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Topological aspects in a two-component Bose condensed system in a neutron star*

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By making use of Duan–Ge's decomposition theory of gauge potential and the topological current theory proposed by Prof. Duan Yi-Shi, we study a two-component superfluid Bose condensed system, which is supposed to be realized in the interior of neutron stars in the form of the coexistence of a neutron superfluid and a protonic superconductor. We propose that this system possesses vortex lines. The topological charges of the vortex lines are characterized by the Hopf indices and the Brower degrees of ϕ-mapping.

Keywords: neutron stars, topological defects, two-component Bose condensed system
PACC: 1110, 0240, 0420

1. Introduction

In the standard model for a neutron star its interior features superfluidity of neutron Cooper pairs and superconductivity of proton Cooper pairs (see, e.g., Refs.[1] and [2]). Both these condensates allow vortices of the $S^1 \to S^1$ map. It was suggested that the phenomenon of glitches in Crab and Vela pulsars is connected with vortex matter in these stars.[3−5] The spin-down of a neutron star can be explained by neutrino radiation from a superfluid vortex.[6] This remains a topic of intensive studies and discussions (for recent developments and citations see Refs.[7] and [8]). Although the standard model for a neutron star is a special two-component superfluid Bose condensed system, it is also a topic of abstract academic interest[9,10] since such a system allows for interesting phenomena with no direct counterparts in, e.g., superconducting metals. Since the two-component superfluid Bose condensed system has played an important role in condensed matter and neutron stars, studies of topological defects in this system become an important subject.[11]

In this paper we mainly focus our attention on the two-component superfluid Bose condensed system, which is believed to be realized in the interior of neutron stars. We start with the free energy density of the this two-component system. By making use of Duan–Ge's decomposition theory of gauge potential and the topological current theory proposed by Prof. Duan Yi-Shi,[12,13] we propose that this system possesses vortex lines, and the topological charges of the vortex lines are characterized by the Hopf indices and the Brower degrees of ϕ-mapping.

2. Two-component Bose condensed system

We consider the effective Landau–Ginzburg free energy density that describes the two-component superfluid Bose condensed system. In this system, we have a proton condensate described by ψ_1 and a neutron condensate described by ψ_2. We do not consider the normal component of the protons and neutrons with their specific excitations, but consider only the superfluid component. The ψ_1 field with electric charge q, which is actually twice the fundamental charge of the proton $q = 2|e|$, interacts with the gauge field $\boldsymbol{A}$, with $\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}$. The free energy density has the following expression:[14]

$$f = \frac{\hbar^2}{2M_1}\left|\left(\boldsymbol{\nabla} - \frac{\mathrm{i}q}{\hbar c}\boldsymbol{A}\right)\psi_1\right|^2 + \frac{\hbar^2}{2M_2}\left|\boldsymbol{\nabla}\psi_2\right|^2 + \frac{\boldsymbol{B}^2}{8\pi} + V(|\psi_1^2|, |\psi_2^2|), \quad (1)$$

where $M_1 = 2m_1$, m_1 being the mass of the proton, and $M_2 = 2m_2$, m_2 being the mass of the neutron. In the free energy density given above, we have ignored

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the term coupling the proton and neutron superfluid velocities, which gives rise to the Andreev–Bashkin effect,[15] for it is not important in our discussion. Indeed, the relevant term in the free energy can be represented as $\sim \int \mathrm{d}^3x \boldsymbol{v}_1\cdot\boldsymbol{v}_2$, where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are velocities of the superfluid components. For neutron stars which do not rotate (the case that is considered in this paper), $\boldsymbol{v}_2 = 0$, and the effect obviously vanishes. We expect that due to the small density of the neutron vortices (compared to the density of the proton vortices) the effect is still negligible for most of the flux tubes in a rotating star.[14] The effect could be important only for a few of the flux tubes situated close to a neutron vortex core, where $\boldsymbol{v}_2$ strongly deviates from the constant value at interflux distance scales. We have also ignored the fact that the neutron condensate has a nontrivial $^3p_2$ order parameter for only the magnitude of the neutron condensate is relevant to the effect described below. The free energy density (1) only describes large distances, it does not describe the gap structure on the Fermi surfaces, but just the superfluid component of the protons and neutrons.

