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Atomic emission spectra and field non-classical properties*

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This paper studies the interaction of a Λ-type three-level atom with a single mode field. It discusses the emission spectrum characteristics of the Λ-type three-level atom driven by the photon-added coherent field. By means of the second-order degree of coherence, it shows some nonclassical properties of the cavity field, such as sub-Poissonian photon-number distribution and the two-time intensity–intensity correlation which violates the Cauchy–Schwarz inequality.

Keywords: emission spectrum, photon-added coherent state, second-order degree of coherence
PACC: 4250, 3280

1. Introduction

The physical spectrum has played an important role in the investigation of the interaction between light and matter.[1−6] It can exhibit not only the characteristics of the interaction, but also the properties of the field and the atomic energy level. The Jaynes–Cummings (J–C) model is an ideal model that describes the interaction of a single atom with a single-mode quantized field. This model has attracted much attention for many years since it is mathematically accurately solvable and experimentally realizable for the improved levels of experimental technique.[7,8] The model and its various extensions predict many interesting effects, such as Rabi splitting,[3] collapse and revivals in population inversion,[9] nonclassical properties of field[7] and entanglement between atom and field.[10]

The quantum mechanical second-order degrees of coherence are usually used to examine field which exhibits nonclassical features, i.e., which cannot be described by a classical statistical theory,[1] like the phenomena of sub-Poissonian photon-number distribution[8] and photon antibunching.[11,12]

On the other hand, the generation of specific quantum states has been a subject of increasing interest.[13] In 1991, Agarwal and Tara[14] defined the photon-added coherent state (PACS) by repeated application of the photon creation operator $\hat{a}^{\dagger}$ on a coherent state. They pointed out that the PACS is intermediate between the Fock state and the coherent state, which demonstrates phase squeezing and the sub-Poissonian statistical character of the photon-added coherent field. That was followed by many investigations about PACS.[13−16] The single-mode field was generated perfectly in an experiment in 2004.[16] This offers the opportunity to closely follow the smooth transition between the particle-like and the wave-like behavior of light. The process of subtracting photons or adding photons can be applied to generate some strongly entangled states,[17−19] which have important applications in quantum information.[20]

In this paper, we pay our attention to the interaction of a Λ-type three-level atom and a single-mode field in an ideal cavity. We investigate the properties of the emission spectrum of a Λ-type three-level atom driven by a photon-added coherent field. Then we calculate the second-order degree of coherence in order to prove some nonclassical properties of the cavity field, such as sub-Poissonian photon-number distribution and the intensity–intensity correlation at different time that violates the Cauchy–Schwarz inequality.

Our paper is organized as follows. In Section 2, we present the emission spectra of the Λ-type three-level atom and discuss the influence of the cavity field initial state on the atomic emission spectra. In Section 3, we make use of the second-order degrees of coherence $g^{(2)}(t,t)$ and $g^{(2)}(0,\tau)$ and demonstrate some nonclassical properties of the cavity field. Finally a brief conclusion is presented in Section 4.

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2. The emission spectra of Λ-type three-level atom

As shown in Fig.1, the Λ-type three-level atom interacts with a single-mode field. In the rotating-wave and dipole approximations the Hamiltonian can be written as

$$\hat{H} = \hat{H}_1 + \hat{H}_2, \tag{1a}$$

$$\hat{H}_1 = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + \hbar\omega|a\rangle\langle a|, \tag{1b}$$

$$\hat{H}_2 = \hbar g\hat{a}\left(\hat{\sigma}_1^\dagger + \hat{\sigma}_2^\dagger\right) + \hbar g\hat{a}^\dagger\left(\hat{\sigma}_1 + \hat{\sigma}_2\right) + \hbar\Delta\left(|b\rangle\langle b| - |c\rangle\langle c|\right). \tag{1c}$$

Here $\hat{a}^\dagger$ and $\hat{a}$ are the creation and annihilation operators for the cavity field of frequency ω, respectively. The state $|i\rangle$ denotes the atomic energy eigenstate with eigen-frequency ω_i $(i = a, b, c)$ and $\Delta = (\omega_b - \omega_c)/2$. The quantity g is the coupling constant, $\hat{\sigma}_1 = |b\rangle\langle a|$ and $\hat{\sigma}_2 = |c\rangle\langle a|$ are the atomic lowering operators.

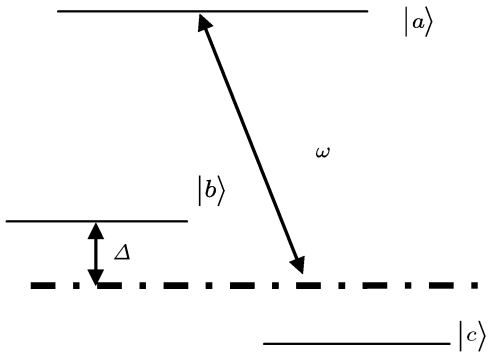


Fig.1. The schematic diagram of a Λ-type three-level atomic energy level. The transitions $|a\rangle$ to $|b\rangle$ and $|a\rangle$ to $|c\rangle$ are dipole allowed.

