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External-cavity birefringence feedback effects of microchip Nd:YAG laser and its application in angle measurement*

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External-cavity birefringence feedback effects of the microchip Nd:YAG laser are presented. When a birefringence element is placed in the external feedback cavity of the laser, two orthogonally polarized laser beams with a phase difference are output. The phase difference is twice as large as the phase retardation in the external cavity along the two orthogonal directions. The variable extra-cavity birefringence, caused by rotation of the external-cavity birefringence element, results in tunable phase difference between the two orthogonally polarized beams. This means that the roll angle information has been translated to phase difference of two output laser beams. A theoretical analysis based on the Fabry–Perot cavity equivalent model and refractive index ellipsoid is presented, which is in good agreement with the experimental results. This phenomenon has potential applications for roll angle measurement.

Keywords: microchip laser, external-cavity birefringence feedback, orthogonally polarized lasers, angle measurement

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1. Introduction

The output intensity of a gas laser can be greatly affected by external optical feedback and has a similar phase sensitivity and modulation depth ratio to the conventional interference, which was first reported by King and Steward.[1] Due to its inherent simplicity, compactness, robustness, and economy, compared with the conventional interference, the optical feedback technique has been widely used for measurement of Doppler velocity,[2,3] displacement,[4,5] distance,[6,7] profile,[8,9] and vibration.[10,11]

Recently, much attention has been paid to optical feedback from an external birefringence cavity,[12,13] in which a birefringence element and an external mirror together formed an anisotropic external cavity,[14,15] in these schemes, with only one optical path, two orthogonally polarized laser beams are output, both modulated by the external cavity length. Fei *et al*[16] found polarization flipping accompanied by intensity transfer between two orthogonally polarized laser modes, while one of the optical axes (o-axis or e-axis) of the birefringence element is set to be parallel to the initial polarization direction of the laser. Consequently, the two orthogonal eigenstates cannot coexist at one time. Tan and Zhang[17] set the laser initial polarization direction at 45° with respect to the two optical axes of the birefringence element. Hence, feedback light beams of the two orthogonal directions whose phases differ would affect the laser intensities at the same time. Consequently, the two orthogonal laser eigenstates with a certain phase difference are output, which can be used to realize high resolution and direction discrimination for precise displacement measurement. However, in Ref.[17], the phase difference between two signals is certain, corresponding to a specific wave plate. To obtain different phase retardation, we need different wave plates, some of which are hard to be fabricated and obtained.

In this paper, we systematically study the optical feedback characteristics of a microchip Nd:YAG laser with a birefringence element rotated in the external cavity. Due to the birefringence effect, the external cavity modulates the laser output intensities in two orthogonal detecting directions (X-axis and Y-axis) with a phase difference. By rotating the external-cavity birefringence element and tuning the angle between X-axis (one of the detecting directions) and o-

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axis (one of the optical axes of the birefringence element), which results in variable extra-cavity birefringence along the two orthogonal directions for detection, two orthogonally polarized laser beams with tunable phase difference are output, both modulated by the external cavity length. The range of their phase difference is found to be from zero to twice as large as that of the birefringence element. A theoretical analysis based on the Fabry–Perot (FP) cavity equivalent model and refractive index ellipsoid is presented. Due to the translation from the roll angle information of the birefringence element in the feedback cavity to phase difference of two output laser beams, this phenomenon is promising for application to the precision measurement of roll angle.

2. Experimental setup

Experiments are carried out on a diode-pumped microchip Nd:YAG laser operating at 1.06 μm with single longitudinal mode. The experimental setup with original coordinates is shown schematically in Fig.1.

