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The effect of stochastic acceleration and delay probability on the velocity and the gap between vehicles in traffic flow*

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This paper proposes a new combined cellular automaton (CA) model considering the driver behavior of stochastic acceleration and delay with the velocity of the preceding vehicle and the gap between the successive vehicles based on the WWH model and the noise-first NaSch model. It introduces the delay probability varying with the gap, adds the anticipation headway and increases the acceleration with a certain probability. Through these simulations, not only can the metastable state and start–stop wave be obtained but also the synchronized flow which the wide moving jam results in. Moreover, the effect of stochastic acceleration and delay on traffic flow is discussed by analyzing the correlation of traffic data. This indicates that synchronized flow easily emerges in the critical area between free flow and synchronized flow when acceleration and delay are synchronized or their probability is close to 0.5.

Keywords: traffic flow, noise-first NaSch model, synchronized flow, long-range correlation
PACC: 0550, 0560

1. Introduction

During the past few decades, scientists have developed a wide range of different theories and models of traffic flow. According to the different aspects of traffic flow operations, the traffic models can be categorized into three classes: macroscopic models (e.g. LWR model), mesoscopic models (e.g. reduced gas-kinetic model) and microscopic models (e.g. cellular automaton model, CA).[1] Compared with other dynamical approaches, CA models are conceptually simpler, and can be easily implemented on computers for numerical investigations. Nagel and Schreckenberg proposed a conservative one-dimensional CA model (NaSch model) to simulate freeway traffic in 1992.[2] This model is able to reproduce the basic phenomena of real traffic, such as spontaneous formation of jams, by using very simple rules. The state of the system at time $t+1$ can be obtained from the state at time t by applying the following rules to all cars at the same time: (i) Acceleration: $v_n \to \min(v_n+1, v_{\max})$; (ii) Deterministic deceleration to avoid accidents: $v_n \to \min(v_n, d_n)$; (iii) Randomization: $v_n \to \max(v_n-1, 0)$ with probability p; (iv) Update of positions: $x_n \to x_n + v_n$. Where $v_n(t)$ and $x_n(t)$ are respectively the velocity and the position of the nth car at the time t step; $d_n(t)$ is the distance to the predecessor and $l_{\rm car}$ is the length of a car, $d_n(t) = x_n(t) - x_{n-1}(t) - l_{\rm car}$, usually $l_{\rm car} = 1$; $v_{\max}$ is the maximum velocity.[2,3]

For the shortcomings of the NaSch model (e.g. it cannot simulate synchronized flow), many scientists have proposed many different CA models considering the characteristics of driver behavior in real traffic. Fukui and Ishibashi adopted the Fukui–Ishibashi (FI) traffic flow model with an aggressive acceleration rule;[4] Wang *et al* proposed the WWH model, which is a combined model between the NaSch model and FI model, and also discussed the effect of delay probability on traffic flow;[5,6] Li and Wu presented the VE (velocity effect) model considering the effect of the preceding car;[7] Knospe *et al* presented the CD (comfort-

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able driving) model according to the driver's wish for comfortable and safe driving;[8] Kerner *et al* proposed the Kerner–Klenov–Wolf (KKW) model based on the three-phase theory (e.g. the KKW-1 model);[9,10] Gao and Jiang presented the Gao–Jiang model with the velocity-difference effect on the randomization of vehicles in the framework of three-phase traffic theory.[11] For these CA models, the update rules are modified in the order of the NaSch model.

On the other hand, Xue *et al* investigated the effects of the orders of the evolutive rule on traffic flow. Changing the evolutive steps of the NaSch model, they presented the SDNaSch model (the randomization is arranged between the acceleration and the deterministic deceleration) and the noise-first NaSch model (the randomization is ahead of the acceleration), and proved that the noise-first NaSch model can simulate synchronized flow.[12] Furthermore, Chen *et al* presented the Chen noise-first NaSch model by introducing anticipation headway. As shown by their simulations, the metastable state can be obtained.[13] However, compared to the KKW model and the Gao–Jiang model, the noise-first model (i.e. the noise-first NaSch model and the Chen noise-first NaSch model, named in the following sections) cannot simulate the synchronized flow which a wide moving jam results in.[10,11]

In this paper, we first propose a new combined CA model considering the driver behavior of stochastic delay and acceleration with the velocity of the preceding vehicle and the gap between successive vehicles based on the WWH model and the noise-first model. In our model, we introduce the delay probability varying with the gap, and add the anticipation headway and increase the acceleration with a certain probability.