In the on-resonant case of $\omega = \omega_a - \omega_b + \Delta$, $[\hat{H}_1, \hat{H}_2] = 0$, so the time-evolution operator can be expressed as

$$\hat{U}(t,0) = \hat{U}_1(t,0)\hat{U}_2(t,0), \tag{2a}$$

$$\hat{U}_1(t,0) = \exp(-\mathrm{i}\,\hat{H}_1 t/\hbar) = \exp\left[-\mathrm{i}\,\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)t\right]\begin{pmatrix}\exp(-\mathrm{i}\,\omega t) & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}, \tag{2b}$$

$$\hat{U}_2(t,0) = \exp(-\mathrm{i}\,\hat{H}_2 t/\hbar) = \begin{pmatrix} \dfrac{\Delta^2 + 2g^2\hat{a}\hat{a}^\dagger\hat{C}}{\hat{\Omega}^2} & g\left[\dfrac{\Delta(\hat{C}-1)}{\hat{\Omega}^2} - \mathrm{i}\,\dfrac{\hat{S}}{\hat{\Omega}}\right]\hat{a} & -g\left[\dfrac{\Delta(\hat{C}-1)}{\hat{\Omega}^2} + \mathrm{i}\,\dfrac{\hat{S}}{\hat{\Omega}}\right]\hat{a} \\ g\left[\dfrac{\Delta(\hat{C}'-1)}{\hat{\Omega}'^2} - \mathrm{i}\,\dfrac{\hat{S}'}{\hat{\Omega}'}\right]\hat{a}^\dagger & \dfrac{g^2\hat{a}^\dagger\hat{a} + (g^2\hat{a}^\dagger\hat{a} + \Delta^2)\hat{C}'}{\hat{\Omega}'^2} - \mathrm{i}\,\Delta g\dfrac{\hat{S}'}{\hat{\Omega}'} & g^2\dfrac{\hat{C}'-1}{\hat{\Omega}'^2}\hat{a}^\dagger\hat{a} \\ -g\left[\dfrac{\Delta(\hat{C}'-1)}{\hat{\Omega}'^2} + \mathrm{i}\,\dfrac{\hat{S}'}{\hat{\Omega}'}\right]\hat{a}^\dagger & g^2\dfrac{\hat{C}'-1}{\hat{\Omega}'^2}\hat{a}^\dagger\hat{a} & \dfrac{g^2\hat{a}^\dagger\hat{a} + (g^2\hat{a}^\dagger\hat{a} + \Delta^2)\hat{C}'}{\hat{\Omega}'^2} + \mathrm{i}\,\Delta g\dfrac{\hat{S}'}{\hat{\Omega}'} \end{pmatrix}. \tag{2c}$$

where

$$\hat{C} = \cos(\hat{\Omega}\,t), \quad \hat{S} = \sin(\hat{\Omega}\,t), \quad \hat{\Omega} = \sqrt{\Delta^2 + 2g^2\hat{a}\hat{a}^\dagger},$$
$$\hat{C}' = \cos(\hat{\Omega}'t), \quad \hat{S}' = \sin(\hat{\Omega}'t), \quad \hat{\Omega}' = \sqrt{\Delta^2 + 2g^2\hat{a}^\dagger\hat{a}}. \tag{2d}$$

If the atom and the field are uncorrelated at the beginning, the initial state of the system is

$$|\psi(0)\rangle = (c_1|a\rangle + c_2|b\rangle + c_3|c\rangle) \otimes \sum_{n=0}^{\infty} F_n|n\rangle. \tag{3}$$

Using the time evolution operator, we obtain

$$
\begin{aligned}
&\left\langle \left[\hat{\sigma}_1^{\dagger}(t_1)+\hat{\sigma}_2^{\dagger}(t_1)\right]\left[\hat{\sigma}_1(t_2)+\hat{\sigma}_2(t_2)\right]\right\rangle \\
&=\langle\psi(0)|\,\hat{U}^{\dagger}(t_1,0)\left(\hat{\sigma}_1^{\dagger}+\hat{\sigma}_2^{\dagger}\right)\hat{U}(t_1,0)\,\hat{U}^{\dagger}(t_2,0)\,(\hat{\sigma}_1+\hat{\sigma}_2)\,\hat{U}(t_2,0)\,|\psi(0)\rangle \\
&=2\,|c_1|^2\sum_{n=0}^{\infty}|F_n|^2\,[D(t_1)\sin(\Omega_n t_1)\,D(t_2)\sin(\Omega_n t_2) \\
&\quad+D(t_1)\cos(\Omega_n t_1)\,D(t_2)\cos(\Omega_n t_2)]\exp[\mathrm{i}\,\omega\,(t_1-t_2)] \\
&\quad+2\sum_{n=0}^{\infty}(n+1)\,|F_{n+1}|^2\,[E^*(t_1)\sin(\Omega_n t_1)\,E(t_2)\sin(\Omega_n t_2) \\
&\quad+E^*(t_1)\cos(\Omega_n t_1)\,E(t_2)\cos(\Omega_n t_2)]\exp[\mathrm{i}\,\omega\,(t_1-t_2)] \\
&\quad+2c_1^*\sum_{n=0}^{\infty}\sqrt{n+1}F_n^*F_{n+1}[D(t_1)\sin(\Omega_n t_1)\,E(t_2)\sin(\Omega_n t_2) \\
&\quad+D(t_1)\cos(\Omega_n t_1)\,E(t_2)\cos(\Omega_n t_2)]\exp[\mathrm{i}\,\omega\,(t_1-t_2)] \\
&\quad+2c_1\sum_{n=0}^{\infty}\sqrt{n+1}F_{n+1}^*F_n[E^*(t_1)\sin(\Omega_n t_1)\,D(t_2)\sin(\Omega_n t_2) \\
&\quad+E^*(t_1)\cos(\Omega_n t_1)\,D(t_2)\cos(\Omega_n t_2)]\exp[\mathrm{i}\,\omega\,(t_1-t_2)]\,,
\end{aligned}
\tag{4a}
$$

