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Traffic of indistinguishable particles in complex networks*

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In this paper, we apply a simple walk mechanism to the study of the traffic of many indistinguishable particles in complex networks. The network with particles stands for a particle system, and every vertex in the network stands for a quantum state with the corresponding energy determined by the vertex degree. Although the particles are indistinguishable, the quantum states can be distinguished. When the many indistinguishable particles walk randomly in the system for a long enough time and the system reaches dynamic equilibrium, we find that under different restrictive conditions the particle distributions satisfy different forms, including the Bose–Einstein distribution, the Fermi–Dirac distribution and the non-Fermi distribution (as we temporarily call it). As for the Bose–Einstein distribution, we find that only if the particle density is larger than zero, with increasing particle density, do more and more particles condense in the lowest energy level. While the particle density is very low, the particle distribution transforms from the quantum statistical form to the classically statistical form, i.e., transforms from the Bose distribution or the Fermi distribution to the Boltzmann distribution. The numerical results fit well with the analytical predictions.

Keywords: complex networks, statistical mechanics of networks

PACC: 0250, 0530, 0175, 0270

1. Introduction

The subject of complex networks[1] has attracted much attention from statistical physicists since the small-world model[2] was put forward by Watts and Strogatz in 1998. Besides the small-world model, another important model was put forward by Albert and Barabási in 1999, which was called the scale-free model or the BA model.[3] In the BA model, the number of neighbours among vertices shows inhomogeneous distribution, which satisfies the power-law form. The study of complex networks has penetrated into various areas, including physical, social, biological ones,[1,4] etc. As for the Internet, which is composed of computers and cables, the computers can be considered as vertices in the network with the cables as edges connecting the vertices. The propagation of information packets among computers can be considered as the traffic of many particles in the network. Because of the finite capacity of every computer, congestion often takes place. One way of quantifying the effects of congestion is by measuring the betweenness centrality.[5−10] Based on this idea, cascades from the failure of a single vertex have already been extensively studied.[11−15] Because macroscopic particles like the information packets are distinguishable, de Moura proposed a simple model[16] to study the traffic of many distinguishable particles in complex networks. He found that if there was at most one particle occupying a vertex, after all the particles walked randomly for a long enough time and the system reached dynamic equilibrium, the particle distribution satisfied the Fermi–Dirac form. Additionally, Germano and de Moura[17] studied the case that there were a finite number of particles occupying a vertex in the classical statistical manner by the method of finding the grand-canonical partition function of the system.[17] They found if the number of particles in a vertex was not restricted, the particle distribution should follow the Boltzmann form; if there was at most one particle occupying a vertex, the particle distribution would return to the Fermi–Dirac form. As we know, one premise of Bose–Einstein distribution or Fermi–Dirac distribution is that particles must be indistinguishable, so in this paper we base our work on the premise that particles are indistinguishable. Since there are many similarities between the complex network and the particle system, e.g., vertices in the network can be considered as spin particles, and edges as interactions, the phenomena of phase transitions can

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be observed when applying the Ising model[18,19] or XY model[20,21] to complex networks. Alternatively, with the same edge-dual method, the phenomenon of critical behaviours in the Ising model when treated with a mean-field method[22] and that of the Bose–Einstein condensation[23] can be observed. In this paper we observe the phenomenon of Bose–Einstein condensation also, but it is very different from the Bose–Einstein condensation in Ref.[23], which will be discussed in Section 4 in detail. Because the theory of Bose statistics or Fermi statistics is ubiquitous, for many particles walking randomly in the network, under specific restrictive conditions, the particle distribution of Bose–Einstein form or Fermi–Dirac form should be observed. By analogy to the work done by de Moura *et al*,[16,17] we propose a simple model to find the distributions followed by many indistinguishable particles after walking randomly in the complex network for a long enough time and with the system reaching a dynamic equilibrium.