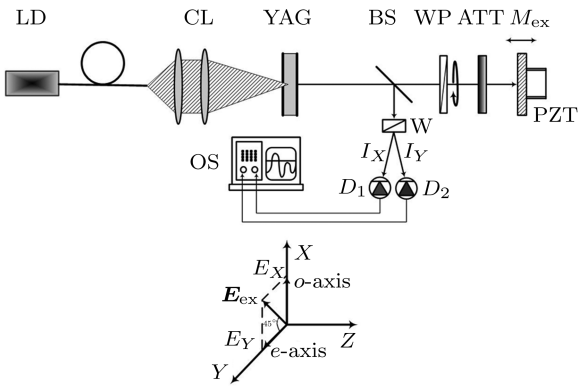


Fig.1. Experimental setup and original coordinates.

The laser chip used consists of a parallel-plane plate of 2 mm in thickness and Φ5 mm in diameter, made from Nd:YAG crystal containing 1.1 at% Nd^{3+} ions. Its pumping surface is dielectric-coated with a reflectivity of $R_1 > 99.8\%$ at the lasing wavelength of $\lambda = 1.064$ μm, but antireflective at the pumping wavelength of $\lambda = 808$ nm, while the output face is coated with a reflectivity of $R_2 = 98.8\%$ at $\lambda = 1.064$ μm. A fiber-coupled laser diode LD is used as a pump, whose output is focused on the laser crystal by collimating and focusing lenses CL. BS is a beam splitter used to divide the output into two parts. One part is used as optical feedback and is reflected by an external feedback mirror M_{ex}, whose reflectivity is $R_3 = 50\%$. WP is a quarter wave plate, rotatable about the optical axis. ATT is a variable attenuator used to adjust the appropriate feedback level to avoid multi-feedback in the external cavity.[18] PZT is a piezoelectric transducer that drives the mirror M_{ex} to modify the external cavity length. The other output is divided by a Wollaston prism W into two orthogonally polarized directions, the X-direction and the Y-direction, and their intensities (I_X and I_Y) are detected by photoelectric detectors D_1 and D_2, respectively. The laser output is in single longitudinal mode with a certain polarization direction E_{ex} without optical feedback. The original angle between the laser polarization direction E_{ex} and the X-axis or Y-axis of W is approximately 45°. The o-axis (fast axis) and e-axis (slow axis) of the wave plate WP are respectively set parallel to the X-axis and Y-axis originally.

3. Experimental results

First, the wave plate WP is removed from the external cavity. We connect the feedback loop and drive the PZT with a triangular wave to change the length of the external feedback cavity. For the isotropic feedback, the laser intensities of I_X and I_Y are in-phase, as shown in Fig.2(a), they are just orthogonal components of the only one output eigenstate, whose polarization direction is the same as that of the original single longitudinal mode. As has been reported many times previously, it is easy to observe the laser intensity modulation in the form of a cosine or quasi-cosine waveform. An undulation fringe of the laser intensity corresponds to only half wavelength ($\lambda/2$) variation of the external cavity length. Then, we put a quarter wave plate WP into the external cavity, and set the o-axis of the wave plate along the X-axis, as shown in Fig.1. Due to birefringence effects, the linearly polarized E_{ex} in the external cavity will be split into two components: E_x and E_y, with a phase difference between them. The external feedback mirror feeds back both of them into the laser active cavity, and modulates the laser intensities in the two orthogonal directions with a phase difference. As can be seen in Fig.2(b), both of the laser intensities I_X and I_Y are cosine waveforms and change periodically with a phase retardation of π.

