Vol 18 No 8, August 2009
1674-1056/2009/18(08)/3453-04

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Robust scheme for implemention of quantum phase gates for two atoms*

Zheng Shi-Biao(郑仕标)†

Department of Physics, Fuzhou University, Fuzhou 350002, *China*

(Received 24 September 2008; revised manuscript received 10 November 2008)

We propose a scheme for implementing conditional quantum phase gates for two four-state atoms trapped in a cavity. The two ground states of the atoms are coupled through two Raman processes induced by the cavity mode and two classical fields. Under certain conditions nonresonant Raman processes lead to two-atom coupling and can be used to produce conditional phase gates. The scheme is insensitive to cavity decay, thermal photons, and atomic spontaneous emission. The scheme does not require individual addressing of the atoms.

Keywords: quantum phase gates, Raman process, two-atom coupling
PACC: 4250

Quantum computation has become an interdisciplinary field across many subjects, such as physics, information science, and mathematics. Employing the fundamental principles of quantum mechanics, a quantum computer can solve some problems that are intractable with a classical computer.[1] The two-qubit logic gates are the building blocks for constructing a quantum computer since any quantum computational network can be decomposed into a series of two-qubit logic gates plus trivial one-qubit gates.[2] Many physical scenarios have been proposed for the implementation of two-qubit logic gates. The elementary quantum information processors have been demonstrated with trapped ions.[3,4] The cavity QED system is an alternative qualified candidate for quantum state engineering and quantum information processing as the atoms are ideal for storage of quantum information and the photons are suitable for transportation of quantum information. Two-qubit quantum gates have been experimentally demonstrated with a cavity QED system.[5] In the experiment, the cavity acts as memory, which stores the information of an electronic system and then transfers back to this electronic system after the conditional dynamics.

Due to the decoherence there exists an error in each quantum gate operation. Quantum error-correcting codes enable quantum computers to operate provided that the noise in individual quantum gates is below a certain constant threshold. The recently estimated threshold is that the individual gate infidelity should be on the order of 10^{-2}.[6] The gate error reported in the experiment of Ref.[5] is far from the required threshold. The main obstacles for the implementation of quantum information in cavity QED is the decoherence arising from atomic spontaneous emission and cavity decay. We have proposed a scheme for implementation of quantum phase gates for two atoms via nonresonant interaction with a cavity mode. As the cavity mode is always in the vacuum state the gate operation is insensitive to cavity decay.[7] However, in the scheme the atoms have a probability of being populated in the excited state and thus the fidelity is affected by atomic spontaneous emission. Schemes have been proposed for suppressing both cavity decay and atomic spontaneous emission.[8−10] Since these schemes succeed under the condition that no photon is emitted either by spontanous emission or by leakage through cavity mirrors, the success probability and gate fidelity depend upon the photodetection efficiency. Furthermore, these schemes require the atoms to be individually addressed.

In this paper we present an alternative cavity QED based scheme for implementing quantum phase gates for two atoms trapped in a cavity. In the scheme two four-state atoms are coupled to the cavity mode and driven by two classical fields. The quantum information is encoded in ground states. Since the fields

*Project supported by the National Natural Science Foundation of China (Grant No 10674025) and the Doctoral Foundation of the Ministry of Education of China (Grant No 20070386002).
†Corresponding author. E-mail: sbzheng@pub5.fz.fj.cn

are highly detuned from the atomic transitions the two ground states are coupled through two Raman processes. When the difference between the detunings of the cavity mode and the classical fields is large enough nonresonant Raman processes lead to two-atom coupling, which can be used to produce conditional two-qubit phase gates. In the scheme the effective Hamiltonians for the two atoms are decoupled from the cavity mode and the gate operation is not affected by cavity decay and thermal photons. During the operation the atoms have no probability of being excited and thus the scheme is insensitive to atomic spontaneous emission. Furthermore, the scheme does not require individual addressing of the atoms.

We consider two identical atoms simultaneously interacting with a single-mode cavity field and driven by two classical fields. The atoms have two excited states $|r\rangle$ and $|s\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The transitions $|r\rangle \rightarrow |g\rangle$ and $|s\rangle \rightarrow |e\rangle$ are coupled to the cavity mode. The transitions $|r\rangle \rightarrow |e\rangle$ and $|s\rangle \rightarrow |g\rangle$ are driven by two classical fields. Assume that the cavity mode is detuned from the respective transitions by Δ, while the two classical fields are detuned from the respective transitions by $\Delta - \delta$, as shown in Fig.1. In the interaction picture the Hamiltonian is