This paper is organized as follows. Section 2 is the introduction to the mechanism of the random walk of the many indistinguishable particles and the corresponding restrictive conditions, along with definitions of the quantum state and the corresponding energy. We also find there are different distributions under different restrictive conditions, including the Bose–Einstein distribution, the Fermi–Dirac distribution and the non-Fermi distribution. In Section 3, we carry out numerical simulations to verify our analytical predictions and find the particle distributions that really satisfy different quantum statistical forms under different restrictive conditions. In Section 4, in the Bose distribution, we will find out under what conditions the phenomenon of Bose–Einstein condensation can be observed. Also, when the particle density is very low, the particle distribution may transform from the quantum statistical form to the classically statistical form, i.e., the particle distributions transform from the Bose distribution and the Fermi distribution to the Boltzmann distribution. The numerical results fit well with the analytical predictions. Section 5 is the conclusion of this paper.

2. Model and statistical distributions

In this paper, we choose the BA model[3] to carry out investigations. Every vertex in the network stands for a quantum state, with the corresponding energy determined by the vertex degree. Initially, there are many indistinguishable particles randomly distributed in the quantum states. Although the particles are indistinguishable, the quantum states can be distinguished. In the following we will give the relations between the complex network and the particle system, and the definitions of the relative parameters.

First we construct a BA network[3] with N vertices as follows. Starting with a few well connected vertices, we carry out at every step two procedures: A new vertex is added to the network with initial degree m_0. Next, the new vertex selects m_0 existing vertices in the network, each of which is attached to it with probability proportional to the degree k_i,

$$\Pi_i = \frac{k_i}{\sum\limits_j k_j}, \tag{1}$$

where the sum is done over all vertices in the network. Repeat the above two procedures until the network's size is in conformity with the requirement. In this paper the energy of a vertex is expressed as[16]

$$\beta\varepsilon_i = -\ln\left(Ck_i\right), \tag{2}$$

where β is the inverse of temperature, and $\beta\varepsilon_i$ cannot be separated, so in this paper we do not consider the effect of temperature. C is a constant which will be determined later. Thus we have

$$k_i = C_0 \mathrm{e}^{-\beta\varepsilon_i}. \tag{3}$$

From graph theory, for a single distinguishable particle walking randomly in the network for a long enough time, the time or the probability it is found in a vertex is proportional to the vertex degree[24]

$$q_i = \frac{k_i}{\sum\limits_j k_j}. \tag{4}$$

Compared with Eq.(3), the probability of a particle found in quantum state i is

$$q_i \propto \mathrm{e}^{-\beta\varepsilon_i}, \tag{5}$$

where $\mathrm{e}^{-\beta\varepsilon_i}$ is the Boltzmann factor.

Now we deal with the particle walking randomly in the network in another way, i.e., consider the particle to be indistinguishable. At each step we choose randomly a vertex having particles in it and reduce the number of particles by one, and we also choose randomly an outgoing edge of the vertex, along which the reduced particle walks to a neighbouring vertex. Since there is only one particle walking in the network, the

case is equivalent to a distinguishable particle walking randomly in the network, and the probability of the particle found in quantum state i is given by Eq.(5).

Next, in the case with many indistinguishable particles in the system, we will consider what distributions the particles will become after they walk randomly for a long enough time and the system reaches a dynamic equilibrium. Initially, there are $n\,(n \gg 1)$ particles distributed randomly among the N vertices of the network. Also, there are three cases for the particles occupying a vertice: infinite particles occupying a vertex, at most one particle occupying a vertex and at most $m\,(m > 1)$ particles occupying a vertex. Then at each step we choose randomly a vertex with particles in it, and reduce the number of particles by one. Then we choose randomly an outgoing edge of the vertex, so we can choose a neighbouring vertex attached to this edge. Here we have to discuss different situations. If there is no restriction on the number of particles occupying a vertex, we increase the number of particles in the neighbouring vertex by one. If there is at most one particle occupying a vertex or at most $m\,(m > 1)$ particles occupying a vertex and the number of particles is not saturated, we can increase the number of particles in the vertex by one. If the number of particles is saturated, we abandon the particle walking and restart another choice of vertex and particle walking. Repeat the above process until the system reaches dynamic equilibrium. We analyse the random walk of many indistinguishable particles in the system in the viewpoint of quantum statistics. We consider the system as an open system, in which the number of particles is not fixed. The probability for an indistinguishable particle walking into quantum state i is proportional to $\mathrm{e}^{-\beta(\varepsilon_i-\mu)}$, where $\beta\varepsilon_i$ is given by Eq.(2), and μ is the chemical potential determined by the number of particles or the particle density in the system. It is well known that for the indistinguishable particle system reaching dynamic equilibrium, if every quantum state can support an infinite number of particles, the particle distribution should satisfy the Bose–Einstein distribution

