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Kinetic theory of (2+4)-level atom in $\sigma^+ - \sigma^-$ laser fields*

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The kinetic theory of (2+4)-level atoms in $\sigma^+ - \sigma^-$ laser fields is presented. We systemically discuss friction coefficient, momentum diffusion tensor and atomic temperature based on the Fokker–Planck equation. This cooling system is much like that of a (1+3)-level atom, and the temperature is still limited to the Doppler temperature. Since this cooling system has not been investigated before, this work may be regarded as a necessary complement to the laser cooling theory.

Keywords: laser cooling, Fokker–Planck equation, quantum optics
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1. Introduction

Laser cooling technologies have been widely applied in many areas.[1−3] They range from a new generation of atomic clocks,[4] which are operating or under construction in many of the world's major standards laboratories, to the achievement of Bose–Einstein condensation in atomic alkali vapours.[5,6] Today, laser cooling and its applications represent one of the major subfields of atomic, molecular and optical physics. Scientists have researched many cooling systems,[7−13] and the kinetic theories of the basic multilevel atomic schemes: (1+3)-, (3+3)-, (3+5)-, (5+7)-, (7+9)-, and (9+11)-level atoms for the same case of a $\sigma^+ - \sigma^-$ cooling field configuration have been summarized in Ref.[13]. However, no one has investigated the dynamics of the interaction between a (2+4)-level atom and $\sigma^+ - \sigma^-$ laser fields so far. Now, we systemically discuss it in this paper.

2. Kinetic theory

A Fokker–Planck type kinetic equation is derived for a one-dimensional model of a (2+4)-level atom interacting with two counterpropagating circular polarized laser beams as shown in Fig.1. We numerically discuss radiation force, diffusion and temperature. Fields of two counter-propagating laser waves constituting a $\sigma^+ - \sigma^-$ field configuration can be expressed as

$$\boldsymbol{E}_1 = \frac{1}{2}\mathcal{E}\left(\boldsymbol{e}_+ \mathrm{e}^{\mathrm{i}(kz-\omega_l t)} - \boldsymbol{e}_- \mathrm{e}^{-\mathrm{i}(kz-\omega_l t)}\right), \quad (1)$$

$$\boldsymbol{E}_2 = \frac{1}{2}\mathcal{E}\left(\boldsymbol{e}_- \mathrm{e}^{-\mathrm{i}(kz+\omega_l t)} - \boldsymbol{e}_+ \mathrm{e}^{\mathrm{i}(kz+\omega_l t)}\right), \quad (2)$$

where $\boldsymbol{e}_\pm = \mp\frac{1}{\sqrt{2}}(\boldsymbol{e}_x \pm \mathrm{i}\,\boldsymbol{e}_y)$ are the spherical unit vectors, ω_l is the frequency of the cooling laser field, $k = \omega_l/c$ is the magnitude of the wave vector, and $\mathcal{E}$ is the amplitude of the field. With respect to quantization axis Oz, the first laser field in expression (1) is a σ^+ polarized wave and the second laser field is a σ^- polarized wave. Since we consider only the temperature in the center of magneto-optical trap (MOT), we can ignore the inhomogeneous static magnetic field, which is introduced to compensate for the Doppler frequency shift.

For the above interaction scheme, the atomic Hamiltonian has a standard form

$$H = H_0 - \left(\hbar^2\nabla^2/2M\right) - \boldsymbol{d}\cdot\boldsymbol{E}, \quad (3)$$

where Hamiltonian H_0 describes the internal atomic states with unperturbed energies $E_{g_{\pm 1/2}}$ and $E_{e_{\pm 1/2,\pm 3/2}}$, the second term on the right-hand side of expression (1) denotes the atomic momentum, and the last term describes the electric-dipole interaction between atom and field $\boldsymbol{E} = \boldsymbol{E}_1 + \boldsymbol{E}_2$. The electric-dipole interaction operator can be re-expressed as

$$\boldsymbol{d}\cdot\boldsymbol{E} = -d_- E_+ + d_0 E_0 - d_+ E_-, \quad (4)$$

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where d_α and E_α with $\alpha = 0, \pm$ are the spherical components of the dipole moments and fields respectively.

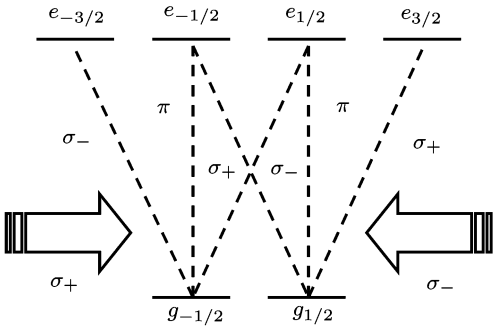


Fig.1. Atoms interacting with laser 1 which has polarization σ^+ traveling along the z-axis and laser 2 which has polarization σ^- traveling in the opposite direction.

All the information on the atomic motion in the laser fields is included in the density-matrix equation. The density matrix operator in the Wigner representation is expressed as $\rho = \rho(\boldsymbol{r}, \boldsymbol{p}, t)$ with atomic position $\boldsymbol{r}$ and momentum $\boldsymbol{p}$, and the density-matrix element is given by

$$\rho_{\alpha\beta}(\boldsymbol{r}, \boldsymbol{p}, t) = \langle\alpha|\rho(\boldsymbol{r}, \boldsymbol{p}, t)|\beta\rangle, \tag{5}$$

where $\alpha, \beta = L, S, J, M_J$ are the quantum numbers for denoting an atomic state. It is convenient to represent the electric field in the form of a plane wave expansion

$$\boldsymbol{E} = \sum_a \boldsymbol{E}^a \mathrm{e}^{\mathrm{i}\,\boldsymbol{k}_a\cdot\boldsymbol{r} - \mathrm{i}\,\omega_l t}. \tag{6}$$

Equations of motion for atomic density matrix elements in the Wigner representation can be expressed as[13]

$$\begin{aligned}
\mathrm{i}\,\hbar\frac{\mathrm{d}}{\mathrm{d}t}\rho_{kl}(\boldsymbol{r}, \boldsymbol{p}) = &-\sum_{a,m}\boldsymbol{d}_{km}\Big\{\boldsymbol{E}^a\rho_{ml}\left(\boldsymbol{r}, \boldsymbol{p} - \frac{1}{2}\hbar\boldsymbol{k}_a\right)\\
&\times \mathrm{e}^{\mathrm{i}\,\boldsymbol{k}_a\cdot\boldsymbol{r} - \mathrm{i}(\omega_l - \omega_{km})t}\\
&+ \boldsymbol{E}^{a*}\rho_{ml}\left(\boldsymbol{r}, \boldsymbol{p} + \frac{1}{2}\hbar\boldsymbol{k}_a\right)\\
&\times \mathrm{e}^{-\mathrm{i}\,\boldsymbol{k}_a\cdot\boldsymbol{r} + \mathrm{i}(\omega_l - \omega_{mk})t}\Big\}\\
&+\sum_{a,n}\boldsymbol{d}_{nl}\Big\{\boldsymbol{E}^a\rho_{kn}\left(\boldsymbol{r}, \boldsymbol{p} + \frac{1}{2}\hbar\boldsymbol{k}_a\right)\\
&\times \mathrm{e}^{\mathrm{i}\,\boldsymbol{k}_a\cdot\boldsymbol{r} - \mathrm{i}(\omega_l - \omega_{nl})t}\\
&+ \boldsymbol{E}^{a*}\rho_{kn}\left(\boldsymbol{r}, \boldsymbol{p} - \frac{1}{2}\hbar\boldsymbol{k}_a\right)\\
&\times \mathrm{e}^{-\mathrm{i}\,\boldsymbol{k}_a\cdot\boldsymbol{r} + \mathrm{i}(\omega_l - \omega_{ln})t}\Big\}\\
&-\mathrm{i}\,\hbar\langle k|\Gamma\rho(\boldsymbol{r}, \boldsymbol{p})|l\rangle,
\end{aligned} \tag{7}$$