The rest of the present paper is organized as follows. In Section 2, we describe our new traffic model. In Section 3, we analyse the simulation results in comparison with the noise-first model and the KKW-1 model. In Section 4, we concentrate on the effect of stochastic acceleration and delay on traffic flow by analyzing the correlation of traffic data with an autocorrelation function and DFA (detrended fluctuation analysis). In Section 5, we given some conclusions.

2. Traffic flow model

In real traffic, a driver usually modulates his velocity according to the velocity and distance of the preceding vehicle. When the distance exceeds $v_{\max}$, and especially if the preceding car is out of the driver's sight, he usually will not decelerate; similarly when it is less than some certain value, according to the slow-start rule for some safety reasons, the delay probability will increase. For different subjective reasons and objective ones (such as calm character of drivers and bad weather conditions), only some drivers can anticipate the headway; in addition, a few aggressive drivers will accelerate when the velocity of the preceding vehicle is larger than his velocity. For the reason, the parallel update rules are as follows:

1) Randomization:

$$v'_n(t) = \begin{cases} v_n(t) - 1 & \text{rand} < \psi \ \psi = \begin{cases} f_{\text{slow}} & v_{\max} - \text{acc_avg} < d_n \le v_{\max} \\ \min(1, \mu f_{\text{slow}}) & d_n \le v_{\max} - \text{acc_avg} \end{cases} \\ v_n(t) & \text{otherwise} . \end{cases} \tag{1}$$

In which, rand is a random number between 0 and 1, f_{slow} is the stochastic delay probability, μ is the amplification parameter when the distance is less than some certain value, acc_max, acc_avg and acc_ min are respectively the maximum, the average and the minimum acceleration. The maximum acceleration is usually related to the vehicle engine, and the minimum is 1 (cell/s^2).

2) Dynamical acceleration:

$$\begin{aligned} & v'_n(t) = \min(v'_n(t) + a_n,\ v_{\max}), \\ & a_n = \begin{cases} \min(\text{acc_ max},\ \max(v_{n+1} - v_n,\ \text{acc_avg})), & v_{n+1} > v_n \quad \text{and} \quad f_{\text{fast}} < \text{rand} < f_r + f_{\text{fast}} \\ \text{acc_ min} & \text{otherwise.} \end{cases} \end{aligned} \tag{2}$$

In which, f_{fast} is the stochastic acceleration probability, f_r is the stochastic aggressive acceleration probability, and is usually less than 0.002.

3) Stochastic anticipation headway:

$$d_{\text{headway}} = \max(\min(d_{n+1} - l_{\text{car}}, v_{\max} - l_{\text{car}}, v_{n+1}), 0) \quad \text{with} \quad \text{rand} < f_{\text{fast}}. \tag{3}$$

In which, d_{headway} is the anticipation headway.

4) Deterministic deceleration:

$$v'_n(t) = \min(v'_n(t),\ d_n + d_{\text{headway}}). \tag{4}$$

5) Update of positions:

$$x_n(t+1) = x_n(t) + v'_n(t). \tag{5}$$

3. Simulation results

In this paper, we consider a one-dimensional lattice of L sites with periodic boundary conditions. Let N denote the total number of vehicles, and the traffic parameters are calculated as follows: the traffic density,

$$\rho = \frac{N \cdot l_{\text{car}}}{L}, \tag{6}$$

the average velocity,

$$V = \frac{1}{N} \sum_{i=1}^{N} v_i(t), \tag{7}$$

the average flow,

$$q = \rho V = \frac{l_{\text{car}}}{L} \sum_{i=1}^{N} v_i(t). \tag{8}$$

In these sections, simulations are carried out on a circular road of 1000 cells ($L = 1000$). Each cell corresponds to 7.5 m and a vehicle has a length of one cell ($l_{\text{car}} = 1$). The parameters are set as $v_{\max} =$ 5 cell/s = 135 km/h, acc_ max = 3, acc_avg = 2, acc_ min = 1 (cell/s^2), $\mu = 1.2$ and $f_r = 0.002$. Under the initial condition, vehicles are arranged with the same headway and their velocity is randomly distributed between 0 and $v_{\max}$. Each run of the simulation takes 3×10^5 time steps, and the first 2×10^5 time steps are discarded to eliminate the transient effect. The final results are averaged over all 30 runs of the simulation. Moreover, we use a virtual loop detector to record the velocity of vehicles for analyzing the correlation (it can fix any position for periodic boundary conditions, we set $x = 600$ cell).