$$\langle n_i\rangle = \frac{1}{\mathrm{e}^{\beta(\varepsilon_i-\mu)}-1}, \tag{6}$$

where $\langle n_i\rangle$ is the average number of particles in quantum state i. Using Eq.(3), we obtain the average number of particles in vertex i with degree k_i as follows:

$$\langle n_i\rangle = \frac{1}{A/k_i-1}, \tag{7}$$

where $A = C_0\mathrm{e}^{-\beta\mu}$, determined by the number of particles or particle density in the system and will be given later. As for the BA network, the number of vertices with degree k is $NP\,(k)$, where $P\,(k) \sim 2m_0^2k^{-\gamma}, \gamma = 3$, with m_0 the initial degree of each vertex.[3] Incorporating this with Eq.(7), we obtain the restrictive condition[16]

$$\sum_{k=m_0}^{k_{\max}} \frac{P\,(k)}{A/k-1} = \nu, \tag{8}$$

where $\nu = n/N$ is the ratio of n particles to N vertices, i.e., particle density. Because the network investigated is of a finite size, we may find a proper value, i.e. the maximum degree $k_{\max}$ of the network, from theoretical analysis, which satisfies $k_{\max} \sim N^{\frac{1}{\gamma-1}}$.[25] But from theoretical analysis we cannot find an exact value of the maximum degree $k_{\max}$ of the BA network, so we have to turn to the numerical method, the exact value of which will be given in Section 3.

When the indistinguishable particle system reaches dynamic equilibrium, if each vertex can support at most one particle, the particle distribution should satisfy the Fermi–Dirac distribution

$$\langle n_i\rangle = \frac{1}{\mathrm{e}^{\beta(\varepsilon_i-\mu)}+1}. \tag{9}$$

From Eq.(3), we obtain the average number of particles in vertex i with degree k_i as follows:

$$\langle n_i\rangle = \frac{1}{A/k_i+1}, \tag{10}$$

under the corresponding restrictive condition

$$\sum_{k=m_0}^{k_{\max}} \frac{P\,(k)}{A/k+1} = \nu. \tag{11}$$

Finally, if each vertex can support at most $m\,(m > 1)$ particles, we may let $m = 3$ as well. The particle distribution should satisfy the non-Fermi distribution (as we call it temporarily). Considering the degeneracy ϖ_l of energy level l, with ϖ_l equal the number of vertices with degree k_l, which is equal to $NP\,(k_l)$, we have the partition function of energy level l

$$\begin{aligned}\Xi_l &= \sum_{a_{l1}=0}^{3}\cdots\sum_{a_{li}=0}^{3}\cdots\sum_{a_{l\varpi_l}}^{3}\mathrm{e}^{-\beta(\varepsilon_l-\mu)\left(a_{l1}+\cdots a_{li}+\cdots a_{l\varpi_l}\right)}\\ &= \left(\sum_{a_{l1}=0}^{3}\mathrm{e}^{-\beta(\varepsilon_l-\mu)a_{l1}}\right)\cdots\left(\sum_{a_{li}=0}^{3}\mathrm{e}^{-\beta(\varepsilon_l-\mu)a_{li}}\right)\\ &\quad\cdots\left(\sum_{a_{l\varpi_l}=0}^{3}\mathrm{e}^{-\beta(\varepsilon_l-\mu)a_{l\varpi_l}}\right)\end{aligned}$$

$$= \left(1 + \mathrm{e}^{-\beta(\varepsilon_l-\mu)} + \mathrm{e}^{-2\beta(\varepsilon_l-\mu)} + \mathrm{e}^{-3\beta(\varepsilon_l-\mu)}\right)^{\varpi_l}, \tag{12}$$