where

$$
\begin{aligned}
&\Omega_n=\sqrt{2g^2n+\Delta^2}, \\
&D(t)=\frac{\Delta^2+2g^2(n+1)\cos(\Omega_{n+1}t)}{2g^2(n+1)+\Delta^2}, \\
&E(t)=(c_2-c_3)\frac{\Delta g\,[\cos(\Omega_{n+1}t)-1]}{2g^2(n+1)+\Delta^2}-\mathrm{i}(c_2+c_3)\frac{g\sin(\Omega_{n+1}t)}{\sqrt{2g^2(n+1)+\Delta^2}}.
\end{aligned}
\tag{4b}
$$

The physical spectrum emitted by a cavity-bound Λ-type three-level atom is given by the expression[2]

$$
S(\nu)=\Gamma\int_0^T \mathrm{d}t_1\int_0^T \mathrm{d}t_2\exp[-(\Gamma-\mathrm{i}\,\nu)(T-t_1)-(\Gamma+\mathrm{i}\,\nu)(T-t_2)]\times\langle(\hat{\sigma}_1^{\dagger}(t_1)+\hat{\sigma}_2^{\dagger}(t_1))(\hat{\sigma}_1(t_2)+\hat{\sigma}_2(t_2))\rangle, \tag{5}
$$

where Γ is the bandwidth of the filter, T is the interaction time.

Substituting Eq.(4) into Eq.(5), and carrying out the integration over the interaction time, we get the expression for the emission spectrum of a cavity-bound Λ-type three-level atom as

$$
\begin{aligned}
S(\nu)=&\,\Gamma\,|c_1|^2\sum_{n=0}^{\infty}\frac{|F_n|^2}{\Omega_{n+1}^4}\left[|G_1(\Omega_n)|^2+|G_1(-\Omega_n)|^2\right]+\frac{\Gamma}{4}\sum_{n=0}^{\infty}\frac{g^2(n+1)\,|F_{n+1}|^2}{\Omega_{n+1}^4}\left[|G_2(\Omega_n)|^2+|G_2(-\Omega_n)|^2\right] \\
&-\frac{\Gamma}{2}\sum_{n=0}^{\infty}\frac{g\sqrt{n+1}}{\Omega_{n+1}^4}\left\{c_1^*F_n^*F_{n+1}\left[G_1^*(\Omega_n)G_2(\Omega_n)+G_1^*(-\Omega_n)G_2(-\Omega_n)\right]+\text{c.c.}\right\}.
\end{aligned}
\tag{6a}
$$

Here

$$
\begin{aligned}
&G_1(x)=\Delta^2F(0,x)+g^2(n+1)F(\Omega_{n+1},x)+g^2(n+1)F(-\Omega_{n+1},x), \\
&G_2(x)=2\Delta(c_2-c_3)F(0,x)-(H_+c_2-H_-c_3)F(\Omega_{n+1},x)-(H_-c_2-H_+c_3)F(-\Omega_{n+1},x), \\
&F(x,y)=\frac{\exp[\mathrm{i}(\nu-\omega-x-y)T]-\exp(-\Gamma T)}{\Gamma+\mathrm{i}(\nu-\omega-x-y)}, \\
&H_{\pm}=\Delta\pm\sqrt{2g^2(n+1)+\Delta^2}.
\end{aligned}
\tag{6b}
$$

When the atom is initially in the upper state, namely, $c_1=1$, $c_2=c_3=0$, the equation (6) can be reduced to

$$
S(\nu)=\Gamma\sum_{n=0}^{\infty}\frac{|F_n|^2}{\Omega_{n+1}^4}\left[|G_1(\Omega_n)|^2+|G_1(-\Omega_n)|^2\right]. \tag{7}
$$

This means that the atomic emission spectrum is only related to the initial photon number distribution of the cavity field in this case. Let the driven field initially be in the PACS which is defined as[14]

$$|\alpha, m\rangle = \frac{\hat{a}^{\dagger m}|\alpha\rangle}{\langle\alpha|\hat{a}^m\hat{a}^{\dagger m}|\alpha\rangle^{1/2}}. \tag{8}$$

Here $\langle\alpha|\hat{a}^m\hat{a}^{\dagger m}|\alpha\rangle = m!\mathrm{L}_m(-|\alpha|^2)$ where $\mathrm{L}_m(x)$ is thevLaguerre polynomial of order m, m is the photon-added number. Apparently, the PACS becomes a Fock state for $\alpha = 0$ and a coherent state for $m = 0$.