where $\mathrm{d}/\mathrm{d}t = \partial/\partial t + \boldsymbol{v}\partial/\partial\boldsymbol{r}$ is the total time derivative, and the quantities $\omega_{ij} = (E_i - E_j)/\hbar$ are defined to be positive. Operator Γ describes spontaneous relaxation from the excited-state sublevel $|J_e, M_e\rangle$ to the ground-state level $|J_g, M_g\rangle$. All the dynamic equations for the atomic variables are derived and listed in Appendix A. The Wigner density-matrix elements are defined with respect to the time-dependent atomic eigenfunctions. Upper indices represent the momentum shifts due to the induced transition,

$$\rho_{ab}^{(\pm)} = \langle a|\rho(\boldsymbol{r}, \boldsymbol{p} \pm \frac{1}{2}\hbar\boldsymbol{k})|b\rangle, \tag{8}$$

where $\boldsymbol{k} = k\boldsymbol{e}_z$, and the notation $\langle\rho_{ll}^q\rangle$ is adopted for the density matrix elements averaged over the angular distribution of the spontaneous photon emission,[12−18]

$$\langle\rho_{ll}^q\rangle = \int \Phi_q(\boldsymbol{n})\langle l|\rho(\boldsymbol{r}, \boldsymbol{p} + \boldsymbol{n}\hbar k, t)|l\rangle\mathrm{d}\boldsymbol{n}. \tag{9}$$

The function $\Phi_q(\boldsymbol{n})$ ($q = 0, \pm 1$) determines the probability of the spontaneous photon emission in the direction of a unit vector $\boldsymbol{n}$,

$$\Phi_0(\boldsymbol{n}) = \frac{3}{8\pi}\left(1 - n_z^2\right), \tag{10}$$

$$\Phi_{\pm 1}(\boldsymbol{n}) = \frac{3}{16\pi}\left(1 + n_z^2\right). \tag{11}$$

Rabi frequency is defined as

$$\hbar\Omega = \langle e||d||g\rangle\mathcal{E}/4, \tag{12}$$

where $\langle e||d||g\rangle$ is the reduced matrix element. The detuning $\delta = \omega_l - \omega_{eg}$ is the difference between the laser frequency and the atomic transition frequency.

In order to follow the Bogolyubov procedure[12−18] we firstly exclude the explicit time dependence from the density matrix equations, and perform the substitutions $\rho_{g_\alpha e_\beta}\mathrm{e}^{-\mathrm{i}\,\delta t} = \sigma_{g_\alpha e_\beta}$. Before cooling, it is assumed that the momentum width of the atomic density-matrix elements is much wider than the photon momentum $\hbar k$. Thus the atomic density matrix elements $\rho_{ij}(\boldsymbol{r}, \boldsymbol{p} + \hbar\boldsymbol{k}, t)$ can be expanded in the photon momentum $\hbar k$. This principal assumption allows one to expand the density-matrix elements in the power of photon momentum $\hbar k$. Considering that the expanded equations are in successively increasing orders in photon momentum $\hbar k$, one can conclude that the diagonal $\rho_{\alpha\alpha}$ and off-diagonal density-matrix elements $\sigma_{\alpha\beta}$ are the functionals of the Wigner distribution function $w(\boldsymbol{r}, \boldsymbol{p}, t)$

$$w(\boldsymbol{r}, \boldsymbol{p}, t) = \sum\rho_{g_\alpha g_\alpha} + \sum\rho_{e_\beta e_\beta}. \tag{13}$$

The general structure of the functional dependence can be found directly from the structure of the expanded equations as follows:

$$\rho_{g_\alpha g_\alpha} = \left(\sum_{n=0,\pm 2,\ldots} H^n_{g_\alpha g_\alpha} \mathrm{e}^{\mathrm{i}\, nkz} \right) w + \left(\sum_{n=0,\pm 2,\ldots} R^n_{g_\alpha g_\alpha} \mathrm{e}^{\mathrm{i}\, nkz} \right) \hbar k \frac{\partial w}{\partial p_z}, \tag{14}$$

$$\sigma_{g_\alpha e_\beta} = \left(\sum_{n=1,\pm 3,\ldots} H^n_{g_\alpha e_\beta} \mathrm{e}^{\mathrm{i}\, nkz} \right) w + \left(\sum_{n=1,\pm 3,\ldots} R^n_{g_\alpha e_\beta} \mathrm{e}^{\mathrm{i}\, nkz} \right) \hbar k \frac{\partial w}{\partial p_z}, \tag{15}$$

$$\rho_{e_\alpha e_\alpha} = \left(\sum_{n=0,\pm 2,\ldots} H^n_{e_\alpha e_\alpha} \mathrm{e}^{\mathrm{i}\, nkz} \right) w + \left(\sum_{n=0,\pm 2,\ldots} R^n_{e_\alpha e_\alpha} \mathrm{e}^{\mathrm{i}\, nkz} \right) \hbar k \frac{\partial w}{\partial p_z}, \tag{16}$$

where $H^n_{\alpha\beta}$ and $R^n_{g_\alpha g_\beta}$ are functions of the atomic momentum $\boldsymbol{p}$. Here in order to exclude the explicit coordinate dependence from the expanded equations, the spatial series representation is introduced. In accordance with the definition of the distribution function $w(\boldsymbol{r}, \boldsymbol{p}, t)$, the unknown diagonal functions satisfy the normalization conditions

$$\sum H^n_{g_\alpha g_\alpha} + \sum H^n_{e_\beta e_\beta} = \delta_{n,0}, \tag{17}$$

and

$$\sum R^n_{g_\alpha g_\alpha} + \sum R^n_{e_\beta e_\beta} = 0. \tag{18}$$

Next, by taking into account the general structure of the solution, one can see from the expanded equations that the Wigner function $w(\boldsymbol{r}, \boldsymbol{p}, t)$ satisfies the closed equations. To the first order in the photon momentum $\hbar k$, the closed equation for the distribution function can be shown to be the Liouville equation,[12–18]

$$\frac{\partial w}{\partial t} + \boldsymbol{v} \frac{\partial w}{\partial \boldsymbol{r}} = -\frac{\partial}{\partial p_z} (F_z w), \tag{19}$$

where the radiation force F_z has a series representation given as

$$F_z = \hbar k \Omega \sum_{n=0,\pm 2,\ldots} f^n_z \mathrm{e}^{\mathrm{i}\, nkz}, \tag{20}$$

and

$$f^n_z = \mathrm{i} \frac{1}{\sqrt{3}} \left(H^{n-1}_{g_{-1}e_1} - H^{n+1}_{e_1 g_{-1}} - H^{n+1}_{g_1 e_{-1}} + H^{n-1}_{e_{-1}g_{-1}} \right) + \mathrm{i} \left(H^{n-1}_{g_1 e_3} - H^{n+1}_{e_3 g_1} - H^{n+1}_{g_{-1}e_{-3}} + H^{n-1}_{e_{-3}g_{-1}} \right). \tag{21}$$

Here we have used the shorthand notations $e_{\pm 1/2,\pm 3/2} \equiv e_{\pm 1,\pm 3}$ and $g_{\pm 1/2} = g_{\pm 1}$. Note that the force is defined by a real equation since complex harmonics of the force satisfy the conditions $f^*_n = f_{-n}$, following the Hermiticity conditions for optical coherence, $H^{n*}_{\alpha\beta} = H^{-n}_{\beta\alpha}$. The harmonics $H^n_{g_\alpha g_\alpha}$, $H^n_{e_\alpha e_\beta}$ and $H^n_{e_\alpha e_\alpha}$ satisfy an infinite set of steady-state equations which come from the set of expanded equations for the zeroth order in the photon momentum (listed in Appendix B).