where a_{li} is the number of particles in quantum state i, which is one of the ϖ_l degenerate states of energy level l. So we obtain the average number of particles in energy level l as follows:

$$\begin{aligned}\langle a_l\rangle &= \frac{1}{\beta}\frac{\partial \ln \Xi_l}{\partial \mu} \\ &= \frac{\varpi_l\left(\mathrm{e}^{-\beta(\varepsilon_l-\mu)} + 2\mathrm{e}^{-2\beta(\varepsilon_l-\mu)} + 3\mathrm{e}^{-3\beta(\varepsilon_l-\mu)}\right)}{1 + \mathrm{e}^{-\beta(\varepsilon_l-\mu)} + \mathrm{e}^{-2\beta(\varepsilon_l-\mu)} + \mathrm{e}^{-3\beta(\varepsilon_l-\mu)}}.\end{aligned} \tag{13}$$

The average number of particles in quantum state i is

$$\langle n_i\rangle = \frac{\langle a_l\rangle}{\varpi_l} = \frac{\mathrm{e}^{-\beta(\varepsilon_i-\mu)} + 2\mathrm{e}^{-2\beta(\varepsilon_i-\mu)} + 3\mathrm{e}^{-3\beta(\varepsilon_i-\mu)}}{1 + \mathrm{e}^{-\beta(\varepsilon_i-\mu)} + \mathrm{e}^{-2\beta(\varepsilon_i-\mu)} + \mathrm{e}^{-3\beta(\varepsilon_i-\mu)}}, \tag{14}$$

which is equivalent to

$$\begin{aligned}\langle n_i\rangle &= \frac{k_i/A + 2\left(k_i/A\right)^2 + 3\left(k_i/A\right)^3}{1 + k_i/A + \left(k_i/A\right)^2 + \left(k_i/A\right)^3} \\ &= \frac{k_iA^2 + 2k_i^2A + 3k_i^3}{A^3 + k_iA^2 + k_i^2A + k_i^3}.\end{aligned} \tag{15}$$

It satisfies the restrictive condition

$$\sum_{k=m_0}^{k_{\max}} \frac{\left(kA^2 + 2k^2A + 3k^3\right)P\left(k\right)}{A^3 + kA^2 + k^2A + k^3} = \nu. \tag{16}$$

3. Numerical simulations

In the following we verify our analytical predictions with numerical simulations. First, for the Bose–Einstein distribution, we assume particles in the system are indistinguishable, and each quantum state can support an infinite number of particles. In the network, we let $N = 5 \times 10^3, \nu = 0.6, m_0 = 3$, incorporated with the numerical result and the restrictive condition Eq.(8), along with a proper choice of constant B satisfying $P\left(k\right) = Bk^{-3}$, then we have $A \approx 199.0006, B = 13.5$. As for the degree distribution $P\left(k\right)$ in the BA model, it is only a mean-field result,[3] so our choice of constant B is acceptable. Substituting A in Eq.(7), we obtain the analytical prediction of the average number of particles in each vertex. Because the number of particles in each vertex should not be negative, we gain the mean value of the maximum degree of the network $k_{\max} = 199 < A$, satisfying $k_{\max} \sim N^{\frac{1}{\gamma-1}}$.[25] Considering the small-world effect of the BA network[26] and only selecting vertices with particles randomly at every step, for each network realization we execute 5×10^5 steps of particle walking. The numerical result is as follows (see Fig.1). When the system reaches dynamic equilibrium, we see the numerical result of particle distribution fits well with the analytical prediction with increasing vertex degree. So it confirms that the choice of $k_{\max}$ as the maximum degree is correct.