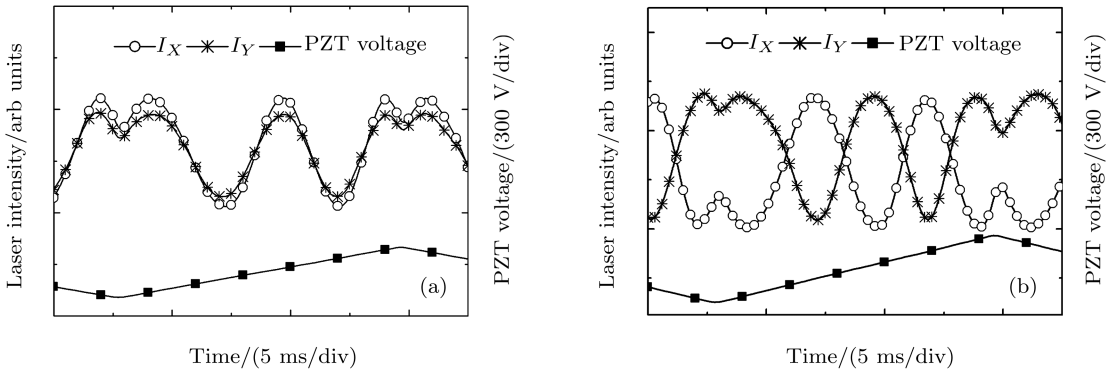


Fig.2. Orthogonally polarized laser lights (a) with conventional optical feedback; (b) with birefringence external cavity feedback.

A series of experiments have been done, the results of which are shown in Fig.3. When the angle between the *o*-axis of the quarter wave plate and the X-axis is tuned from 0° to 90°, the two output orthogonally polarized laser beams are both of cosine wave form and the phase difference between them is tuned from –180° to 180°. The maximal phase difference is twice as large as the phase retardation of the wave plate in the external cavity, when the optical axes are parallel to the X-axis and Y-axis.

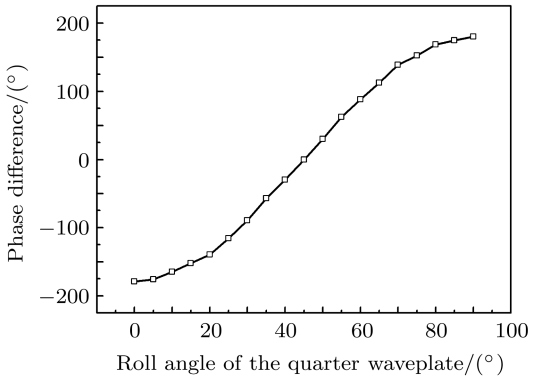


Fig.3. Phase difference between the two orthogonally polarized laser lights, when the angle between *o*-axis of the quarter wave plate and X-axis is tuned from 0° to 90°.

4. Theoretical analysis

The model of an equivalent cavity of an FP interferometer[19,20] has been developed and widely used for explaining intensity modulation of laser with optical feedback. The intensity modulation mechanism of optical feedback has been considered as the modulation of the threshold gain of a laser.

First, the wave plate is removed from the external cavity. The initial electric vector $E_0(t)$ exiting from output surface of the Nd:YAG laser travels a round trip in the laser active cavity and the external feedback cavity, respectively. After the round trips, $E_0(t)$ turns into the final electric vector $E(t)$, which can be written as

$$E(t) = r_1 r_2 \exp\left(j4\pi\nu\frac{nd}{c} + 2gd\right) E_0(t) + r_1 T_2 r_3 \xi \exp\left(j4\pi\nu\frac{nd+l}{c} + 2gd\right) E_0(t), \tag{1}$$

where r_1 and r_2 are the amplitude reflection coefficients of both end faces of Nd:YAG crystal; $T_2 = 1-r_2^2$ is the transmission coefficient of the laser coupled from the laser cavity to the external cavity; r_3 is the amplitude reflection coefficient of the external feedback mirror, ξ is the feedback ratio of the external optical field coupled into the laser cavity, l is the external cavity length, n is the effective refractive index of the laser cavity material, d is the length of gain medium, and g is the linear gain per unit length due to the stimulated emission inside the laser cavity in the presence of feedback.