Firstly, we obtain the fundamental diagram of different traffic models. Figure 1 clearly shows a comparison among the new model, the noise-first model and the KKW-1 model. As proposed by Xue and Chen, the noise-first model can simulate real traffic better than NaSch model with the same delay probability.[12,13] So we discard the comparison between the NaSch model and our new model. As shown in Fig.1, when $f_{\text{fast}} = f_{\text{slow}} = 0.5$, the fluctuation of average flow of the new model is similar to the noise-first model, and is evidently different from the KKW-1 model. When $0.18 \le \rho \le 0.54$, because the vehicles cannot decelerate when the distance exceeds $v_{\max}$ and because of the effect of f_r on the drivers, the new model allows a fast vehicle to run as fast as possible when the density of road is in the medium level. Thereby the number of fast vehicles in the new model is larger than that in the Chen noise-first model even if only some vehicles anticipate headway in the new model. For this reason, the average flow of the new model is slightly larger than that of the noise-first model.

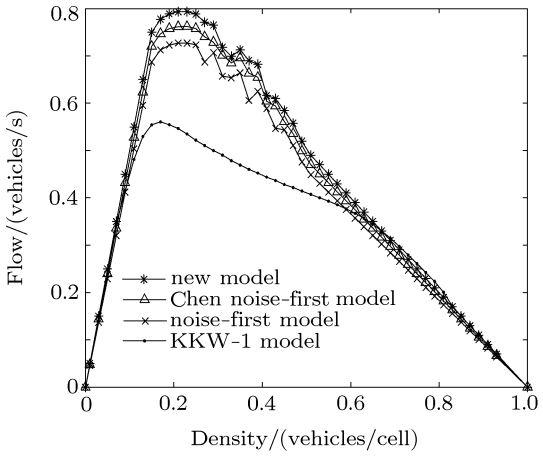


Fig.1. Fundamental diagram of different traffic models: set $\rho = 0.35$, $f_{\text{slow}} = 0.5$, $f_{\text{fast}} = 0.5$, the other parameters of the KKW-1 model are the same as those in Ref.[10].

Moreover, when $0.18 \le \rho \le 0.54$, in comparison with the new model and the noise-first model, the average flow of the KKW-1 model has a steady fluctuation and is less than the others. The reasons are as follows: for the effect of a small cell (1 cell=0.5 m<7.5 m) and the cruise control function on the KKW-1 model, the velocity decreases slowly when the density increases, thus the flow has a steady state for a long time. In addition, the acceleration of the KKW-1 model is much less than that of the other models (a=4 cell/s^2=2 m/s^2 <7.5 m/s^2). Thereby the average velocity is less than that of the other models, and the flow is lower.