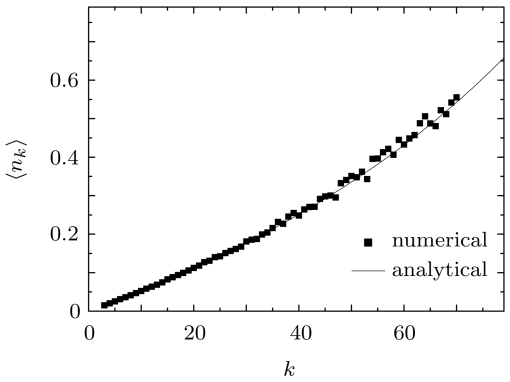


Fig.1. A plot of $\langle n_k\rangle$ versus k for particles in the Bose–Einstein distribution. $N = 5 \times 10^3, \nu = 0.6, m_0 = 3$. The solid squares are the numerical results while the solid line is the analytical prediction. The numerical result is the average of 5×10^3 different network realizations. The number of particles each vertex can support is infinite. We see that the numerical result of the average particles $\langle n_k\rangle$ in the vertex fits well with the curve of analytical prediction versus k.

Next we consider the Fermi–Dirac distribution. We also suppose that particles in the system are indistinguishable, and each quantum state can support at most one particle. In the network we still keep $N = 5 \times 10^3$, $\nu = 0.6, m_0 = 3$, $k_{\max} = 199$ and $P\left(k\right) = 13.5 \times k^{-3}$ as before. Under the restrictive condition (11), we obtain $A \approx 3.004$. Substituting A in Eq.(10), we gain the analytical prediction of the average number of particles in each vertex. After a long enough time when the system reaches dynamic equilibrium, we see that the numerical result fits well with the analytical prediction (see Fig.2).

Finally, we consider the non-Fermi distribution. Particles in the system are indistinguishable, and each quantum state can support at most $m\,(m = 3)$ particles. In the network we keep the same constants $N = 5 \times 10^3, \nu = 0.6, m_0 = 3$, and $k_{\max} = 199$ and $P\left(k\right) = 13.5 \times k^{-3}$ as before. Using the restrictive condition (16), we obtain $A \approx 12.74$, with the analytical prediction of the average number of particles in each vertex given by Eq.(15). In Fig.3, we see that when the system reaches dynamic equilibrium, the numerical result fits well with the analytical prediction.

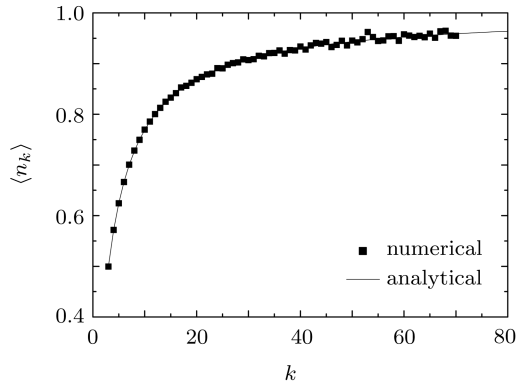


Fig.2. A plot of $\langle n_k \rangle$ versus k for particles in Fermi–Dirac distribution. $N = 5 \times 10^3, \nu = 0.6, m_0 = 3$. The solid squares are the numerical results while the solid line is the analytical prediction. The numerical result is obtained by taking the average of 3×10^3 different network realizations. There is at most one particle occupying a vertex.

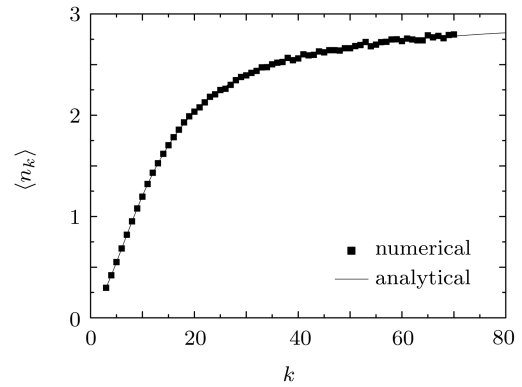


Fig.3. The $\langle n_k \rangle$ versus k plot for non-Fermi distribution (the temporary name). $N = 5 \times 10^3, \nu = 0.6, m_0 = 3$. The solid squares are the numerical results while the solid line is the analytical prediction. The numerical result is obtained by taking the average of 3×10^3 different network realizations. Each vertex can support at most three particles.