According to the laser equilibrium requirement $E(t) = E_0(t)$, The following equation can be obtained:

$$r_1 r_2 \exp\left(j4\pi\nu\frac{nd}{c} + 2gd\right)\left[1 + \beta\exp\left(j4\pi\nu\frac{l}{c}\right)\right] = 1 \tag{2}$$

with $\beta = Tr_3\xi/r_2$. From Eq.(2), the excess required gain Δg in the presence of optical feedback can be

expressed as

$$\Delta g = -\frac{\beta}{2nd}\cos\left(4\pi\nu\frac{l}{c}\right). \tag{3}$$

The laser intensity can be expressed as:

$$I = I_0(1 - k\Delta g) = I_0(1 + \varsigma\cos(\varphi)), \tag{4}$$

where k is a constant related to the laser operation parameters, $\varphi = 4\pi\nu l/c$, $\zeta = \beta k/2nd$. And I_0 is the initial laser intensity without optical feedback.

Then, we consider the birefringence feedback. With a wave plate in the external cavity, the cavity can be decomposed into two different physical external cavities because of the birefringence effect of the wave plate. According to our coordinates, we define the lengths of the two physical cavities in which the polarization directions are along the X-axis and Y-axis respectively, as l_X and l_Y. For a birefringence wave plate, the following equation can be obtained

$$l_Y - l_X = \left|\frac{\lambda\delta}{2\pi}\right|, \tag{5}$$

where δ is the phase retardation between X- and Y-directions in the external cavity.

The electric field $E_{\rm ex}$ in the external cavity can also be divided into two orthogonal electric fields E_X and E_Y along the X-axis and the Y-axis respectively, as shown in Fig.1. These two orthogonal electric fields, with different phases, would affect the electric field in the laser active cavity at the same time. Thus with the optical feedback of two different kinds of laser light at the same time, the output light of the laser is composed of two parts that are affected by the E_X and E_Y respectively. According to the above theory, the required excess gain in the presence of anisotropic external feedback is also divided into two orthogonal parts, which can be expressed as

$$\begin{aligned}\Delta g_X &= -\frac{\beta}{2nd}\cos\left(4\pi\nu\frac{l_X}{c}\right),\\ \Delta g_Y &= -\frac{\beta}{2nd}\cos\left(4\pi\nu\frac{l_Y}{c}\right).\end{aligned} \tag{6}$$

By substituting Eq.(6) into Eq.(4), the two orthogonally polarized laser beams with anisotropic external feedback can be written as

$$\begin{aligned}I_X &= I_{0X}(1 + \varsigma\cos(\varphi)),\\ I_Y &= I_{0Y}(1 + \varsigma\cos(\varphi + 2\delta)).\end{aligned} \tag{7}$$

From Eq.(7), we can see that these two orthogonally polarized laser beams both are of cosine waveforms and have a phase difference of 2δ between them; δ can be expressed as

$$\delta = \frac{2\pi}{\lambda}(n_X - n_Y)d_{\rm wp}, \tag{8}$$

where n_X and n_Y are refractive-indices of the wave-plate along X- and Y-axes, respectively; and $d_{\rm wp}$ is the thickness of the wave plate.

The optical axis of the wave plate is parallel to its surface. Hence, the cross section of the refractive index ellipsoid of the wave plate in the X–Y plane is an ellipse, whose minor axis and major axis equal the ordinary refractive index $n_{\rm o}$ and the extraordinary refractive index $n_{\rm e}$ of the uniaxial crystal (the wave plate is made from quartz), that is why we usually call them the o-axis and e-axis, respectively, as shown in Fig.4.

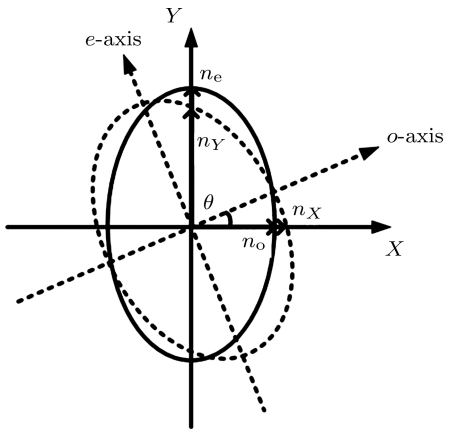


Fig.4. Refractive index ellipsoid of the wave plate in the X–Y plane.