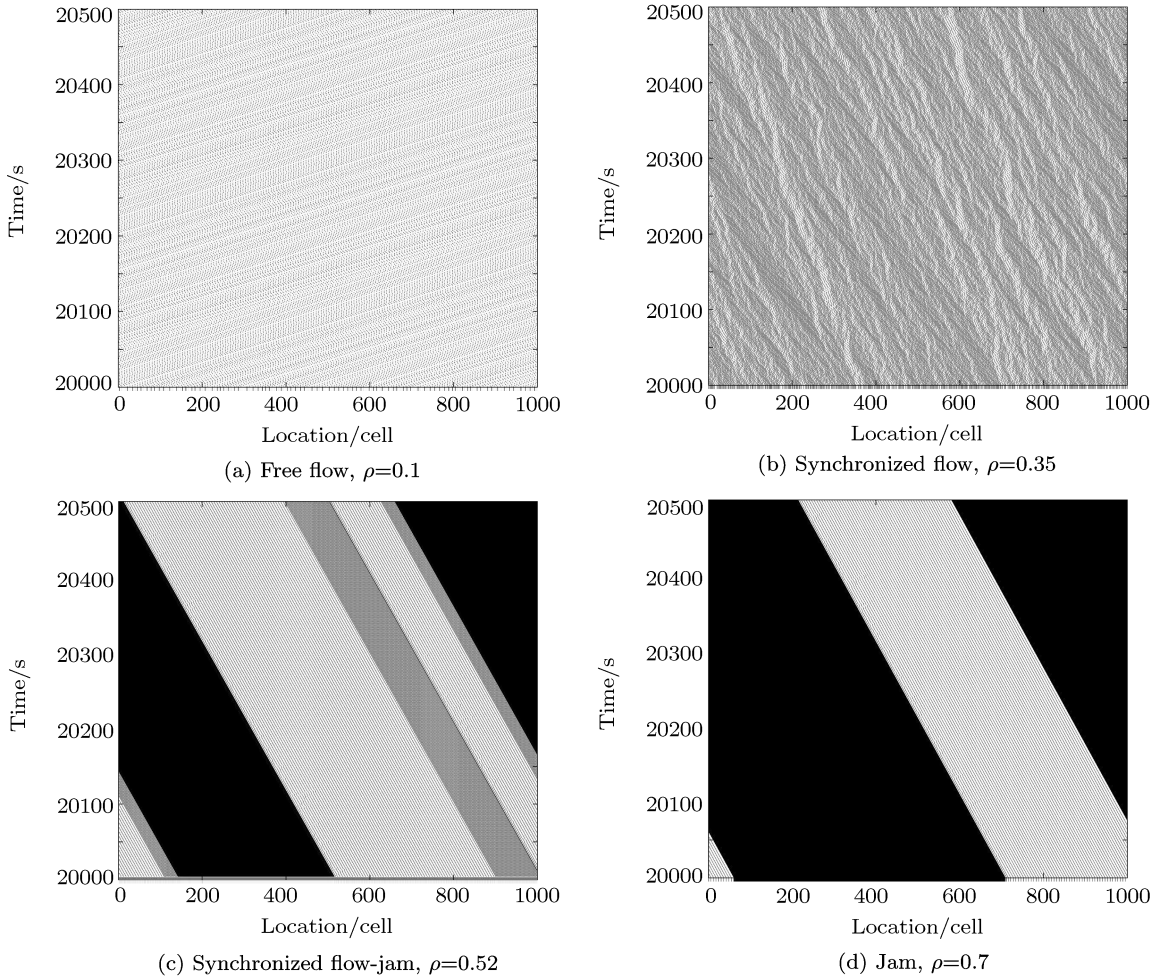


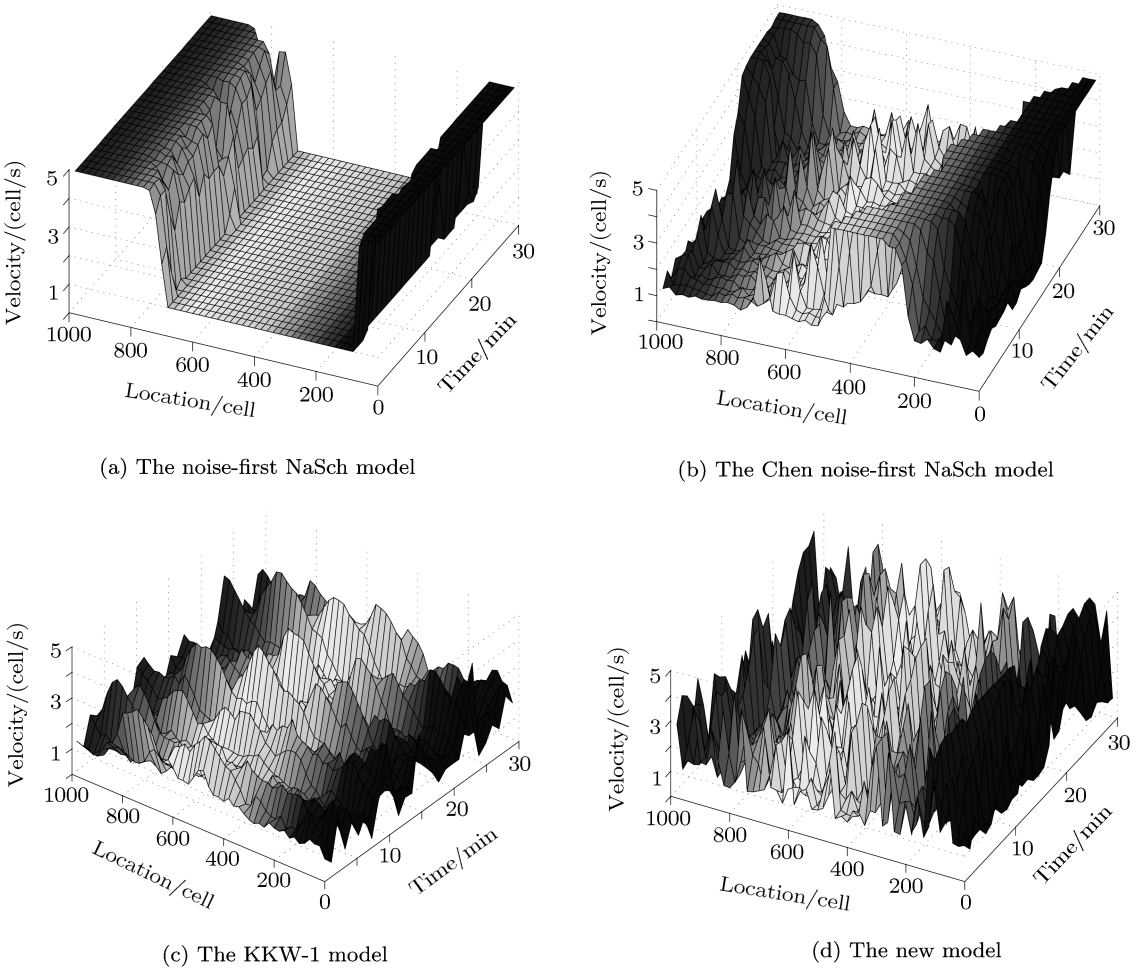
Fig.2. Space–time spot plot describing the evolution of traffic flow of the new model in the case of different densities, $f_{\text{slow}} = 0.5$, $f_{\text{fast}} = 0.5$.