To discuss the detail of Eq.(1), we start by introducing variables χ_1, χ_2 and ρ as

$$\psi_1 = \sqrt{2M_1}\rho\chi_1, \qquad \psi_2 = \sqrt{2M_2}\rho\chi_2, \tag{2}$$

where the complex $\chi_1 = |\chi_1|\mathrm{e}^{\mathrm{i}\phi_1}$ and $\chi_2 = |\chi_2|\mathrm{e}^{\mathrm{i}\phi_2}$ are chosen such that $|\chi_1|^2 + |\chi_2|^2 = 1$. The modulus ρ then has the following expression:

$$\rho^2 = \frac{1}{2}\left[\frac{|\psi_1|^2}{M_1} + \frac{|\psi_2|^2}{M_2}\right]. \tag{3}$$

By using formula (2), the free energy density (1) can be rewritten as

$$\begin{aligned} f \;=\; & \hbar^2(\boldsymbol{\nabla}\rho)^2 + \hbar^2\rho^2\left|\left(\boldsymbol{\nabla} - \frac{\mathrm{i}q}{\hbar c}\boldsymbol{A}\right)\chi_1\right|^2 \\ & + \hbar^2\rho^2\left|\boldsymbol{\nabla}\chi_2\right|^2 + \frac{\boldsymbol{B}^2}{8\pi} + V. \end{aligned} \tag{4}$$

Then we can introduce current density

$$\begin{aligned} \boldsymbol{J} \;=\; & \frac{\mathrm{i}\hbar}{2M_1}(\psi_1\boldsymbol{\nabla}\psi_1^* - \psi_1^*\boldsymbol{\nabla}\psi_1) \\ & + \frac{\mathrm{i}\hbar}{2M_2}(\psi_2\boldsymbol{\nabla}\psi_2^* - \psi_2^*\boldsymbol{\nabla}\psi_2) \\ & - \frac{q}{M_1 c}\boldsymbol{A}|\psi_1|^2 \end{aligned} \tag{5}$$

and electrical current density

$$\boldsymbol{J}_q = \frac{\mathrm{i}\hbar q}{2M_1}(\psi_1\boldsymbol{\nabla}\psi_1^* - \psi_1^*\boldsymbol{\nabla}\psi_1) - \frac{q^2}{M_1 c}\boldsymbol{A}|\psi_1|^2. \tag{6}$$

And we can obtain

$$\boldsymbol{J} = \hbar\rho^2\boldsymbol{j} - \frac{2\rho^2 q}{c}\boldsymbol{A}|\chi_1|^2, \tag{7}$$

where

$$\boldsymbol{j} = \mathrm{i}(\chi_1\boldsymbol{\nabla}\chi_1^* - \chi_1^*\boldsymbol{\nabla}\chi_1 + \chi_2\boldsymbol{\nabla}\chi_2^* - \chi_2^*\boldsymbol{\nabla}\chi_2). \tag{8}$$

We introduce a gauge-invariant vector field $\boldsymbol{C}$, directly related to the current density by

$$\boldsymbol{C} = \frac{\boldsymbol{J}}{\rho^2}. \tag{9}$$

We can also obtain the electrical current density

$$\boldsymbol{J}_q = \hbar q\rho^2[\mathrm{i}(\chi_1\boldsymbol{\nabla}\chi_1^* - \chi_1^*\boldsymbol{\nabla}\chi_1) - \frac{2q}{c\hbar}\boldsymbol{A}|\chi_1|^2]. \tag{10}$$