When the wave plate is turned an angle θ, according to analytical geometry, n_X and n_Y can be expressed as

$$\begin{aligned}n_X &= \frac{n_{\rm o}n_{\rm e}}{\sqrt{n_{\rm o}^2\sin\theta^2 + n_{\rm e}^2\cos\theta^2}},\\ n_Y &= \frac{n_{\rm o}n_{\rm e}}{\sqrt{n_{\rm o}^2\cos\theta^2 + n_{\rm e}^2\sin\theta^2}}.\end{aligned} \tag{9}$$

From Eqs.(7) and (9), we can see that these two orthogonally polarized laser beams have a phase difference of 2δ between them, which is related to the roll angle between o-axis of the wave plate and the X-axis. The simulated results of phase difference between the two orthogonally polarized beams of the laser with a birefringence element of different phase retardation rotated in external cavity are shown in Fig.5.

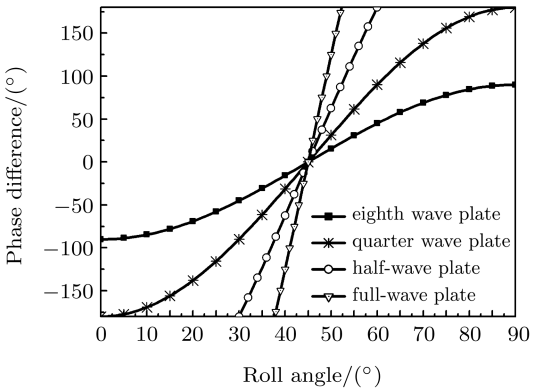


Fig.5. Simulation of phase difference between the two orthogonally polarized laser beams of the laser with a birefringence element rotated in external cavity.

At the beginning, the *o*-axis and *e*-axis are set parallel to the X-axis and Y-axis, respectively; $n_X = n_{\rm o}$, $n_Y = n_{\rm e}$, $\delta = \dfrac{2\pi}{\lambda}(n_X - n_Y)d_{\rm wp} = \dfrac{2\pi}{\lambda}(n_{\rm o} - n_{\rm e})d_{\rm wp}$ has the largest absolute value. As for a quarter wave plate, $\delta = -90°$, which means that the phase difference between the two orthogonal beams is $-\pi$, as shown in Fig.2(b). When the quarter wave plate is turned an angle θ, the intercept of the ellipse with X-axis, namely n_X, increases, and the intercept of it with Y-axis, namely n_Y, decreases, as shown in Fig.4. The absolute value of δ decreases and thus the phase difference between the two orthogonal beams declines correspondingly. When $\theta = 45°$, $n_X = n_Y$, $\delta = 0°$, the two channel signals are in-phase, as shown in Fig.2(a). Continuing to rotate the wave plate, we see that, when the roll angle of the wave plate is tuned from 0° to 90°, the result is to make the variable extra-cavity birefringence δ change from $-\pi/2$ to $\pi/2$, and in turn, the phase difference of two channel orthogonal laser eigenstates is changed from $-\pi$ to π. And the theoretical analyses are in good agreement with the experimental results.

Accurate measurement of the roll angle of the worktable in experiments plays a significant role to correct the Abbe-error of coordinate measuring machines and machine tools. The phenomenon we have just found seems to have potential application in this area. From Fig.5, we can see that based on the external birefringence feedback, the information of the roll angle of the birefringence element, which is placed in the external cavity of laser, can be translated into the phase difference between the two output orthogonally polarized laser beams. At the roll angle around 45°, there is a range of linearity that could be utilized for angle measurement. When the phase retardation of the birefringence element gets larger, the linearity of this range, as well as the sensibility, increases. For example, when an external-cavity full wave plate rotates an angle of ±7.5° in the linear range, the phase difference changes 2π, resulting in a resolution of 0.000125° (0.45 arcsec) obtained by using a phase meter with a resolution of 0.003°.[21] A roll-angle sensor based on the external-cavity birefringence feedback will expect to have high sensitivity, high stability, and a sufficiently wide linear range. Moreover, the optical configuration of this system is simple and compact, which can make the sensor light and small.