For the second-order in the photon momentum $\hbar k$, the kinetic equation for the function $w(\boldsymbol{r}, \boldsymbol{p}, t)$ reduces to the Fokker–Planck equation

$$\frac{\partial w}{\partial t} + \boldsymbol{v} \frac{\partial w}{\partial \boldsymbol{r}} = -\frac{\partial}{\partial p_z} (F_z w) + \sum \frac{\partial^2}{\partial p_i \partial p_j} (D_{ij} w), \tag{22}$$

where $i, j = x, y, z$, and D_{ij} is the momentum diffusion tensor. The diffusion tensor has the series representation

$$D_{ij} = \hbar^2 k^2 \gamma \sum_{n=0,\pm 2,\ldots} d^n_{ij} \mathrm{e}^{\mathrm{i}\, nkz}. \tag{23}$$

Each spatial harmonic of the diffusion tensor includes two parts $d^n_{ij} = \delta_{iz} d^{dn}_{zz} + d^{an}_{ij}$. The first part is proportional to the optical coherence $R^n_{g_0 e_\alpha}$ and can be expressed as

$$d^{dn}_{zz} = \mathrm{i} \frac{\Omega}{2\gamma} \left(-R^{n-1}_{g_1 e_3} + R^{n+1}_{e_3 g_1} + R^{n+1}_{g_{-1}e_{-3}} - R^{n-1}_{e_{-3}g_{-1}} \right) + \mathrm{i} \frac{\Omega}{2\sqrt{3}\gamma} \left(-R^{n-1}_{g_{-1}e_1} + R^{n+1}_{e_1 g_{-1}} + R^{n+1}_{g_1 e_{-1}} - R^{n-1}_{e_{-1}g_1} \right). \tag{24}$$

It comes from the fluctuation in the number of scattered photons. The second part of the diffusion tensor is proportional to the angular anisotropy coefficients α^σ_{ii} and α^π_{ii}. It originates from the fluctuation in the direction of spontaneous photon emission. In this case the coefficients α^σ_{ii} and α^π_{ii}, which determine the angular anisotropy of the spontaneous photon emission,

$$\alpha^\sigma_{ii} = \int \Phi_\sigma(\boldsymbol{n}') n'^2_i d^2 n', \tag{25}$$

$$\alpha^\pi_{ii} = \int \Phi_\pi(\boldsymbol{n}') n'^2_i d^2 n', \tag{26}$$

become

$$\alpha^\sigma_{xx} = \alpha^\sigma_{yy} = \frac{3}{10}, \quad \alpha^\sigma_{zz} = \frac{2}{5},$$

$$\alpha^\pi_{xx} = \alpha^\pi_{xx} = \frac{2}{5}, \quad \alpha^\pi_{zz} = \frac{1}{5},$$

and the second part of the diffusion tensor reduces to the approximate expression

$$d_{ii}^{an} = \frac{2}{5}\left(H^n_{e_3e_3} + \frac{1}{3}H^n_{e_{-1}e_{-1}} + \frac{1}{3}H^n_{e_1e_1} + H^n_{e_{-3}e_{-3}}\right) + \frac{1}{5}\left(\frac{2}{3}H^n_{e_{-1}e_{-1}} + \frac{2}{3}H^n_{e_1e_1}\right). \quad (27)$$

The harmonics of the diffusion tensor satisfy the Hermiticity conditions, $d^{n*}_{ii} = d^{-n}_{ii}$. Steady-state equations of the function R^n_{ab} expanded to the first-order photon momentum are listed in Appendix C.

Kinetic coefficients F_z and D_{ii} each have a well-known physical meaning. The dipole force F_z comes, as usual, from the photon momentum exchange between an atom and a laser field, and determines the rate of change of the expectation value of atomic momentum. In the considered case where the dipole interaction of an atom with two plane laser waves takes place, the force $F_{\rm z}$ means the radiation pressure force. The quantity D_{ii} is the momentum diffusion tensor that determines the rates of broadening the atomic momentum distribution due to the quantum fluctuations in atomic momentum. Below, we will discuss $F_{\rm z}$, D_{ii}, and atomic temperature T.

To exclude the explicit coordinate dependence, the matrix elements should be expressed as

$$H_{g_\alpha g_\beta} \equiv H^0_{g_\alpha g_\beta} + H^{-2}_{g_\alpha g_\beta}\mathrm{e}^{-\mathrm{i}\,2kz} + H^2_{g_\alpha g_\beta}\mathrm{e}^{\mathrm{i}\,2kz} + \ldots, \quad (28)$$

$$H_{g_\alpha e_\beta} \equiv H^{-1}_{g_\alpha e_\beta}\mathrm{e}^{-\mathrm{i}\,kz} + H^1_{g_\alpha e_\beta}\mathrm{e}^{\mathrm{i}\,kz} + \ldots, \quad (29)$$

$$H_{e_\alpha e_\beta} \equiv H^0_{e_\alpha e_\beta} + H^{-2}_{e_\alpha e_\beta}\mathrm{e}^{-\mathrm{i}\,2kz} + H^2_{e_\alpha e_\beta}\mathrm{e}^{\mathrm{i}\,2kz} + \ldots. \quad (30)$$

The above decompositions show that the multi-photon process plays an important role in the atom-field interaction. Identification of the processes follows most clearly from the equations for different orders of the rate equation approximation (REA). As usual, the calculation of atomic functions in the $2n$-th order REA implies that the ground- and the upper-state populations and coherence are calculated in the $2n$-th order REA while the optical coherences are calculated in the $(2n-1)$-th order REA. Considering atomic populations and coherences in the second order REA, one takes into account the direct one-photon and two-photon processes and stepwise process composed of the direct processes. When atomic functions are considered in the fourth order REA, one takes into account the direct one-, two-, three-, and four-photon processes, and stepwise processes composed of the above direct process. In this paper, we calculate all the variables to the 4-th order REA.

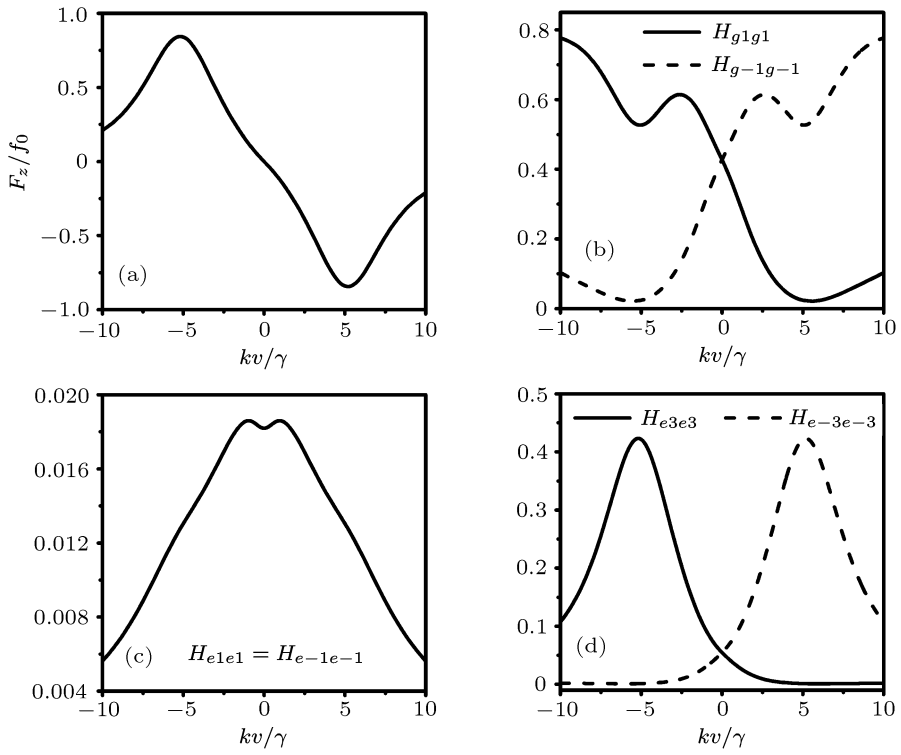


Fig.2. Radiation force F_z and atomic distributions each as a function of kv with $\delta = -5\gamma$ and $\Omega = 2\gamma$. $f_o = \hbar k\gamma$ is the unit of F_z.