In a PACS, the photon number distribution depends on both α and m. If α is small enough, the photon number fluctuation of the PACS is very small, so the atomic emission spectrum is very similar to that for the Fock state as shown in Fig.2. The average photon number $\bar{n}$ and the photon number fluctuation Δn increase with m, the emission spectrum presents three peaks instead of six peaks step by step; the two sidebands move away from each other and broaden as shown in Figs.3(a)–3(c), so the difference between the atomic emission spectra for the PACS and the Fock state is evident for a sufficiently large m; at the same time the two sidebands are higher and narrower than that for the coherent state in the case of the same average photon number as shown in Fig.3(c) and 3(d).

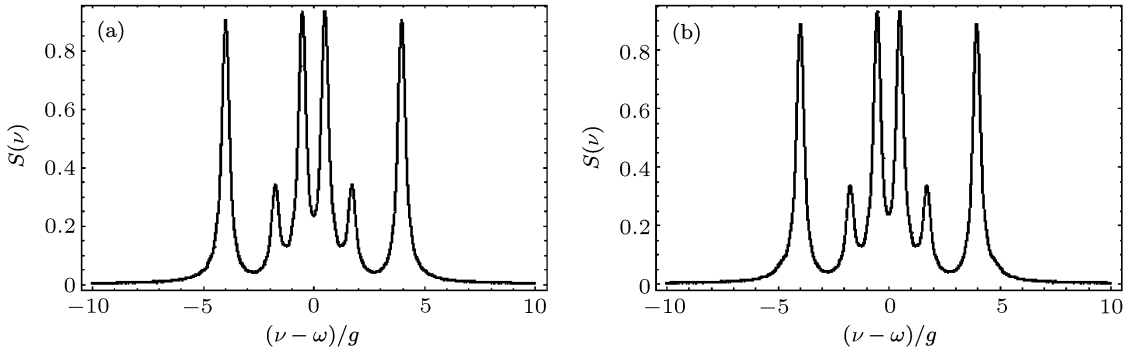


Fig.2. The atomic spectra $S(\nu)$. $c_1 = 1$, $c_2 = c_3 = 0$; $\Gamma = 20/g$; $\Delta/g = 1$; $T = 20/g$; (a) $m = 1$ and $\alpha = 0.1$; (b) $m = 1$ and $\alpha = 0$.

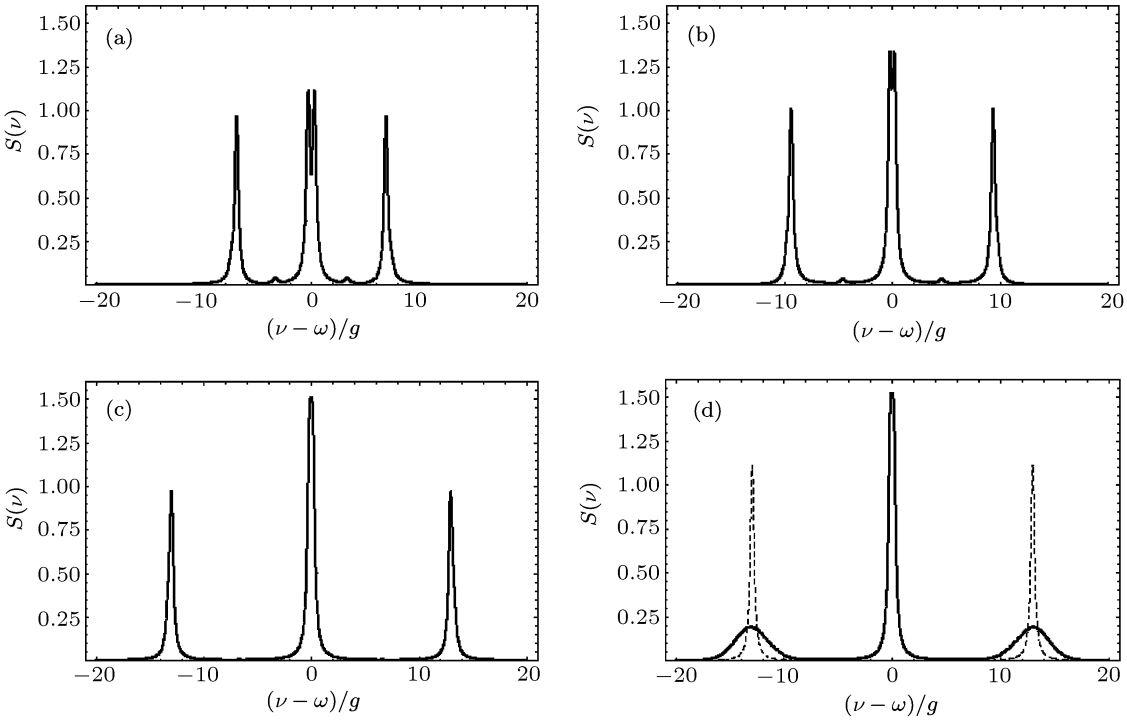


Fig.3. Same as Fig.2. but (a) $m = 5$, $\alpha = 0.1$; (b) $m = 10$, $\alpha = 0.1$; c. m=20, $\alpha = 0.1$ ($\bar{n} = 20.1915$); (d) $m = 20$, $\alpha = 0$ (the dashed line), $m = 0$, $\alpha = \sqrt{20.1915}$ (the solid line).