$$\begin{aligned} H = & \sum_{j=1}^{2}[\Omega\, \mathrm{e}^{\mathrm{i}(\Delta-\delta)t}\, |r_j\rangle \langle e_j| + g\, \mathrm{e}^{\mathrm{i}\Delta t} a\, |r_j\rangle \langle g_j| \\ & + \Omega\, \mathrm{e}^{\mathrm{i}(\Delta-\delta)t}\, |s_j\rangle \langle g_j| \\ & + g\, \mathrm{e}^{\mathrm{i}\Delta t} a\, |s_j\rangle \langle e_j|] + \mathrm{H.c.}, \end{aligned} \tag{1}$$

where a is the annihilation operator of the cavity mode, g is the atom-cavity coupling strength, and Ω is the Rabi frequency of the classical fields. In the case $\Delta \gg \Omega$, g, δ the upper levels $|r\rangle$ and $|s\rangle$ can be adiabatically eliminated. The classical fields and cavity mode leads to Stark shifts and Raman couplings between the two ground states. No matter whether each atom is in the state $|e\rangle$ or $|g\rangle$ the Stark shifts for each atom induced by the cavity mode and the corresponding classical field are $(g^2/\Delta)a^\dagger a$ and Ω^2/Δ, respectively. Therefore, the Stark shifts are independent of the atomic state. The corresponding effective Hamiltonian is

$$\begin{aligned} H = & -2\nu a^\dagger a - \lambda \sum_{j=1}^{2}[(\mathrm{e}^{\mathrm{i}\delta t} a S_j^+ + \mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^-) \\ & + (\mathrm{e}^{\mathrm{i}\delta t} a S_j^- + \mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^+)], \end{aligned} \tag{2}$$

where $\nu = g^2/\Delta$, $\lambda = \Omega g/\Delta$, $S_j^+ = |e_j\rangle \langle g_j|$, and $S_j^- = |g_j\rangle \langle e_j|$. We here have discarded the constant Stark shift $2\Omega^2/\Delta$, which only induces a trivial common phase shift. The Raman coupling $\lambda(\mathrm{e}^{\mathrm{i}\delta t} a S_j^+ + \mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^-)$ is induced by the virtual excitation of the state $|r\rangle$, while the Raman coupling $\lambda(\mathrm{e}^{\mathrm{i}\delta t} a S_j^- + \mathrm{e}^{-\mathrm{i}\delta t} a^\dagger S_j^+)$ is induced by the virtual excitation of the state $|s\rangle$.

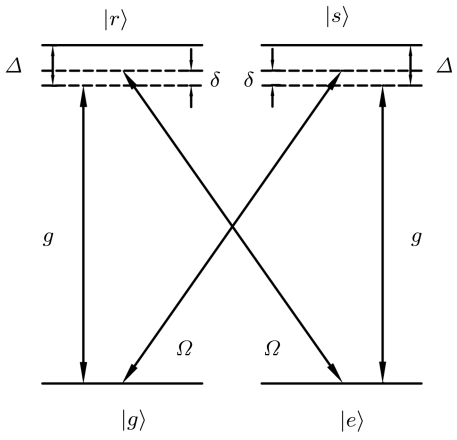


Fig.1. The configuration of the atomic level. The transitions $|r\rangle \rightarrow |g\rangle$ and $|s\rangle \rightarrow |e\rangle$ are coupled to the cavity mode with the coupling g and detuning Δ. The transitions $|r\rangle \rightarrow |e\rangle$ and $|s\rangle \rightarrow |g\rangle$ are driven by two classical fields with the Rabi frequency Ω and detuning $\Delta - \delta$.

The time evolution of this system is decided by Schrödinger's equation:

$$\mathrm{i}\frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H|\psi(t)\rangle. \tag{3}$$

Perform the unitary transformation

$$|\psi(t)\rangle = \mathrm{e}^{-\mathrm{i}H_0 t}|\psi'(t)\rangle \tag{4}$$

with

$$H_0 = -2\nu a^\dagger a. \tag{5}$$

Then we obtain

$$\mathrm{i}\frac{\mathrm{d}|\psi'(t)\rangle}{\mathrm{d}t} = H_i'|\psi'(t)\rangle, \tag{6}$$

where

$$\begin{aligned} H_i' = & -\sum_{j=1}^{2}\lambda\{[\mathrm{e}^{\mathrm{i}(\delta+2\nu)t} a S_j^+ + \mathrm{e}^{-\mathrm{i}(\delta-2\nu)t} a^\dagger S_j^-] \\ & + [\mathrm{e}^{\mathrm{i}(\delta+2\nu)t} a S_j^- + \mathrm{e}^{-\mathrm{i}(\delta-2\nu)t} a^\dagger S_j^+]\}. \end{aligned} \tag{7}$$