By numerically solving the equations listed in Appendix B, we can obtain the radiation force F_z. In Fig.2, we show F_z and atomic distribution among the levels each as a function of speed v. One can see that the transitions $|e_{\pm1}\rangle \leftrightarrow |g_{\mp1}\rangle$ do not contribute to the radiation force. F_z is almost from the transitions $|e_{\pm3}\rangle \leftrightarrow |g_{\pm1}\rangle$. Thus, a (2+4)-level system is much like a (1+3)-level system, since in both systems the middle Zeeman sublevels of the upper state nearly do not join the atomic-field interaction. In a small speed region the radiation force is reduced to the friction in the form of

$$F_z = -\beta v, \tag{31}$$

where $v = v_z$ and β is a friction coefficient, which is defined as the slope of the radiation force near zero speed. In Fig.3, we show β as a function of detuning δ and saturation parameter $s = \Omega^2/2\gamma$ separately.

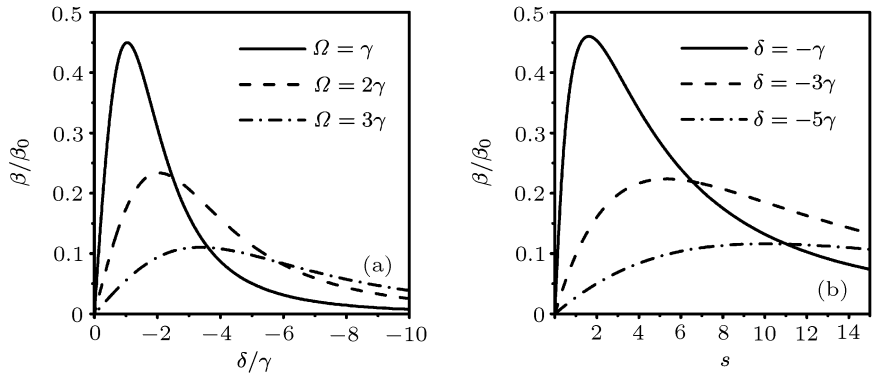


Fig.3. Friction coefficient β as a function of detuning δ (a) and saturation coefficient s (b) separately. $\beta_0 = \hbar k^2$ is the unit of β.

The next important kinetic quantity that determines the time evolution of an atom in a laser field is the diffusion longitudinal coefficient $D_{zz}(v = 0)$ at position $z = 0$. By numerically solving the equations listed in Appendix C, we obtain diffusion coefficient $D_{zz}(v = 0)$. Figure 4 shows $D_{zz}(v = 0)$ as a function of detuning δ and saturation parameter s separately.

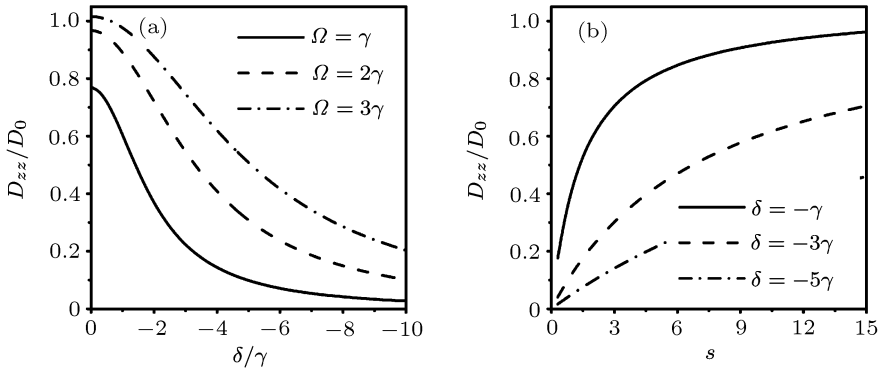


Fig.4. Diffusion coefficient D_{zz} as a function of detuning δ (a) and saturation coefficient s (b) separately. $D_0 = \hbar^2 k^2 \gamma$ is the unit of D_{zz}.

Explicit expressions for the coefficients of the Fokker–Planck equations can be directly applied to estimating the achievable temperatures in the scheme of laser cooling of atoms. The effective temperature T is defined by the Einstein relation as

$$T = \frac{D_{zz}(v = 0)}{\beta k_B}, \tag{32}$$

where $D_{zz}(v = 0)$ is the diffusion coefficient at zero speed. In Fig.5, we show T as a function of detuning δ and the saturation parameter s separately. As mentioned above, the (2+4)-level system is much like the (1+3)-level system. The temperature T has the same changing trend in both cases, and it is still limited to the Doppler

temperature. This is because the (2+4)-level atom is composed of two V-systems. There is no Λ-system, which causes the sub-Doppler cooling as in (3+3)-, (3+5)-, (5+7)-level atoms.

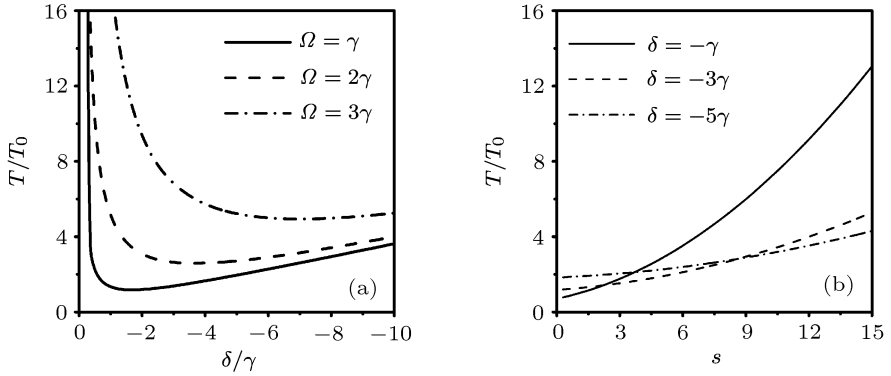


Fig.5. Atomic temperature T as a function of detuning δ (a) and saturation coefficient s (b) separately. $T_0 = \hbar\gamma/k_B$ is the unit of β.

3. Conclusion

We have systemically investigated the kinetic theory of (2+4)-level atom in $\sigma^+ - \sigma^-$ laser fields based on the Fokker–Planck equation. The limit of atomic temperature is still Doppler temperature. Since this cooling system has not been investigated before, this work may be regarded as a necessary complement to the laser cooling theory.