5. Conclusions

In summary, we have presented the optical feedback characteristics of a single mode Nd:YAG laser with a birefringence external cavity. Due to the birefringence effect of the wave plate placed in the external cavity, two kinds of orthogonally polarized laser beams with a phase difference can be fed back into the laser active cavity at the same time. Consequently, the two orthogonally polarized laser beams are output simultaneously, with a phase difference which is twice as large as the phase retardation between these two orthogonal directions in the external cavity. By rotating a quarter wave plate in the external cavity, and the roll angle being tuned from 0° to 90°, there appears a variable extra-cavity birefringence along the two orthogonal detecting directions, and the phase difference of these two channel orthogonally polarized laser eigenstates is changed from $-\pi$ to π. With only one optical path and one wave plate, the phase difference between these two channel cosine signals can be conveniently obtained and changed without the need for expensive electric devices and careful alignment of experimental setup, compared with common methods for retardation measurement.[22,23]

Furthermore, according to the relationship between roll angle and phase difference, the novel laser feedback system described in this paper offers a potential method for angle measurement, which can be simple, compact and on-line.

References

[1] King P G R and Steward G J 1963 *New Sci.* **17** 180

[2] Shinohara S, Mochizuki A, Yoshida H and Sumi M 1986 *Appl. Opt.* **25** 1417

[3] Kawai R, Asakawa Y and Otsuka K 1999 *IEEE Photon. Technol. Lett.* **11** 706

[4] Giulian G, Norgia M, Donati S and Bosch T 2002 *J. Opt.* A: *Pure Appl. Opt.* **4** S283

[5] Tan Y D, Zhang S L, Liu W X and Mao W 2007 *Chin. Phys.* **16** 1020

[6] Gouaux F, Servagent N and Bosch T 1998 *Appl. Opt.* **37** 6684

[7] De Groot P 2001 *Opt. Eng.* **40** 28

[8] Wang M and Lai G 2004 *Opt. Commun.* **238** 237

[9] Lu C, Wang J and Deng K 1995 *Appl. Phys. Lett.* **66** 2022

[10] Castellini P, Revel G M and Tomasini E P 1998 *Shock Vib. Dig.* **30** 443

[11] Giuliani G, Bozzi-Pietra S and Donati S 2003 *Meas. Sci. Technol.* **14** 24

[12] Sciamanna M, Panajotov K, Thienpont H, Veretennicoff I, Megret P and Blondel M 2003 *Opt. Lett.* **28** 1543

[13] Li H, Hohi A, Gavrielides A, Hou H and Choquette K D 1998 *Appl. Phys. Lett.* **70** 2355

[14] Ropars G, Le Floch A and Le Naour R 1992 *Phys. Rev.* A **46** 623

[15] Besnard P, Jia X, Dalgliesh R, May A D and Stephan G 1993 *J. Opt. Soc. Am.* B **10** 1605

[16] Fei L G, Zhang S L and Zong X B 2005 *Opt. Commun.* **246** 505

[17] Tan Y D and Zhang S L 2007 *Appl. Opt.* **46** 6064

[18] Mao W and Zhang S L 2006 *Chin. Phys.* **15** 340

[19] Bearden A, O'Neill M P, Osborne L C and Wong T L 1993 *Opt. Lett.* **18** 238

[20] Wang W M, Boyle W J O, Grattan K T V and Palmer A W 1993 *Appl. Opt.* **32** 1551

[21] Liu Z, Lin D J, Jiang H and Yin C Y 2003 *Sensors and Actuators* A **104** 127

[22] Shyu L H, Chen C L and Su D C 1993 *Appl. Opt.* **32** 4228

[23] Zhang Y, Zhang S and Li Y 2001 *Opt. Eng.* **40** 1071