Under the condition $\lambda \ll \delta + 2\nu$, the Raman coupling terms osillate rapidly and thus the probability that the cavity mode exchanges energy with the atoms is negligible. However, the nonresonant Raman couplings can lead to the transitions $|e_j g_k n\rangle \longleftrightarrow |g_j e_k n\rangle$

and $|e_je_kn\rangle \longleftrightarrow |g_jg_kn\rangle$, where n is the photon-number of the cavity mode.[11,12] The coupling between the two atoms is induced by the virtual excitation of the cavity mode. The transition $|e_jg_kn\rangle \longleftrightarrow |g_je_kn\rangle$ is mediated by $|g_jg_kn \pm 1\rangle$ and $|e_je_kn \pm 1\rangle$. The contributions of $|g_jg_kn \pm 1\rangle$ are equal to those of $|e_je_kn \pm 1\rangle$. The effective coupling strength for the transition $|e_jg_kn\rangle \longleftrightarrow |g_je_kn\rangle$ is given by[12]

$$\begin{aligned}\xi &= 2\frac{\langle e_jg_kn|\, H_i'\,|g_jg_kn+1\rangle\,\langle g_jg_kn+1|\, H_i'\,|g_je_kn\rangle}{\delta+2\nu} \\ &\quad + 2\frac{\langle e_jg_kn|\, H_i'\,|g_jg_kn-1\rangle\,\langle g_jg_kn-1|\, H_i'\,|g_je_kn\rangle}{-(\delta+2\nu)} \\ &= \frac{2\lambda^2}{\delta-2\nu}. \end{aligned} \tag{8}$$

Since the transition paths interfere destructively the effective coupling strength ξ is independent of the photon-number of the cavity mode. The effective coupling strength for the transition $|e_je_kn\rangle \longleftrightarrow |g_jg_kn\rangle$, mediated by $|e_jg_kn \pm 1\rangle$ and $|g_je_kn \pm 1\rangle$, is also equal to ξ. The further Stark shift for the state $|e_j\rangle$ induced by the nonresonant Raman coupling is

$$\begin{aligned}\eta &= \frac{\langle e_jn|\, H_i'\,|g_jn+1\rangle\,\langle g_jn+1|\, H_i'\,|e_jn\rangle}{\delta+2\nu} \\ &\quad + \frac{\langle e_jn|\, H_i'\,|g_jn-1\rangle\,\langle g_jn-1|\, H_i'\,|e_jn\rangle}{-(\delta+2\nu)} \\ &= \xi/2. \end{aligned} \tag{9}$$

The Stark shift for $|g_j\rangle$ is also η. Then the effective Hamiltonian is given by

$$\begin{aligned}H_e &= \xi\left[\frac{1}{2}\sum_{j=1}^{2}(|e_j\rangle\langle e_j| + |g_j\rangle\langle g_j|)\right. \\ &\quad \left. + (S_1^+S_2^+ + S_1^+S_2^- + \text{H.c.})\right] \\ &= 2\xi\left(\sum_{j=1}^{2}S_{1,x}\right)^2, \end{aligned} \tag{10}$$

where

$$S_{j,x} = \frac{1}{2}(S_j^+ + S_j^-). \tag{11}$$

The distinct feature of the effective Hamiltonian is that it is independent of the photon-number of the cavity field. The evolution operator of the system is given by $U(t) = \mathrm{e}^{-\mathrm{i}H_et}$. Define the new atomic basis states

$$\begin{aligned}|+_j\rangle &= \frac{1}{\sqrt{2}}(|g_j\rangle + |e_j\rangle), \\ |-_j\rangle &= \frac{1}{\sqrt{2}}(|g_j\rangle - |e_j\rangle), \end{aligned} \tag{12}$$

which are the eigenstates of $S_{j,x}$ with the eigenvalues $1/2$ and $-1/2$, respectively. The quantum information is encoded in the states $|+\rangle$ and $|-\rangle$. The evolution operator $U(t)$ leads to

$$\begin{aligned}&|+_1\rangle\,|+_2\rangle \to \mathrm{e}^{-\mathrm{i}2\xi t}\,|+_1\rangle\,|+_2\rangle\,, \\ &|+_1\rangle\,|-_2\rangle \to |+_1\rangle\,|-_2\rangle\,, \\ &|-_1\rangle\,|+_2\rangle \to |-_1\rangle\,|+_2\rangle\,, \\ &|-_1\rangle\,|-_2\rangle \to \mathrm{e}^{-\mathrm{i}2\xi t}\,|-_1\rangle\,|-_2\rangle\,. \end{aligned} \tag{13}$$