Appendix A

In the rotating-wave approximation, all the dynamic equations for the density-matrix elements can be expressed as follows:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_1g_1} = {} & \mathrm{i}\,\Omega\left(\varepsilon_{-+}\rho^{(+)}_{e_3g_1} - \varepsilon_{+-}\rho^{(+)}_{g_1e_3}\right) \\ & + \mathrm{i}\,\Omega'\left(\varepsilon_{++}\rho^{(-)}_{e_{-1}g_1} - \varepsilon_{--}\rho^{(-)}_{g_1e_{-1}}\right) \\ & + (2\gamma/3)\left\langle\rho^{(-1)}_{e_{-1}e_{-1}}\right\rangle + (4\gamma/3)\left\langle\rho^{(0)}_{e_1e_1}\right\rangle \\ & + 2\gamma\left\langle\rho^{(+1)}_{e_3e_3}\right\rangle, \end{aligned} \tag{A1}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_{-1}g_{-1}} = {} & \mathrm{i}\,\Omega\left(\varepsilon_{++}\rho^{(-)}_{e_{-3}g_{-1}} - \varepsilon_{--}\rho^{(-)}_{g_{-1}e_{-3}}\right) \\ & - \mathrm{i}\,\Omega'\left(\varepsilon_{+-}\rho^{(+)}_{g_{-1}e_1} - \varepsilon_{-+}\rho^{(+)}_{e_1g_{-1}}\right) \\ & + (4\gamma/3)\left\langle\rho^{(0)}_{e_{-1}e_{-1}}\right\rangle + (2\gamma/3)\left\langle\rho^{(+1)}_{e_1e_1}\right\rangle \\ & + 2\gamma\left\langle\rho^{(-1)}_{e_{-3}e_{-3}}\right\rangle, \end{aligned} \tag{A2}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_1e_1} = -2\gamma\rho_{e_1e_1} + \mathrm{i}\,\Omega'\left(\varepsilon_{+-}\rho^{(-)}_{g_{-1}e_1} - \varepsilon_{-+}\rho^{(-)}_{e_1g_{-1}}\right), \tag{A3}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_{-1}e_{-1}} = \mathrm{i}\,\Omega'\left(\varepsilon_{--}\rho^{(+)}_{g_1e_{-1}} - \varepsilon_{++}\rho^{(+)}_{e_{-1}g_1}\right) - 2\gamma\rho_{e_{-1}e_{-1}}, \tag{A4}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_3e_3} = -2\gamma\rho_{e_3e_3} + \mathrm{i}\,\Omega\left(\varepsilon_{+-}\rho^{(-)}_{g_1e_3} - \varepsilon_{-+}\rho^{(-)}_{e_3g_1}\right), \tag{A5}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_{-3}e_{-3}} = \mathrm{i}\,\Omega\left(\varepsilon_{--}\rho^{(+)}_{g_{-1}e_{-3}} - \varepsilon_{++}\rho^{(+)}_{e_{-3}g_{-1}}\right) - 2\gamma\rho_{e_{-3}e_{-3}}, \tag{A6}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_1e_{-1}} = -\gamma\rho_{g_1e_{-1}} + \mathrm{i}\,\Omega'\varepsilon_{++}\left(\rho^{(-)}_{e_{-1}e_{-1}} - \rho^{(+)}_{g_1g_1}\right) + \mathrm{i}\,\Omega\varepsilon_{-+}\rho^{(+)}_{e_3e_{-1}}, \tag{A7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_{-1}e_1} = -\gamma\rho_{g_{-1}e_1} + \mathrm{i}\,\Omega'\varepsilon_{-+}\left(\rho^{(+)}_{e_1e_1} - \rho^{(-)}_{g_{-1}g_{-1}}\right) + \mathrm{i}\,\Omega\varepsilon_{++}\rho^{(-)}_{e_{-3}e_1}, \tag{A8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_{-1}e_{-1}} = \mathrm{i}\,\Omega'\left(\varepsilon_{-+}\rho^{(+)}_{e_1e_{-1}} - \varepsilon_{++}\rho^{(+)}_{g_{-1}g_1}\right) + \mathrm{i}\,\Omega\varepsilon_{++}\rho^{(-)}_{e_{-3}e_{-1}} - \gamma\rho_{g_{-1}e_{-1}}, \tag{A9}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_1e_1} = -\gamma\rho_{g_1e_1} + \mathrm{i}\,\Omega'\left(\varepsilon_{++}\rho^{(-)}_{e_{-1}e_1} - \varepsilon_{-+}\rho^{(-)}_{g_1g_{-1}}\right) + \mathrm{i}\,\Omega\varepsilon_{-+}\rho^{(+)}_{e_3e_1}, \tag{A10}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_1e_3} = -\gamma\rho_{g_1e_3} + \mathrm{i}\,\Omega\varepsilon_{-+}\left(\rho^{(+)}_{e_3e_3} - \rho^{(-)}_{g_1g_1}\right) + \mathrm{i}\,\Omega'\varepsilon_{++}\rho^{(-)}_{e_{-1}e_3}, \tag{A11}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_{-1}e_{-3}} = -\gamma\rho_{g_{-1}e_{-3}} + \mathrm{i}\,\Omega\varepsilon_{++}\left(\rho^{(-)}_{e_{-3}e_{-3}} - \rho^{(+)}_{g_{-1}g_{-1}}\right) + \mathrm{i}\,\Omega'\varepsilon_{-+}\rho^{(+)}_{e_1e_{-3}}, \tag{A12}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_3e_1} = -2\gamma\rho_{e_3e_1} + \mathrm{i}\,\Omega\varepsilon_{+-}\rho^{(-)}_{g_1e_1} - \mathrm{i}\,\Omega'\varepsilon_{-+}\rho^{(-)}_{e_3g_{-1}}, \tag{A13}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_{-3}e_{-1}} = -2\gamma\rho_{e_{-3}e_{-1}} + \mathrm{i}\,\Omega\varepsilon_{--}\rho^{(+)}_{g_{-1}e_{-1}} - \mathrm{i}\,\Omega'\varepsilon_{++}\rho^{(+)}_{e_{-3}g_1}, \tag{A14}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_1e_{-1}} = -2\gamma\rho_{e_1e_{-1}} + \mathrm{i}\,\Omega'\varepsilon_{+-}\rho^{(-)}_{g_{-1}e_{-1}} - \mathrm{i}\,\Omega'\varepsilon_{++}\rho^{(+)}_{e_1g_1}, \tag{A15}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_3e_{-1}} = -2\gamma\rho_{e_3e_{-1}} + \mathrm{i}\,\Omega\varepsilon_{+-}\rho^{(-)}_{g_1e_{-1}} - \mathrm{i}\,\Omega'\varepsilon_{++}\rho^{(+)}_{e_3g_1}, \tag{A16}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_{-3}e_1} = -2\gamma\rho_{e_{-3}e_1} + \mathrm{i}\,\Omega\varepsilon_{--}\rho^{(+)}_{g_{-1}e_1} - \mathrm{i}\,\Omega'\varepsilon_{-+}\rho^{(-)}_{e_{-3}g_{-1}}, \tag{A17}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{e_3e_{-3}} = -2\gamma\rho_{e_3e_{-3}} + \mathrm{i}\,\Omega\varepsilon_{+-}\rho^{(-)}_{g_1e_{-3}} - \mathrm{i}\,\Omega\varepsilon_{++}\rho^{(+)}_{e_3g_{-1}}, \tag{A18}$$

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_1g_{-1}} = {}& \mathrm{i}\,\Omega'\left(\varepsilon_{++}\rho^{(-)}_{e_{-1}g_{-1}} - \varepsilon_{+-}\rho^{(+)}_{g_1e_1}\right) \\ & + \mathrm{i}\,\Omega\left(\varepsilon_{-+}\rho^{(+)}_{e_3g_{-1}} - \varepsilon_{--}\rho^{(-)}_{g_1e_{-3}}\right) \\ & + (4\gamma/3)\left\langle\rho^{(0)}_{e_{-1}e_1}\right\rangle + (2\gamma/\sqrt{3})\left\langle\rho^{(-1)}_{e_{-3}e_{-1}}\right\rangle \\ & + (2\gamma/\sqrt{3})\left\langle\rho^{(+1)}_{e_1e_3}\right\rangle,\end{aligned} \tag{A19}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_1e_{-3}} = -\gamma\rho_{g_1e_{-3}} + \mathrm{i}\,\Omega\left(\varepsilon_{-+}\rho^{(+)}_{e_3e_{-3}} - \varepsilon_{++}\rho^{(+)}_{g_1g_{-1}}\right) + \mathrm{i}\,\Omega'\varepsilon_{++}\rho^{(-)}_{e_{-1}e_{-3}}, \tag{A20}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{g_{-1}e_3} = -\gamma\rho_{g_{-1}e_3} + \mathrm{i}\,\Omega\left(\varepsilon_{++}\rho^{(-)}_{e_{-3}e_3} - \varepsilon_{-+}\rho^{(-)}_{g_{-1}g_1}\right) + \mathrm{i}\,\Omega'\varepsilon_{-+}\rho^{(+)}_{e_1e_3}, \tag{A21}$$

where we have used the shorthand notations: $e_{\pm3/2,\pm1/2} = e_{\pm3,\pm1}$, $g_{\pm1/2} = g_{\pm1}$, $\Omega' = \Omega/\sqrt{3}$, and $\varepsilon_{\pm\pm} = \exp(\pm\mathrm{i}\,kz \pm \mathrm{i}\,\delta t)$.