Then rearrange the terms in Eq.(4) as follows: add to and subtract from Eq.(4) a term $\frac{1}{4}\hbar^2\rho^2\boldsymbol{j}$ and observe that the following expression

$$\hbar^2\rho^2\left[|\boldsymbol{\nabla}\chi_1|^2 + |\boldsymbol{\nabla}\chi_2|^2 - \frac{\boldsymbol{j}}{4}^2\right] \tag{11}$$

is also gauge-invariant. Indeed, if we introduce the gauge-invariant field

$$\boldsymbol{n} = \overline{\chi}\boldsymbol{\sigma}\chi, \tag{12}$$

where $\overline{\chi} = (\chi_1^*, \chi_2^*)$ and $\boldsymbol{\sigma}$ are Pauli matrices, then $\boldsymbol{n}$ is a unit vector $|\boldsymbol{n}| = 1$ and we can write expression (11) as follows:

$$\hbar^2\rho^2\left[|\boldsymbol{\nabla}\chi_1|^2 + |\boldsymbol{\nabla}\chi_2|^2 - \frac{\boldsymbol{j}}{4}^2\right] = \frac{1}{4}\hbar^2\rho^2(\boldsymbol{\nabla}\boldsymbol{n})^2. \tag{13}$$

Now we consider the remaining terms in Eq.(4). For the magnetic field we can have

$$\begin{aligned} \boldsymbol{B} \;&=\; \boldsymbol{\nabla}\times\boldsymbol{A} \\ &=\; \frac{\hbar c}{2q|\chi_1|^2}\left(\boldsymbol{\nabla}\times\boldsymbol{j} - \frac{1}{\hbar}\boldsymbol{\nabla}\times\boldsymbol{C}\right), \end{aligned} \tag{14}$$

where $\boldsymbol{\nabla}\times\boldsymbol{j}$ can be rewritten by using the unit vector $\boldsymbol{n}$ as follows:

$$\boldsymbol{\nabla}\times\boldsymbol{j} = \frac{1}{2}\boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}. \tag{15}$$

Combining these we can obtain our main result: the free energy density becomes

$$\begin{aligned} f \;=\; & \hbar^2(\boldsymbol{\nabla}\rho)^2 + \frac{1}{4}\hbar^2\rho^2(\boldsymbol{\nabla}\boldsymbol{n})^2 \\ & + \frac{\hbar^2c^2}{128\pi q^2|\chi_1|^4} \\ & \times\left[\frac{1}{\hbar}(\partial_\mu C_\nu - \partial_\nu C_\mu) - \boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}\right]^2 \\ & - \frac{\hbar}{2q|\chi_1|^2}\boldsymbol{J}_q\cdot\left(\boldsymbol{j} - \frac{\boldsymbol{C}}{\hbar}\right) \\ & + \frac{1}{4}\hbar^2\rho^2\left[\boldsymbol{j}^2 - \frac{1}{|\chi_1|^2}\left(\boldsymbol{j} - \frac{\boldsymbol{C}}{\hbar}\right)^2\right] \\ & + V(\rho, |\chi_1|^2, |\chi_2|^2). \end{aligned} \tag{16}$$

Next we discuss the topological defects allowed in Eq.(16).

3. Topological vortex lines in the two-component Bose condensed system

As shown in Eq.(16), the magnetic field of the system can be divided into two parts: one is the contribution of field C_μ, which is introduced by the current density and can only be present with the topological defects such as those in a single-condensate system. The other part, the contribution of $\boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}$ to the magnetic field term in Eq.(16), is a fundamental topological property of this system. Here we emphasize that another nontrivial topological configuration is allowed, which originates from the contribution of the term $\boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}$. Thus we will investigate this term in detail.

It is easy to prove that the term $\boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}$ can be re-expressed in an Abelian field tensor form

$$\boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}=\partial_\mu b_\nu-\partial_\nu b_\mu, \tag{17}$$

where b_μ is the Wu–Yang potential[16]

$$b_\mu=\boldsymbol{e}_1\cdot\partial_\mu\boldsymbol{e}_2. \tag{18}$$

Here $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are two perpendicular unit vectors normal to $\boldsymbol{n}$, and $(\boldsymbol{e}_1,\boldsymbol{e}_2,\boldsymbol{n})$ forms an orthogonal frame:

$$\begin{aligned}&\boldsymbol{e}_1\cdot\boldsymbol{e}_2=\boldsymbol{e}_1\cdot\boldsymbol{n}=\boldsymbol{e}_2\cdot\boldsymbol{n}=0,\\&\boldsymbol{e}_1\cdot\boldsymbol{e}_1=\boldsymbol{e}_2\cdot\boldsymbol{e}_2=\boldsymbol{n}\cdot\boldsymbol{n}=1.\end{aligned} \tag{19}$$