For the same average photon number $\bar{n}$, the photon number fluctuation Δn of the PACS increases with α; therefore the emission spectra of the atom driven by the photon-added coherent field evolves gradually into those for the coherent field, as shown in Figs.3(c), 3(d) and 4.

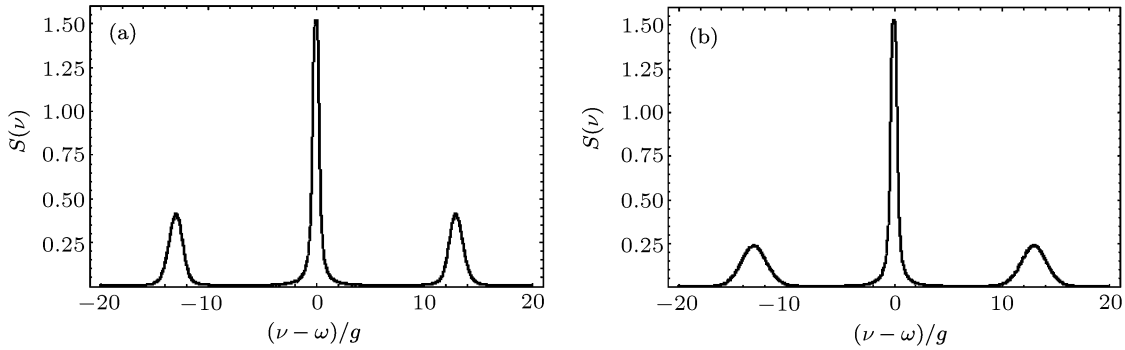


Fig.4. Same as Fig.2, but $\bar{n} = 20.1915$. (a) $m = 15$, $\alpha = 1.19152$; (b) $m = 5$, $\alpha = 3.38654$.

3. Nonclassical properties of the cavity field

As for a single mode, the second-order degree of coherence is defined as[1]

$$g^{(2)}(t, t+\tau) = \frac{\langle \hat{a}^\dagger(t)\hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\hat{a}(t)\rangle}{\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle \langle \hat{a}^\dagger(t+\tau)\hat{a}(t+\tau)\rangle}. \tag{9}$$

The second-order degree of coherence depends on both the time t and the delay time τ.

When $\tau = 0$, the second-order degree of coherence $g^{(2)}(t, t)$ denotes the intensity–intensity autocorrelation. Utilizing the commutation relation of the creation and annihilation operators, $g^{(2)(t,t)}$ can be written as

$$g^{(2)}(t, t) = \frac{\Delta n^2 - \bar{n}}{\bar{n}^2} + 1, \tag{10}$$

so that $g^{(2)}(t, t)$ reflects the relation between the photon number fluctuation Δn and the average photon number $\bar{n}$. If $g^{(2)}(t, t) < 1$, i.e. $\Delta n < \sqrt{\bar{n}}$, the field possesses sub-Poissonian statistics, which is a quantum mechanical phenomenon without any classical analogue. If $g^{(2)}(t, t) = 1$, i.e. $\Delta n = \sqrt{\bar{n}}$, the field exhibits Poissonian statistics, just as the coherent field. When the field takes on sub-Poissonian statistics, $g^{(2)}(t, t) > 1$, i.e. $\Delta n > \sqrt{\bar{n}}$.

When $t = 0$, $g^{(2)}(0, \tau)$ denotes the two-time (the initial time and time τ) intensity–intensity correlation. If $\lambda(\tau) = g^{(2)}(0, 0) \cdot g^{(2)}(\tau, \tau) - [g^{(2)}(0, \tau)]^2 \geq 0$, the Cauchy–Schwarz (C–S) inequality $|I(0)I(\tau)|^2 \leq \langle I^2(0)\rangle\langle I^2(\tau)\rangle$ is obeyed. So while the C–S inequality is violated, i.e. $\lambda(\tau) < 0$, the cavity field emerges with a nonclassical character.

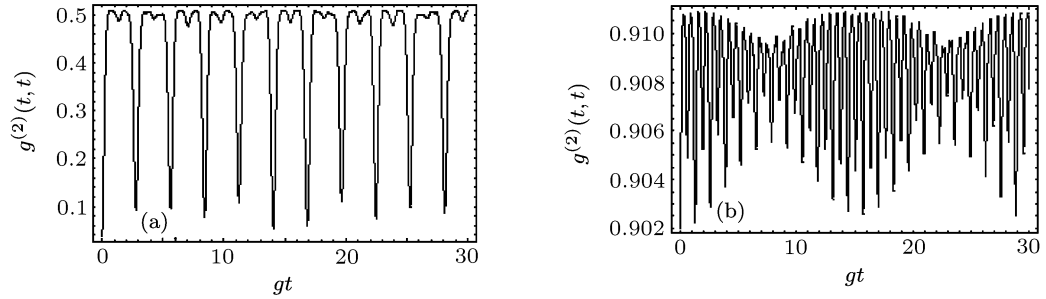


Fig.5. The second-order degree of coherence $g^{(2)}(t, t)$, $\Delta/g = 1$, $c_1 = 1$, $c_2 = c_3 = 0$, $\alpha = 0.1$. (a) $m = 1$; (b) $m = 10$.