Appendix B

Below we list an infinite set of linear algebraic equations for the variables $H^n_{g_\alpha g_\beta}$, $H^n_{e_\alpha e_\beta}$, and $H^{n\pm1}_{g_\alpha e_\beta}$. The set of equations originates from Appendix A by considering the zeroth order in the photon momentum,

$$H^n_{g_1g_1} + H^n_{g_{-1}g_{-1}} + H^n_{e_3e_3} + H^n_{e_1e_1} + H^n_{e_{-1}e_{-1}} + H^n_{e_{-3}e_{-3}} = \delta_{n0}, \tag{B1}$$

$$(2\gamma/3)H^n_{e_{-1}e_{-1}} + (4\gamma/3)H^n_{e_1e_1} + 2\gamma H^n_{e_3e_3} + \mathrm{i}\,\Omega\left(H^{n+1}_{e_3g_1} - H^{n-1}_{g_1e_3}\right) + \mathrm{i}\,\Omega'\left(H^{n-1}_{e_{-1}g_1} - H^{n+1}_{g_1e_{-1}}\right) - \mathrm{i}\,nkvH^n_{g_1g_1} = 0, \tag{B2}$$

$$(4\gamma/3)H^n_{e_{-1}e_{-1}} + (2\gamma/3)H^n_{e_1e_1} + 2\gamma H^n_{e_{-3}e_{-3}} + \mathrm{i}\,\Omega\left(H^{n-1}_{e_{-3}g_{-1}} - H^{n+1}_{g_{-1}e_{-3}}\right) - \mathrm{i}\,\Omega'\left(H^{n-1}_{g_{-1}e_1} - H^{n+1}_{e_1g_{-1}}\right) - \mathrm{i}\,nkvH^n_{g_{-1}g_{-1}} = 0, \tag{B3}$$

$$-(2\gamma + \mathrm{i}\,nkv)H^n_{e_1e_1} + \mathrm{i}\,\Omega'\left(H^{n-1}_{g_{-1}e_1} - H^{n+1}_{e_1g_{-1}}\right) = 0, \tag{B4}$$

$$-(2\gamma + \mathrm{i}\,nkv)H^n_{e_{-1}e_{-1}} + \mathrm{i}\,\Omega'\left(H^{n+1}_{g_1e_{-1}} - H^{n-1}_{e_{-1}g_1}\right) = 0, \tag{B5}$$

$$-(2\gamma + \mathrm{i}\,nkv)H^n_{e_3e_3} + \mathrm{i}\,\Omega\left(H^{n-1}_{g_1e_3} - H^{n+1}_{e_3g_1}\right) = 0, \tag{B6}$$

$$-(2\gamma + \mathrm{i}\,nkv)H^n_{e_{-3}e_{-3}} + \mathrm{i}\,\Omega\left(H^{n+1}_{g_{-1}e_{-3}} - H^{n-1}_{e_{-3}g_{-1}}\right) = 0, \tag{B7}$$

$$-(\gamma + \mathrm{i}\,\delta + \mathrm{i}(n+1)kv)H^{n+1}_{g_1e_{-1}} + \mathrm{i}\,\Omega H^{n+2}_{e_3e_{-1}} + \mathrm{i}\,\Omega'\left(H^n_{e_{-1}e_{-1}} - H^n_{g_1g_1}\right) = 0, \tag{B8}$$

$$-(\gamma + \mathrm{i}\,\delta + \mathrm{i}(n-1)kv)H^{n-1}_{g_{-1}e_1} + \mathrm{i}\,\Omega H^{n-2}_{e_{-3}e_1} + \mathrm{i}\,\Omega'\left(H^n_{e_1e_1} - H^n_{g_{-1}g_{-1}}\right) = 0, \tag{B9}$$

$$-(\gamma + \mathrm{i}\,\delta + \mathrm{i}(n+1)kv)H^{n+1}_{g_{-1}e_{-1}} + \mathrm{i}\,\Omega H^n_{e_{-3}e_{-1}} + \mathrm{i}\,\Omega'\left(H^{n+2}_{e_1e_{-1}} - H^n_{g_{-1}g_1}\right) = 0, \tag{B10}$$

$$-(\gamma + \mathrm{i}\,\delta + \mathrm{i}(n-1)kv)H^{n-1}_{g_1e_1} + \mathrm{i}\,\Omega H^n_{e_3e_1} + \mathrm{i}\,\Omega'\left(H^{n-2}_{e_{-1}e_1} - H^n_{g_1g_{-1}}\right) = 0, \tag{B11}$$

$$-(\gamma + \mathrm{i}\,\delta + \mathrm{i}(n+1)kv)H^{n+1}_{g_1e_3} + \mathrm{i}\,\Omega' H^n_{e_{-1}e_3} + \mathrm{i}\,\Omega\left(H^{n+2}_{e_3e_3} - H^{n+2}_{g_1g_1}\right) = 0, \tag{B12}$$

$$-(\gamma + \mathrm{i}\,\delta + \mathrm{i}(n-1)kv)H^{n-1}_{g_{-1}e_{-3}} + \mathrm{i}\,\Omega' H^n_{e_1e_{-3}} + \mathrm{i}\,\Omega\left(H^{n-2}_{e_{-3}e_{-3}} - H^{n-2}_{g_{-1}g_{-1}}\right) = 0, \tag{B13}$$

$$-(2\gamma + \mathrm{i}\,nkv)H^n_{e_3e_1} + \mathrm{i}\,\Omega H^{n-1}_{g_1e_1} - \mathrm{i}\,\Omega' H^{n+1}_{e_3g_{-1}} = 0, \tag{B14}$$

$$-(2\gamma + \mathrm{i}\,nkv)H^n_{e_{-3}e_{-1}} + \mathrm{i}\,\Omega H^{n+1}_{g_{-1}e_{-1}} - \mathrm{i}\,\Omega' H^{n-1}_{e_{-3}g_1} = 0, \tag{B15}$$

$$-(2\gamma+\mathrm{i}\,nkv)H^n_{e_1e_{-1}}+\mathrm{i}\,\Omega'\left(H^{n-1}_{g_{-1}e_{-1}}-H^{n-1}_{e_1g_1}\right)=0, \tag{B16}$$

$$-(2\gamma+\mathrm{i}\,nkv)H^n_{e_3e_{-1}}+\mathrm{i}\,\Omega H^{n-1}_{g_1e_{-1}}-\mathrm{i}\,\Omega' H^{n-1}_{e_3g_1}=0, \tag{B17}$$

$$-(2\gamma+\mathrm{i}\,nkv)H^n_{e_{-3}e_1}+\mathrm{i}\,\Omega H^{n+1}_{g_{-1}e_1}-\mathrm{i}\,\Omega' H^{n+1}_{e_{-3}g_{-1}}=0, \tag{B18}$$

$$-(2\gamma+\mathrm{i}\,nkv)H^n_{e_3e_{-3}}+\mathrm{i}\,\Omega\left(H^{n-1}_{g_1e_{-3}}-H^{n-1}_{e_3g_{-1}}\right)=0, \tag{B19}$$

$$\begin{aligned}&(4\gamma/3)H^n_{e_{-1}e_1}+(2\gamma/\sqrt{3})H^n_{e_{-3}e_{-1}}+(2\gamma/\sqrt{3})H^n_{e_1e_3}\\&+\mathrm{i}\,\Omega'\left(H^{n-1}_{e_{-1}g_{-1}}-H^{n-1}_{g_1e_1}\right)+\mathrm{i}\,\Omega\left(H^{n+1}_{e_3g_{-1}}-H^{n+1}_{g_1e_{-3}}\right)\\&-\mathrm{i}\,nkvH^n_{g_1g_{-1}}=0,\end{aligned} \tag{B20}$$

$$\begin{aligned}&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n+1)kv)H^{n+1}_{g_1e_{-3}}+\mathrm{i}\,\Omega' H^n_{e_{-1}e_{-3}}\\&+\mathrm{i}\,\Omega\left(H^{m+2}_{e_3e_{-3}}-H^n_{g_1g_{-1}}\right)=0,\end{aligned} \tag{B21}$$

$$\begin{aligned}&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n-1)kv)H^{n-1}_{g_{-1}e_3}+\mathrm{i}\,\Omega' H^n_{e_1e_3}\\&+\mathrm{i}\,\Omega\left(H^{n-2}_{e_{-3}e_3}-H^n_{g_{-1}g_1}\right)=0.\end{aligned} \tag{B22}$$

Integer n denotes the order of the spatial harmonic, $n=0,\ \pm1,\ \pm2,\ \ldots$. Optical coherences H^n_{ab} satisfy the relation: $H^{n*}_{ab}=H^{-n}_{ba}$.