Now, consider a two-component vector field $\boldsymbol{\phi}=(\phi^1,\phi^2)$ residing in the plane formed by $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$:

$$\begin{aligned}&e_1^a=\frac{\phi^a}{||\phi||},\quad e_2^a=\epsilon^{ab}\frac{\phi^b}{||\phi||}\\&(||\phi||^2=\phi^a\phi^a,\quad a,b=1,2)\end{aligned} \tag{20}$$

It can be proved that relations for $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ satisfy above restrictions (19). Obviously the zero points of $\boldsymbol{\phi}=(\phi^1,\phi^2)$ are just the two-dimensional singular points of $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$.

Using the $\boldsymbol{\phi}$ field, the Wu–Yang potential can be expressed as

$$b_\mu=\epsilon_{ab}\frac{\phi^a}{||\phi||}\partial_\mu\frac{\phi^b}{||\phi||}. \tag{21}$$

Comparing Eq.(21) with the expression of Duan–Ge's decomposition theory of gauge potential,[17,18] we find that b_μ satisfies the $U(1)$ gauge transformation. Now introduce a second-order tensor

$$H_{\mu\nu}=\boldsymbol{n}\cdot\partial_\mu\boldsymbol{n}\times\partial_\nu\boldsymbol{n}, \tag{22}$$

which describes the magnetic field induced in the system due to a non-trivial electromagnetic interaction in this system.

By using the $\boldsymbol{\phi}$ field, the second-order tensor $H_{\mu\nu}$ can be rewritten as

$$H_{\mu\nu}=2\epsilon_{ab}\partial_\mu\frac{\phi^a}{||\phi||}\partial_\nu\frac{\phi^b}{||\phi||}. \tag{23}$$

Because the tensor $H_{\mu\nu}$ plays an important role in the topological feature in this system, here we will use Duan's topological current theory to investigate the properties hidden in this tensor. To do so, we introduce a topological tensor current $K^{\mu\nu}$,[13] which is given by

$$\begin{aligned}K^{\mu\nu}&=\frac{1}{8\pi}\epsilon^{\mu\nu\lambda\rho}H_{\lambda\rho}\\&=\frac{1}{4\pi}\epsilon^{\mu\nu\lambda\rho}\epsilon_{ab}\partial_\mu\frac{\phi^a}{||\phi||}\partial_\nu\frac{\phi^b}{||\phi||}.\end{aligned} \tag{24}$$

According to Ref.[12] and using $\partial_\mu\dfrac{\phi^a}{||\phi||}=\dfrac{\partial_\mu\phi^a}{||\phi||}+\phi^a\partial_\mu\dfrac{1}{||\phi||}$ and the Green function relation in ϕ-space, we have

$$\partial_a\partial_a\ln||\phi||=2\pi\delta^2(\boldsymbol{\phi}),\qquad\left(\partial_a=\frac{\partial}{\partial\phi^a}\right). \tag{25}$$

It can be proved that

$$K^{\mu\nu}=\delta^2(\boldsymbol{\phi})D^{\mu\nu}\left(\frac{\phi}{x}\right), \tag{26}$$

where $D^{\mu\nu}\left(\dfrac{\phi}{x}\right)=\dfrac{1}{2}\epsilon^{\mu\nu\lambda\rho}\epsilon_{ab}\partial_\lambda\phi^a\partial_\rho\phi^b$ is the Jacobian between the ϕ space and the Cartesian coordinates. Denoting the spacial component of $K^{\mu\nu}$ by K^i, we obtain

$$K^i=K^{0i}=\delta^2\left(\boldsymbol{\phi}\right)D^i\left(\frac{\phi}{x}\right)\qquad(i=1,2,3), \tag{27}$$

where $D^i\left(\dfrac{\phi}{x}\right)=D^{0i}\left(\dfrac{\phi}{x}\right)$.