It is assumed that the initial state of the system is still denoted by $|\psi(0)\rangle$. Utilizing the time evolution operator we get

$$\begin{aligned}\left\langle a^{\dagger}(0)a^{\dagger}(t)a(t)a(0)\right\rangle =& |c_1|^2\sum_{n=0}^{\infty}|F_n|^2\, n[(n-1)+K_n(t)]+\left(|c_2|^2+|c_3|^2\right)\sum_{n=0}^{\infty}|F_{n+1}|^2\,(n+1)[(n-1)+R_n(t)]\\ &+\sum_{n=0}^{\infty}\left\{(c_1c_3^*F_nF_{n+1}^*-c_3c_1^*F_{n+1}F_n^*)\sqrt{n(n+1)}J_n(t)+\text{c.c}\right\}\\ &-2\sum_{n=0}^{\infty}\left\{c_2c_3^*|F_{n+1}|^2\,(n+1)\left[W_n(t)+\text{i}\,\frac{\Delta g^2 n}{\Omega_n^3}(n-1)\cos(\Omega_n t)\sin(\Omega_n t)\right]+\text{c.c}\right\},\end{aligned}\tag{11a}$$

$$\begin{aligned}\left\langle a^{\dagger}(t)a(t)\right\rangle =& |c_1|^2\sum_{n=0}^{\infty}|F_n|^2\,[n+K_{n+1}(t)]+\left(|c_2|^2+|c_3|^2\right)\sum_{n=0}^{\infty}|F_{n+1}|^2\,[n+R_{n+1}(t)]\\ &+\sum_{n=0}^{\infty}\left\{(c_1c_3^*F_nF_{n+1}^*-c_3c_1^*F_{n+1}F_n^*)J_{n+1}(t)+\text{c.c}\right\}\\ &-2\sum_{n=0}^{\infty}\left\{c_2c_3^*|F_{n+1}|^2\left[W_{n+1}(t)+\text{i}\,\frac{\Delta g^2(n+1)}{\Omega_{n+1}^3}n\cos(\Omega_{n+1}t)\sin(\Omega_{n+1}t)\right]+\text{c.c}\right\},\end{aligned}\tag{11b}$$

$$\begin{aligned}\langle a^{\dagger}(t)a^{\dagger}(t)a(t)a(t)\rangle =& |c_1|^2\sum_{n=0}^{\infty}|F_n|^2\, n[(n-1)+2K_{n+1}(t)]\\ &+\left(|c_2|^2+|c_3|^2\right)\sum_{n=0}^{\infty}|F_{n+1}|^2\, n[(n-1)+2R_{n+1}(t)]\\ &+2\sum_{n=0}^{\infty}\left\{(c_1c_3^*F_nF_{n+1}^*-c_3c_1^*F_{n+1}F_n^*)nJ_{n+1}(t)+\text{c.c}\right\}\\ &-2\sum_{n=0}^{\infty}\left\{c_2c_3^*|F_{n+1}|^2\,n\left[2W_{n+1}(t)+\text{i}\,\frac{\Delta g^2(n+1)}{\Omega_{n+1}^3}(n-1)\cos(\Omega_{n+1}t)\sin(\Omega_{n+1}t)\right]+\text{c.c}\right\},\end{aligned}\tag{11c}$$

where

$$\begin{aligned}K_n(t) &= 4\frac{g^2n}{\Omega_n^4}[\Delta^2+g^2n-\Delta^2\cos(\Omega_nt)-g^2n\cos^2(\Omega_nt)],\\ R_n(t) &= \frac{1}{\Omega_n^4}[\Delta^2\Omega_n^2+2g^4n^2+2\Delta^2g^2n\cos(\Omega_nt)+2g^4n^2\cos^2(\Omega_nt)],\\ J_n(t) &= \frac{g\sqrt{n}}{\Omega_n^4}\left\{\Delta^3+\Delta(2g^2n-\Delta^2)\cos(\Omega_nt)-2\Delta g^2n\cos^2(\Omega_nt)-\text{i}[\Delta^2+2g^2n\cos(\Omega_nt)]\Omega_n\sin(\Omega_nt)\right\},r\\ W_n(t) &= \frac{g^2n}{\Omega_n^4}\left\{g^2n+\Delta^2\cos(\Omega_nt)-(\Delta^2+g^2n)\cos^2(\Omega_nt)-\text{i}[1-\cos(\Omega_nt)]\Delta\Omega_n\sin(\Omega_nt)\right\}.\end{aligned}\tag{12}$$