Appendix C

Below we list an infinite set of linear algebraic equations for variables $R^n_{g_\alpha g_\beta}$, $R^n_{e_\alpha e_\beta}$, and $R^{n\pm1}_{g_\alpha e_\beta}$. This set of equations originates from Appendix A by considering the first order in the photon momentum,

$$R^n_{g_1g_1}+R^n_{g_{-1}g_{-1}}+R^n_{e_3e_3}+R^n_{e_1e_1}+R^n_{e_{-1}e_{-1}}+R^n_{e_{-3}e_{-3}}=0, \tag{C1}$$

$$\begin{aligned}&(2\gamma/3)R^n_{e_{-1}e_{-1}}+(4\gamma/3)R^n_{e_1e_1}+2\gamma R^n_{e_3e_3}\\&+\mathrm{i}\,\Omega\left(R^{n+1}_{e_3g_1}-R^{n-1}_{g_1e_3}\right)+\mathrm{i}\,\Omega'\left(R^{n-1}_{e_{-1}g_1}-R^{n+1}_{g_1e_{-1}}\right)\\&+\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n+1}_{e_3g_1}-H^{n-1}_{g_1e_3}\right)-\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n-1}_{e_{-1}g_1}-H^{n+1}_{g_1e_{-1}}\right)\\&-\mathrm{i}\,nkvR^n_{g_1g_1}+\sum H^{n-l}_{g_1g_1}f^l_z=0,\end{aligned} \tag{C2}$$

$$\begin{aligned}&(4\gamma/3)R^n_{e_{-1}e_{-1}}+(2\gamma/3)R^n_{e_1e_1}+2\gamma R^n_{e_{-3}e_{-3}}\\&+\mathrm{i}\Omega\left(R^{n-1}_{e_{-3}g_{-1}}-R^{n+1}_{g_{-1}e_{-3}}\right)-\mathrm{i}\,\Omega'\left(R^{n-1}_{g_{-1}e_1}-R^{n+1}_{e_1g_{-1}}\right)\\&-\mathrm{i}\frac{\Omega}{2}\left(H^{n-1}_{e_{-3}g_{-1}}-H^{n+1}_{g_{-1}e_{-3}}\right)-\mathrm{i}\frac{\Omega'}{2}\left(H^{n-1}_{g_{-1}e_1}-H^{n+1}_{e_1g_{-1}}\right)\\&-\mathrm{i}\,nkvR^n_{g_{-1}g_{-1}}+\sum H^{n-l}_{g_{-1}g_{-1}}f_l=0,\end{aligned} \tag{C3}$$

$$\begin{aligned}&-(2\gamma+\mathrm{i}\,nkv)R^n_{e_1e_1}+\mathrm{i}\,\Omega'\left(R^{n-1}_{g_{-1}e_1}-R^{n+1}_{e_1g_{-1}}\right)\\&-\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n-1}_{g_{-1}e_1}-H^{n+1}_{e_1g_{-1}}\right)\\&+\sum H^{n-l}_{e_1e_1}f_l=0,\end{aligned} \tag{C4}$$

$$\begin{aligned}&-(2\gamma+\mathrm{i}\,nkv)R^n_{e_{-1}e_{-1}}+\mathrm{i}\,\Omega'\left(R^{n+1}_{g_1e_{-1}}-R^{n-1}_{e_{-1}g_1}\right)\\&+\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n+1}_{g_1e_{-1}}-H^{n-1}_{e_{-1}g_1}\right)\\&+\sum H^{n-l}_{e_{-1}e_{-1}}f_l=0,\end{aligned} \tag{C5}$$

$$\begin{aligned}&-(2\gamma+\mathrm{i}\,nkv)R^n_{e_3e_3}+\mathrm{i}\,\Omega\left(R^{n-1}_{g_1e_3}-R^{n+1}_{e_3g_1}\right)\\&-\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n-1}_{g_1e_3}-H^{n+1}_{e_3g_1}\right)\\&+\sum H^{n-l}_{e_3e_3}f_l=0,\end{aligned} \tag{C6}$$

$$\begin{aligned}&-(2\gamma+\mathrm{i}\,nkv)R^n_{e_{-3}e_{-3}}+\mathrm{i}\,\Omega\left(R^{n+1}_{g_{-1}e_{-3}}-R^{n-1}_{e_{-3}g_{-1}}\right)\\&+\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n+1}_{g_{-1}e_{-3}}-H^{n-1}_{e_{-3}g_{-1}}\right)\\&+\sum H^{n-l}_{e_{-3}e_{-3}}f_l=0,\end{aligned} \tag{C7}$$

$$\begin{aligned}&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n+1)kv)R^{n+1}_{g_1e_{-1}}+\mathrm{i}\,\Omega R^{n+2}_{e_3e_{-1}}\\&+\mathrm{i}\,\Omega'\left(R^n_{e_{-1}e_{-1}}-R^n_{g_1g_1}\right)+\mathrm{i}\,\frac{1}{2}\Omega H^{n+2}_{e_3e_{-1}}\\&-\mathrm{i}\,\frac{1}{2}\Omega'\left(H^n_{e_{-1}e_{-1}}+H^n_{g_1g_1}\right)\\&+\sum H^{n+1-l}_{g_1e_{-1}}f_l=0,\end{aligned} \tag{C8}$$

$$\begin{aligned}&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n+1)kv)R^{n+1}_{g_{-1}e_1}+\mathrm{i}\,\Omega R^n_{e_{-3}e_1}\\&+\mathrm{i}\,\Omega'\left(R^{n+2}_{e_1e_1}-R^{n+2}_{g_{-1}g_{-1}}\right)-\mathrm{i}\,\frac{1}{2}\Omega H^n_{e_{-3}e_1}\\&+\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n+2}_{e_1e_1}+H^{n+2}_{g_{-1}g_{-1}}\right)\\&+\sum H^{n+1-l}_{g_{-1}e_1}f_l=0,\end{aligned} \tag{C9}$$

$$\begin{aligned}&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n+1)kv)R^{n+1}_{g_{-1}e_{-1}}+\mathrm{i}\,\Omega R^n_{e_{-3}e_{-1}}\\&+\mathrm{i}\,\Omega'\left(R^{n+2}_{e_1e_{-1}}-R^n_{g_{-1}g_1}\right)-\mathrm{i}\,\frac{1}{2}\Omega H^n_{e_{-3}e_{-1}}\\&+\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n+2}_{e_1e_{-1}}-H^n_{g_{-1}g_1}\right)\\&+\sum H^{n+1-l}_{g_{-1}e_{-1}}f_l=0,\end{aligned} \tag{C10}$$

$$\begin{aligned}&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n-1)kv)R^{n-1}_{g_1e_1}+\mathrm{i}\,\Omega R^n_{e_3e_1}\\&+\mathrm{i}\,\Omega'\left(R^{n-2}_{e_{-1}e_1}-R^n_{g_1g_{-1}}\right)+\mathrm{i}\,\frac{1}{2}\Omega H^n_{e_3e_1}\\&-\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n-2}_{e_{-1}e_1}-H^n_{g_1g_{-1}}\right)\\&+\sum H^{n-1-l}_{g_1e_1}f_l=0,\end{aligned} \tag{C11}$$