4. Bose–Einstein condensation and the transition from quantum statistics to classical statistics

In Section 3, we have let the particle density $\nu = 0.6$, in order that the quantum effect could be easily seen. In this paper, we do not consider the effect of temperature (see Eq.(2)). In addition, we consider that if the particle density is very low, the particle distribution should transform from the quantum statistical form to the classical statistical form, i.e., from the Bose distribution or the Fermi distribution to the Boltzmann distribution. As we do not take into account the effect of temperature, for the Bose distribution, we assume there is a critical particle density $\nu_{\rm c}$, beyond which the phenomenon of Bose–Einstein condensation could be observed. As the particle density satisfies $0 < \nu < +\infty$, based on Eq.(8), we obtain

$$\lim_{A \to +\infty} \sum_{k=m_0}^{k_{\max}} \frac{P(k)}{A/k - 1} = 0, \tag{17}$$

and

$$\lim_{A \to k_{\max}} \sum_{k=m_0}^{k_{\max}} \frac{P(k)}{A/k - 1} = +\infty. \tag{18}$$

So the relationship between A and the particle density ν is a monotonically decreasing function. From Eq.(7), for the number of particles in the vertex with the maximum degree, we have

$$\lim_{A \to k_{\max}} \langle n_{\max} \rangle = \lim_{A \to k_{\max}} \frac{1}{A/k_{\max} - 1} = +\infty. \tag{19}$$

Thus with increasing number of particles, more and more particles would occupy the lowest energy level. From Eq.(8), the total number of particles in other energy levels is given by

$$n_0 = \lim_{A \to k_{\max}} N \sum_{k=m_0}^{k_{\max}-1} \frac{P(k)}{A/k - 1} \approx 0.03N, \tag{20}$$

where we assume the degree distribution $P(k)$ is given as before. We can see that most particles accumulate in the lowest energy level. Because the BA network[3] investigated is of finite size, there are a few vertices with very large degrees in the network. For the random walk of particles, we choose randomly a vertex with particles at each step, and the probability for a particle walking to a vertex is proportional to the vertex degree. After a long enough random walk time, most particles should accumulate in those vertices with a very large degree. But by choosing different particle densities, we find that the constants A and B must be changed. Only for the particle density $\nu > 0$ will a portion of particles accumulate in those vertices with a very large degree in the network. Also, with increasing particle density, the ratio of particles in the vertices with very large degree to the total particles increases accordingly (see Fig.4).

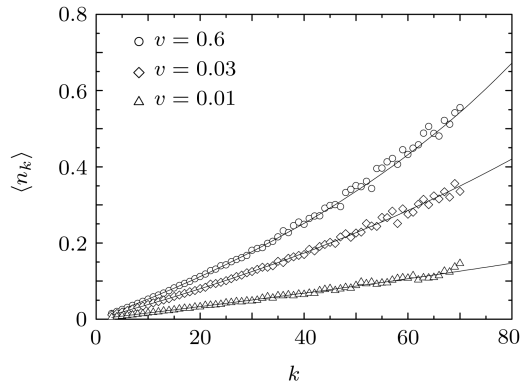


Fig.4. A plot of $\langle n_k \rangle$ for Bose–Einstein distribution. $N = 5 \times 10^3, m_0 = 3$. The numerical result is given by taking the average of 5×10^3 different network realizations. The number of particles each vertex can support is infinite.

Next, we analyse the differences in Bose–Einstein condensation between the result in this paper and that in Ref.[23]. In Ref.[23], the edges are mapped to particles and the structure of the network changes with temperature. So we can say the dynamics dictates the topology. When the temperature is lower than a critical value, the phenomenon of Bose–Einstein condensation can be observed. That is, a fraction of the total edges will attach to one vertex of the network, while in this paper, many indistinguishable particles walk randomly in the network with the network structure unchanged. That is to say, the topology predetermines the dynamics. Besides, in this paper the temperature is not considered (of Eq.(2)). Only if the particle density $\nu > 0$, can the phenomenon of Bose–Einstein condensation be observed. That is, nearly all the particles occupy the lowest energy level. In Ref.[23], the particle distribution is static, i.e., when particles reach an energy level, they stay there. Besides, the ratio of particles in the lowest energy level to the total particles is at most 1/2, while in this paper, the particle distribution is dynamic, i.e., particles keep walking among different energy levels. With increasing particle density, this ratio in the system goes to unity. In addition, the lowest energy level in Ref.[23] is of the vertex with the largest degree, while in this paper, the lowest energy level or the vertex with the largest degree $k_{\max}$ is only an average value determined by theoretical prediction.