It is easy to obtain $g^{(2)}(t,t)$ and $g^{(2)}(0,\tau)$ applying Eqs.(11a), (11b) and (11c) according to the definition. The PACS has sub-Poissonian statistics just as the Fock state does.[13] When α is sufficiently small, $g^{(2)}(t,t)$ is always less than 1 and presents quasi-periodic oscillation. When the photon-added number m increases, the evolution of $g^{(2)}(t,t)$ becomes more complicated for the augmentation of the photon number fluctuation Δn, whereas the range of $g^{(2)}(t,t)$ is reduced by the reason of the increase of $g^{(2)}(0,0)$, as shown in Fig.5. When α is large to some extent, $g^{(2)}(t,t)$ is no longer less than 1 all the time. When $\bar{n}$ is sufficiently large, the evolution of $g^{(2)}(t,t)$ would present the phenomena of "collapse" and "revival", as shown in Fig.6. Comparing Fig.6 with Fig.7, in the first instance, the oscillating amplitude of the average photon number $\bar{n}(t)$ is gradually reduced to zero; at the same interval which is called the collapse time t_{c}, the oscillating amplitude of $g^{(2)}(t,t)$ rises first, then falls to zero little by little. The collapse time t_{c} is not only dependent on the initial average photon number $\bar{n}$, but also decreases with the increase of the initial photon number fluctuation Δn. The maximum oscillating

amplitude of $g^{(2)}(t,t)$ is likewise sensitive to the initial photon number fluctuation Δn. When t increases further, the revivals of $\bar{n}(t)$ and $g^{(2)}(t,t)$ take place, but when the oscillating amplitude of $\bar{n}(t)$ reaches the maximum the oscillating amplitude of $g^{(2)}(t,t)$ is very small. This interval, which is called the revival time t_r, is hardly sensitive to the photon number fluctuation Δn, but increases evidently with the initial average photon number $\bar{n}$.

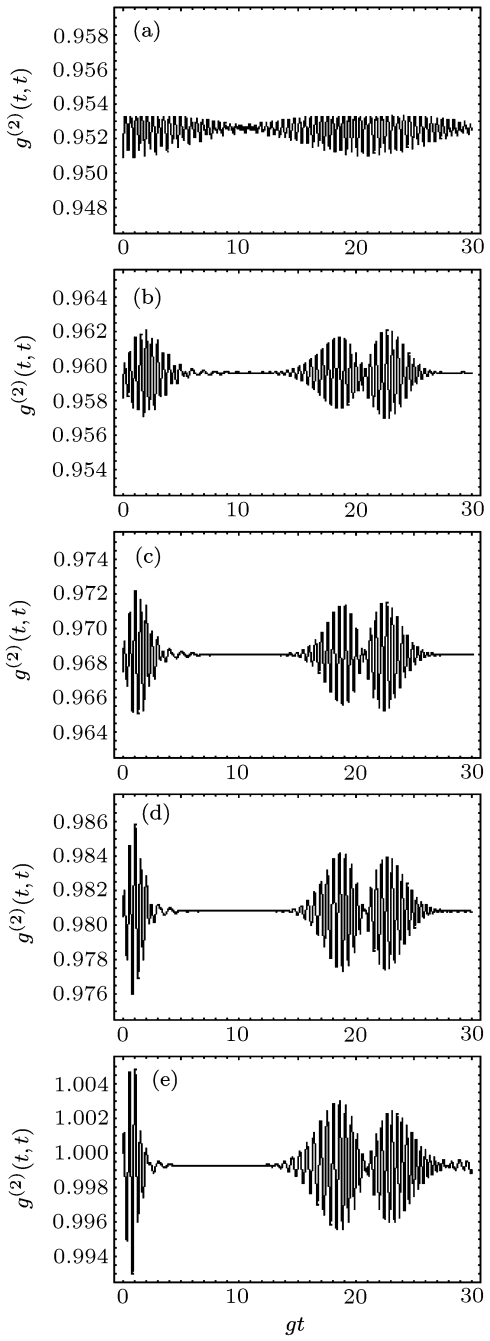


Fig.6. Same as Fig.5. But $\bar{n} = 20.1915$, (a) $m = 20$, $\alpha = 0.1$; (b) $m = 15$, $\alpha = 1.19152$; (c) $m = 10$; $\alpha = 2.28684$; (d) $m = 5$; $\alpha = 3.38654$; (e) $m = 0$; $\alpha = \sqrt{20.1915}$.