After performing the single-qubit rotation $|+_j\rangle \to \mathrm{e}^{\mathrm{i}\xi t}\,|+_1\rangle$ and $|-_j\rangle \to \mathrm{e}^{-\mathrm{i}\xi t}\,|-_j\rangle$ we obtain

$$\begin{aligned}&|+_1\rangle\,|+_2\rangle \to |+_1\rangle\,|+_2\rangle\,, \\ &|+_1\rangle\,|-_2\rangle \to |+_1\rangle\,|-_2\rangle\,, \\ &|-_1\rangle\,|+_2\rangle \to |-_1\rangle\,|+_2\rangle\,, \\ &|-_1\rangle\,|-_2\rangle \to \mathrm{e}^{-\mathrm{i}4\xi t}\,|-_1\rangle\,|-_2\rangle\,. \end{aligned} \tag{14}$$

In this way we obtain a quantum phase gate, in which the system undergoes a phase shift $-4\xi t$ if and only if it is initially in the state $|-_1\rangle\,|-_2\rangle$.

We now briefly address the experimental feasibility of the proposed scheme. Set $\Delta = 20g$, $\delta = \Omega = g$, and $\Gamma = \kappa = 0.01g$,[13] where Γ and κ are the atomic spontaneous decay rate and cavity decay rate, respectively. Then we have $\lambda = g/20$ and $\xi = g/220$. The time needed to produce a conditional phase π is $t = \pi/4\xi = 55\pi/g$. As the atoms and cavity mode are only virtually excited during the evolution, the effective decoherence rates due to the atomic spontaneous emission and cavity decay are $\Gamma_e = \Gamma\Omega^2/\Delta^2 = 2.5 \times 10^{-5}g$ and $\kappa_e = \kappa\lambda^2/(\delta+2\nu)^2 = 2.07 \times 10^{-5}g$. Thus the infidelity induced by the atomic spontaneous emission is about $t(\Gamma_e + \kappa_e) = 7.89 \times 10^{-3}$. The scheme requires the Lamb–Dicke limit. In recent experiments,[14,15] single $^{40}Ca^+$ ions were localized at a fixed position in a cavity with a high precision for long times and the spatial variation of the coupling strength can be neglected, allowing the achievement of the Lamb–Dicke limit. On the other hand, the localization to the Lamb–Dicke limit of the axial motion was demonstrated for a single atom trapped in an optical cavity.[16]

In conclusion, we have proposed a scheme for implementing quantum phase gates for two atoms with two excited states and two ground states. In the scheme the quantum information is encoded in ground states. Under the large detuning condition the two atoms are coupled via two nonresonant Raman processes induced by a cavity mode and two classical

fields. During the operation the atoms are always populated in the ground states and the cavity mode is only virtually excited and thus the scheme is insensitive to atomic spontaneous emission, cavity decay and thermal photons. In addition, individual addressing of the atoms is unnecessary

References

[1] Shor P W 1994 *Proceedings of the 35th Annual Symposium on Foundations of Computer Science* (Los Alamitos CA: IEEE Computer Society Press) p116

[2] Sleator T and Weinfurter H 1995 *Phys. Rev. Lett.* **74** 4087

[3] Schmidt-Kaler F, Haffner H, Riebe M, Gulde S, Lancaster G P T, Deuschle T, Becher C, Roos C F, Eschner J and Blatt R 2003 *Nature* **422** 408

[4] Leibfried D, DeMarco B, Meyer V, Lucas D, Barrett M, Britton J, Itano W M, Jelenkovic B, Langer C, Rosenhand T and Wineland D J 2003 *Nature* **422** 412

[5] Rauschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M, and Haroche S 1999 *Phys. Rev. Lett.* **83** 5166

[6] Knill E 2005 *Nature* **434** 39

[7] Zheng S B and Guo G C 2000 *Phys. Rev. Lett.* **85** 2392

[8] Pellizzari T, Gardiner S A, Cirac J I and Zoller P 1995 *Phys. Rev. Lett.* **75** 3788

[9] Beige A, Braun D, Tregenna B and Knight P L 2000 *Phys. Rev. Lett.* **85** 1762

[10] Pachos J and Walther H 2002 *Phys. Rev. Lett.* **89** 187903

[11] Sϕrensen A and Mϕlmer K 1999 *Phys. Rev. Lett.* **82** 1971

[12] Zheng S B 2003 *Phys. Rev.* A **68** 035801

[13] Serafini A, Mancini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503

[14] Mundt A B, Kreuter A, Becher C, Leibfried D, Eschner J, Schmidt-Kaler F and Blatt R 2002 *Phys. Rev. Lett.* **89** 103001

[15] M. Keller, Lange B, Hayasaka K, Lange W and Walther H 2004 *New J. Phys.* **6** 95

[16] Boozer A D, Boca A, Miller R, Northup T E and Kimble H J 2006 quant-ph/0606104