It is well known that for the indistinguishable particle system if the temperature is very high and the particle density is very low, the description of particle distribution should be transformed from the quantum statistical form to the classical statistical form. Since in this paper the effect of temperature is neglected, through the analysis of Eqs.(17) and (18), with decreasing particle density ν, A will increases. When A is large enough, from Eq.(7), the Bose distribution is expressed as

$$\langle n_i \rangle = \frac{1}{A/k_i - 1} \approx \frac{k_i}{A} = \mathrm{e}^{-\beta(\varepsilon_i - \mu)}, \qquad (21)$$

while from Eq.(10) the Fermi distribution can be expressed as

$$\langle n_i \rangle = \frac{1}{A/k_i + 1} \approx \frac{k_i}{A} = \mathrm{e}^{-\beta(\varepsilon_i - \mu)}. \qquad (22)$$

So the Bose distribution and the Fermi distribution all change into the Boltzmann distribution. In Fig.4, for $\nu = 0.6, 0.03, 0.01$, along with a proper choice of A and B, we find that the particle distribution really satisfies the Bose distribution (of Eq.(7)). Also, with increasing A, the curve changes into a straight line gradually, i.e., the particle distribution changes from the Bose form into the Boltzmann form gradually. In Fig.5, for $\nu = 0.6, 0.03, 0.01$, with a proper choice of A and B, we see that the particle distribution still satisfies the Fermi form (of Eq.(10)), and with increasing A the curve changes into a straight line gradually, i.e., the particle distribution changes from the Fermi distribution into the Boltzmann distribution gradually.

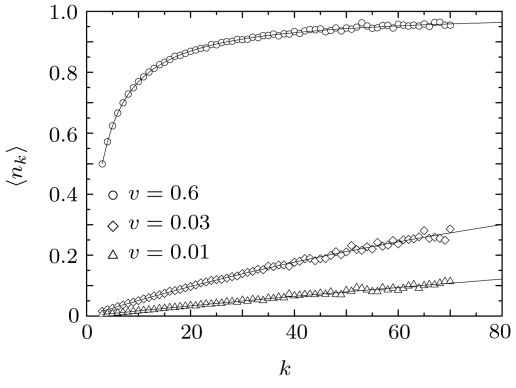


Fig.5. A plot of $\langle n_k \rangle$ versus k for Fermi–Dirac distribution. $N = 5 \times 10^3, m_0 = 3$. The numerical result is obtained by taking the average of 3×10^3 different network realizations. There is at most one particle occupying a vertex.

5. Conclusion

We have studied the traffic of many indistinguishable particles in a complex network. In the simulation we put some restrictions on the number of particles in each quantum state, which can be divided into three

cases: each quantum state can support infinite particles, each quantum state can support at most one particle and each quantum state can support at most $m\,(m > 1)$ particles. When the system reaches dynamic equilibrium, the particle distributions can satisfy the forms of Bose–Einstein distribution, Fermi–Dirac distribution or non-Fermi distribution. Additionally, for the Bose distribution, with increasing particle density, the ratio of particles in the lowest energy level to the total particles increases. When the particle density is very low, the particle distribution changes from the quantum statistical form into the classical statistical form, i.e., from the Bose distribution or the Fermi distribution into the Boltzmann distribution. It is well known that for the gas in a container reaching thermodynamic equilibrium due to the collision of molecules at a specific temperature, the molecule distribution is of Bose–Einstein form or Fermi–Dirac form, depending on whether they are bosons or fermions. In this paper, we find that, in the networks, though there is no collision of particles but random walk, under different restrictive conditions, i.e. the number of particles each vertex can support, the particle distributions may follow different quantum statistical forms. We further verify the ubiquity of the quantum statistics, such as the Bose–Einstein statistics and the Fermi–Dirac statistics, etc. Also, we further realize the important role played by statistical mechanics in the study of complex networks.

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