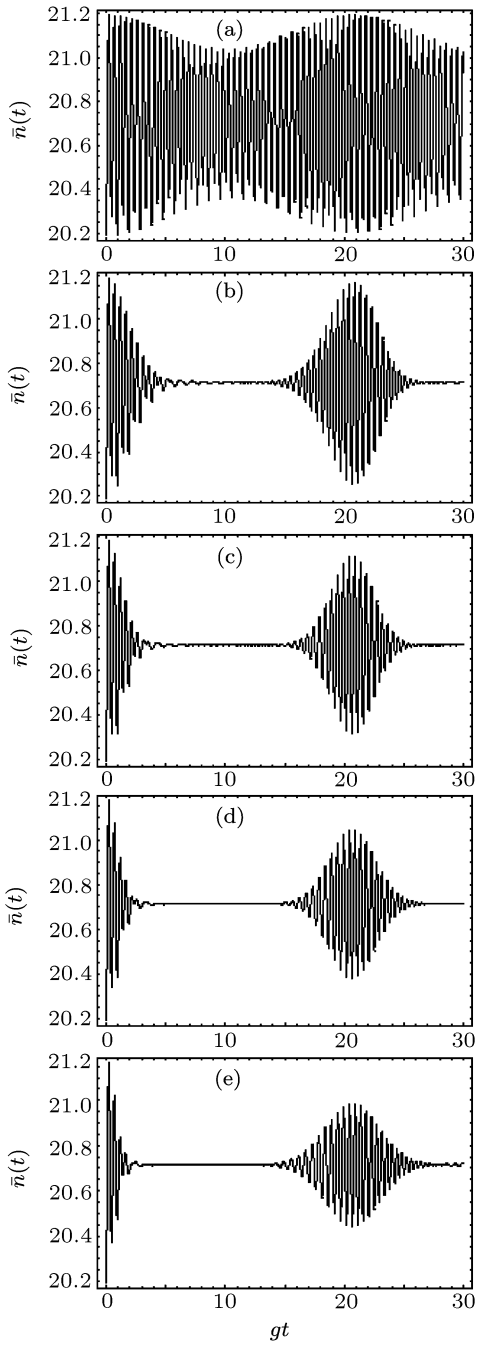


Fig.7. The average photon number $\bar{n}(t)$ of the cavity field. The parameters are the same as those in Fig.6.

It is easy to obtain $g^{(2)}(0,\tau)$ from the definition for $m = 0$. Thus $\lambda(\tau) = g^{(2)}(\tau,\tau) - 1$; the cavity field inevitably violates the C–S inequality when it has sub-Poissonian statistics. This relation between the two nonclassical characteristics is maintained when α is sufficiently large but the added photon number m is very small, as shown in Fig.8. The two-time intensity–intensity correlation $g^{(2)}(0,\tau)$ of the cavity field is no longer a constant while increasing m or decreasing α, so the evolutions of $\lambda(\tau)$ and $g^{(2)}(t,t)$ are not in step, as shown

in Fig.9; the smaller α is, the more obvious the differences between them are. The cavity field does not always violate the C–S inequality when it has sub-Poissonian statistics.

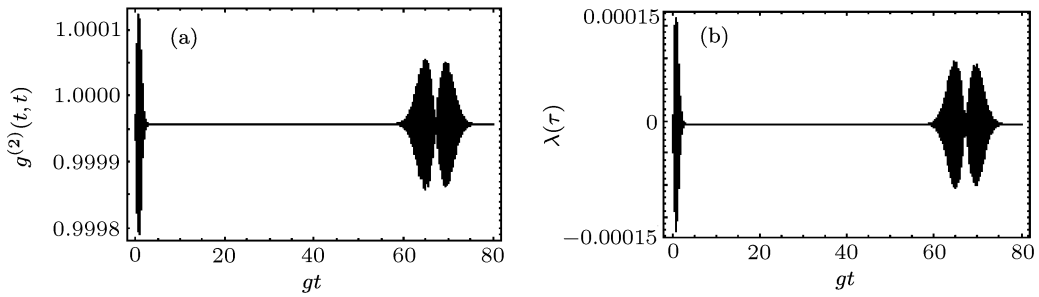


Fig.8. The second-order degree of coherence $g^{(2)}(t,t)$ (figure (a)) and the function $\lambda(\tau)$ (figure (b)). The parameters are the same as those in Fig.5, but $m = 1$ and $\alpha = 15$.

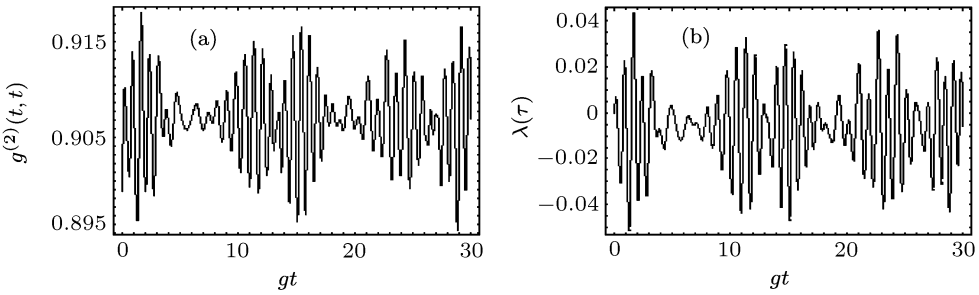


Fig.9. The same as Fig.8, but $m = 5$ and $\alpha = 2$.

4. Conclusions

In an ideal cavity, the Λ-type three-level atomic emission spectra driven by photon-added coherent field are dependent on α and the added photon number m. When the average photon number $\bar{n}$ is sufficiently large, the atomic emission spectra take on three peaks, and the two sidebands become higher and narrower with increasing m and decreasing α. When the average photon number $\bar{n}$ is small, α and the added photon number m influence not only the height and width but also the position and the amount of the peaks. The parameters α and m of the cavity field initial state also play an importance role for the dynamic behavior of the system. The cavity field presents sub-Poissonian statistics and violates the C–S inequality simultaneity only for $|\alpha|^2 \gg m$, otherwise, sub-Poissonian statistics and the C–S inequality being violated do not always occur together. Therefore, the two nonclassical characteristics are different and independent.[21]

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