An important conclusion from formula (27) is

$$\begin{cases}K^i=0 & \text{if and only if}\quad\boldsymbol{\phi}\neq0,\\K^i\neq0 & \text{if and only if}\quad\boldsymbol{\phi}=0.\end{cases} \tag{28}$$

So it is necessary to study the zero point of ϕ to determine the nonzero solution of K^i. The implicit

function theory[19] shows that under the regular condition $D^i\left(\frac{\phi}{x}\right)\neq 0$, the general solution of

$$\phi^1(t,x^1,x^2,x^3)=0,\quad \phi^1(t,x^1,x^2,x^3)=0, \tag{29}$$

can be expressed as

$$\begin{aligned} x^1 &= x_k^1(s,t), \qquad x^2=x_k^2(s,t),\\ x^3 &= x_k^3(s,t), \qquad (k=1,2,3,\cdots,N), \end{aligned} \tag{30}$$

which represent the world surfaces of N moving isolated singular strings L_k with string parameter s. This indicates that in this system there are vortex lines located at the zero points of the $\boldsymbol{\phi}$ field.

Now we will study the topological charges of the vortex lines and their properties by using Duan's topological current theory. In δ-function theory,[20] one can prove that in three-dimensional space

$$\delta^2(\boldsymbol{\phi})=\sum_{k=1}^{N}\beta_k\int_{L_k}\frac{\delta^3(\boldsymbol{x}-\boldsymbol{x}_k(s))}{\left|D\left(\frac{\phi}{u}\right)\right|_{\Sigma_k}}, \tag{31}$$

where

$$D\left(\frac{\phi}{u}\right)_{\Sigma_k}=\frac{1}{2}\epsilon^{\mu\nu}\epsilon_{mn}\left(\frac{\partial\phi^m}{\partial u^\mu}\right)\left(\frac{\partial\phi^n}{\partial u^\nu}\right), \tag{32}$$

and Σ_k is the k-th planar element transverse to L_k with local coordinates (u^1,u^2). The positive integer β_k is the Hopf index of ϕ-mapping, which indicates that when the point $\boldsymbol{x}$ covers the neighborhood of the zero point $\boldsymbol{x}_k$ once, the vector field $\boldsymbol{\phi}$ covers the corresponding region in ϕ-space β_k times. Meanwhile, the direction vector of L_k is given by[12]

$$\left.\frac{\mathrm{d}x^i}{\mathrm{d}s}\right|_{\boldsymbol{x}_k}=\left.\frac{D^i(\phi/x)}{D(\phi/u)}\right|_{\boldsymbol{x}_k}, \tag{33}$$

which leads to

$$\begin{aligned} \left.\frac{\mathrm{d}x^1}{\mathrm{d}x^3}\right|_{\boldsymbol{x}_k} &= \left.\frac{D^1(\phi/x)}{D^3(\phi/x)}\right|_{\boldsymbol{x}_k},\\ \left.\frac{\mathrm{d}x^2}{\mathrm{d}x^3}\right|_{\boldsymbol{x}_k} &= \left.\frac{D^2(\phi/x)}{D^3(\phi/x)}\right|_{\boldsymbol{x}_k}. \end{aligned} \tag{34}$$

From Eqs.(31) and (33), we can find the inner topological structure

$$\begin{aligned} K^i &= \delta^2(\boldsymbol{\phi})D^i(\phi/x)\\ &= \sum_{k=1}^{N}\beta_k\eta_k\int_{L_k}\frac{\mathrm{d}x^i}{\mathrm{d}s}\delta^3(\boldsymbol{x}-\boldsymbol{x}_k(s))\mathrm{d}s, \end{aligned} \tag{35}$$

in which η_k is the Brouwer degree of the ϕ-mapping, with

$$\eta_k=\mathrm{sgn}\ D(\phi/u)_{\Sigma_k}=\pm 1. \tag{36}$$

Hence the topological charge of the vortex line L_k is

$$Q_k=\int_{\Sigma_k}K^i d\sigma_i=W_k, \tag{37}$$

where $W_k=\beta_k\eta_k$ is the winding number of $\boldsymbol{\phi}$ around L_k. And the total topological number on surface Σ is

$$Q=\int_{\Sigma}K^i\mathrm{d}\sigma_i=\sum_{k=1}^{n}W_k. \tag{38}$$

Then we come to the conclusion that, under the regular condition $D^i(\phi/x)\neq 0$, there exist vortex lines in this system, whose topological charges are just the winding numbers of the ϕ-mapping.