$$
\begin{aligned}
&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n+1)kv)R^{n+1}_{g_1e_3}+\mathrm{i}\,\Omega' R^{n}_{e_{-1}e_3}\\
&+\mathrm{i}\,\Omega\left(R^{n+2}_{e_3e_3}-R^{n+2}_{g_1g_1}\right)-\mathrm{i}\,\frac{1}{2}\Omega' H^{n}_{e_{-1}e_3}\\
&+\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n+2}_{e_3e_3}+H^{n+2}_{g_1g_1}\right)\\
&+\sum H^{n+1-l}_{g_1e_3}f_l=0, \qquad \text{(C12)}
\end{aligned}
$$

$$
\begin{aligned}
&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n-1)kv)R^{n-1}_{g_{-1}e_{-3}}+\mathrm{i}\,\Omega' R^{n}_{e_1e_{-3}}\\
&+\mathrm{i}\,\Omega\left(R^{n-2}_{e_{-3}e_{-3}}-R^{n-2}_{g_{-1}g_{-1}}\right)+\mathrm{i}\,\frac{1}{2}\Omega' H^{n}_{e_1e_{-3}}\\
&-\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n-2}_{e_{-3}e_{-3}}-H^{n-2}_{g_{-1}g_{-1}}\right)\\
&+\sum H^{n-1-l}_{g_{-1}e_{-3}}f_l=0, \qquad \text{(C13)}
\end{aligned}
$$

$$
\begin{aligned}
&-(2\gamma+\mathrm{i}\,nkv)R^{n}_{e_3e_1}+\mathrm{i}\,\Omega R^{n-1}_{g_1e_1}-\mathrm{i}\,\Omega' R^{n+1}_{e_3g_{-1}}\\
&-\mathrm{i}\,\frac{1}{2}\Omega H^{n-1}_{g_1e_1}+\mathrm{i}\,\frac{1}{2}\Omega' H^{n+1}_{e_3g_{-1}}\\
&+\sum H^{n-l}_{e_3e_1}f_l=0, \qquad \text{(C14)}
\end{aligned}
$$

$$
\begin{aligned}
&-(2\gamma+\mathrm{i}\,nkv)R^{n}_{e_{-3}e_{-1}}+\mathrm{i}\,\Omega R^{n+1}_{g_{-1}e_{-1}}-\mathrm{i}\,\Omega' R^{n-1}_{e_{-3}g_1}\\
&+\mathrm{i}\,\frac{1}{2}\Omega H^{n+1}_{g_{-1}e_{-1}}-\mathrm{i}\,\frac{1}{2}\Omega' H^{n-1}_{e_{-3}g_1}\\
&+\sum H^{n-l}_{e_{-3}e_{-1}}f_l=0, \qquad \text{(C15)}
\end{aligned}
$$

$$
\begin{aligned}
&-(2\gamma+\mathrm{i}\,nkv)R^{n}_{e_1e_{-1}}+\mathrm{i}\,\Omega'\left(R^{n-1}_{g_{-1}e_{-1}}-R^{n-1}_{e_1g_1}\right)\\
&-\mathrm{i}\,\frac{1}{2}\Omega'\left(H^{n-1}_{g_{-1}e_{-1}}+H^{n-1}_{e_1g_1}\right)\\
&+\sum H^{n-l}_{e_1e_{-1}}f_l=0, \qquad \text{(C16)}
\end{aligned}
$$

$$
\begin{aligned}
&-(2\gamma+\mathrm{i}\,nkv)R^{n}_{e_3e_{-1}}+\mathrm{i}\,\Omega R^{n-1}_{g_1e_{-1}}-\mathrm{i}\,\Omega' R^{n-1}_{e_3g_1}\\
&-\mathrm{i}\,\frac{1}{2}\Omega H^{n-1}_{g_1e_{-1}}-\mathrm{i}\,\frac{1}{2}\Omega' H^{n-1}_{e_3g_1}\\
&+\sum H^{n-l}_{e_3e_{-1}}f_l=0, \qquad \text{(C17)}
\end{aligned}
$$

$$
\begin{aligned}
&-(2\gamma+\mathrm{i}\,nkv)R^{n}_{e_{-3}e_1}+\mathrm{i}\,\Omega R^{n+1}_{g_{-1}e_1}-\mathrm{i}\,\Omega' R^{n+1}_{e_{-3}g_{-1}}\\
&+\mathrm{i}\,\frac{1}{2}\Omega H^{n+1}_{g_{-1}e_1}+\mathrm{i}\,\frac{1}{2}\Omega' H^{n+1}_{e_{-3}g_{-1}}\\
&+\sum H^{n-l}_{e_{-3}e_1}f_l=0, \qquad \text{(C18)}
\end{aligned}
$$

$$
\begin{aligned}
&-(2\gamma+\mathrm{i}\,nkv)R^{n}_{e_3e_{-3}}+\mathrm{i}\,\Omega\left(R^{n-1}_{g_1e_{-3}}-R^{n-1}_{e_3g_{-1}}\right)\\
&-\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n-1}_{g_1e_{-3}}+H^{n-1}_{e_3g_{-1}}\right)\\
&+\sum H^{n-l}_{e_3e_{-3}}f_l=0, \qquad \text{(C19)}
\end{aligned}
$$

$$
\begin{aligned}
&(4\gamma/3)R^{n}_{e_{-1}e_1}+(2\gamma/\sqrt{3})R^{n}_{e_{-3}e_{-1}}+(2\gamma/\sqrt{3})R^{n}_{e_1e_3}\\
&+\mathrm{i}\,\Omega'\left(R^{n-1}_{e_{-1}g_{-1}}-R^{n-1}_{g_1e_1}\right)+\mathrm{i}\,\Omega\left(R^{n+1}_{e_3g_{-1}}-R^{n+1}_{g_1e_{-3}}\right)\\
&-\mathrm{i}\,\frac{\Omega'}{2}\left(H^{n-1}_{e_{-1}g_{-1}}+H^{n-1}_{g_1e_1}\right)+\mathrm{i}\,\frac{\Omega}{2}\left(H^{n+1}_{e_3g_{-1}}+H^{n+1}_{g_1e_{-3}}\right)\\
&-\mathrm{i}\,nkvR^{n}_{g_1g_{-1}}+\sum H^{n-l}_{g_1g_{-1}}f_l=0, \qquad \text{(C20)}
\end{aligned}
$$

$$
\begin{aligned}
&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n+1)kv)R^{n+1}_{g_1e_{-3}}+\mathrm{i}\,\Omega' R^{n}_{e_{-1}e_{-3}}\\
&+\mathrm{i}\,\Omega\left(R^{n+2}_{e_3e_{-3}}-R^{n}_{g_1g_{-1}}\right)+\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n+2}_{e_3e_{-3}}-H^{n}_{g_1g_{-1}}\right)\\
&-\mathrm{i}\,\frac{1}{2}\Omega' H^{n}_{e_{-1}e_{-3}}+\sum H^{n+1-l}_{g_1e_{-3}}f_l=0, \qquad \text{(C21)}
\end{aligned}
$$

$$
\begin{aligned}
&-(\gamma+\mathrm{i}\,\delta+\mathrm{i}(n-1)kv)R^{n-1}_{g_{-1}e_3}+\mathrm{i}\,\Omega' R^{n}_{e_1e_3}\\
&+\mathrm{i}\,\Omega\left(R^{n-2}_{e_{-3}e_3}-R^{n}_{g_{-1}g_1}\right)-\mathrm{i}\,\frac{1}{2}\Omega\left(H^{n-2}_{e_{-3}e_3}-H^{n}_{g_{-1}g_1}\right)\\
&+\mathrm{i}\,\frac{1}{2}\Omega' H^{n}_{e_1e_3}+\sum H^{n-1-l}_{g_{-1}e_3}f_l=0. \qquad \text{(C22)}
\end{aligned}
$$

The above set can be solved when the inhomogeneous terms have been determined from the solution of the set of equations listed in Appendix B.

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