been reported yet. Furthermore we have investigated their main properties and the interaction scenario between two neighbouring solitons by using direct computer simulation. We have found that the soliton amplitude, width and centre all vary with GVD parameter $d(z)$ value. Hence, we believe that the novel bright soliton solutions reported here can be used for propagating the 'sech' form of pulses in the DM fibre system equation considering optical loss (gain).

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