4. Conclusion

In conclusion, based on Duan–Ge's decomposition theory of gauge potential and Duan's topological current theory, we point out that vortex lines are allowed in the two-component superfluid Bose condensed system, which is believed to be realized in the interior of neutron stars. Under the regular condition $D^i(\phi/x)\neq 0$, vortex lines originate from the zero points of the $\boldsymbol{\phi}$ field, and the topological charges of vortex lines are characterized by the winding numbers of the ϕ-mapping.

In this paper we treat the vortex lines as mathematical lines, i.e., the width of a line is zero. This description is obtained by means of the approximation that the radius of curvature of a line is much larger than the width of the line.[21]

Finally, it should be pointed out that in the present paper we have just considered the case in which the neutron star does not rotate, but the effect of the neutron star's rotation is significant. We will study the topological aspects of a rotating neutron star in our next work.

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References

[1] Hoffberg H, Glassgold A E, Richardson R W and Ruderman M 1970 *Phys. Rev. Lett.* **24** 775

[2] Baym G, Pethick C and Pines D 1969 *Nature* (London) **224** 673

[3] Anderson P W and Itoh N 1975 *Nature* **256** 25
Alpar M A 1977 *Astroph. J.* **213** 527

[4] Anderson P W, Alpar M A, Pines D and Shaham J 1982 *Phil. Mag.* A **45** 227
Pines D, Shaham J, Alpar M A and Anderson P W 1980 *Prog. Theor. Phys. Suppl.* **69** 376

[5] Alpar M A, Anderson P W, Pines D and Shaham J 1981 *Astrophys. J.* (Letters) **249** L33

[6] Peng Q H, Huang K L and Huang J H 1982 *Astron. & Astrophys.* **107** 258

[7] Link B 2003 *Phys. Rev. Lett.* **91** 101101
Buckley K B W, Metlitski M A and Zhitnitsky A R 2004 *Phys. Rev.* C **69** 055803

[8] Cui C X, Zuo W and Schulze H J 2008 *Chin. Phys.* B **17** 3289
Li Z H, Li Z H, Zuo W and Luo P Y 2006 *Acta Phys. Sin.* **55** 84 (in Chinese)

[9] Carter B and Langlois D 1998 *Nucl. Phys.* B **531** 478

[10] Carter B 2001 astro-ph/0101257
Langlois D 2000 astro-ph/0008161

[11] Babaev E, Faddeev L D and Niemi A J 2002 *Phys. Rev.* B **65** 100512(R)
Babaev E 2004 *Phys. Rev.* D **70** 043001

[12] Duan Y S, Zhang H and Li S 1998 *Phys. Rev.* B **58** 125
Duan Y S, Li S and Yang G H 1998 *Nucl. Phys.* B **514** 705

[13] Jiang Y and Duan Y S 2000 *J. Math. Phys.* **24** 6463
Duan Y S, Zhang S L and Feng S S 1994 *J. Math. Phys.* **35** 4463

[14] Buckley K B W, Metlitski M A and Zhitnitsky A R 2004 *Phys. Rev. Lett.* **92** 151102

[15] Andreev A F and Bashkin E 1975 *Sov. Phys. JETP* **42** 164

[16] Wu T T and Yang C N 1975 *Phys. Rev.* D **12** 3845
Wu T T and Yang C N 1975 *Phys. Rev.* D **14** 437

[17] Duan Y S, Liu X and Zhang P M 2003 *J. Phys. A: Math. Gen.* **36** 563

[18] Duan Y S, Zhang X H, Liu Y X and Zhao L 2006 *Phys. Rev.* B **74** 144508

[19] Goursat È 1904 *A Course in Mathematical Analysis* translated by Hedrick E R (New York: Dover) Vol.I

[20] Schouten J A 1951 *Tensor Analysis for Physics* (Oxford: Clarendon)

[21] Nielsen H B and Olesen P 1973 *Nucl. Phys.* B **61** 45