Secondly we capture the influence of our model on traffic flow through the space–time spot plot. Figure 2 clearly shows four states of traffic flow in the case of different densities. Similar to the noise-first model, the gray region represents uniform low-speed flow, the dark regions represent static vehicles which cluster to form the jam, and the dark line and the white region represent stop-and-go traffic. In Fig.2(b), in comparison with the figure of the noise-first model (see Fig.5 in Ref.[12] and Fig.5 in Ref.[13]) in the same density, uniform low-speed flow and the free flow frequently emerge alternately. Apparently, this is different from the noise-first model and similar to the figure of the KKW-1 model in the three-phase theory (see Fig.3(b) in Ref.[11]). As shown in Fig.2, it is indicated that we can obtain the wide moving jam of synchronized flow when employing a stochastic noise function similar to the KKW model on the noise-first model.

Next, we study the difference of the space–time plots. In the simulation of the velocity distribution, we set $\rho = 0.35$, $f_{\text{slow}} = 0.5$, $f_{\text{fast}} = 0.5$, other parameters are the same as Refs.[10], [12] and [13]. Figures 3(a) and 3(b) are the same as the related figures in Refs.[12] and [13]. Compared with the noise-first model, we find that the velocity of the new model and the KKW-1 model fluctuates irregularly. The fluctuation of velocity is apparently in accord with the description of synchronized flow in Fig.2. In addition, for the effect of the anticipation headway and f_r on the velocity, the fluctuation of the new model is more irregular than that of the KKW-1 model.

Finally we discuss the velocity ratio and density relation. As shown in Fig.4, the variation of the new model is similar to the noise-first model in free flow and in a jam: the velocity tends to the maximum when

$0.18 \leq \rho$, and it tends to the maximum or zero when $\rho > 0.5$ in agreement with stop-and-go traffic in the Fig.2.



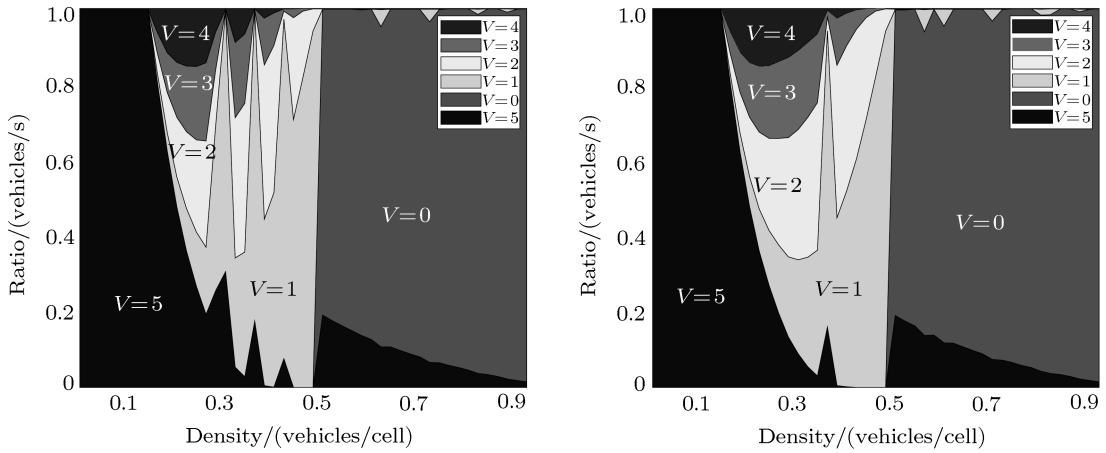
(a) The noise-first NaSch model

(b) The Chen noise-first NaSch model

(c) The KKW-1 model

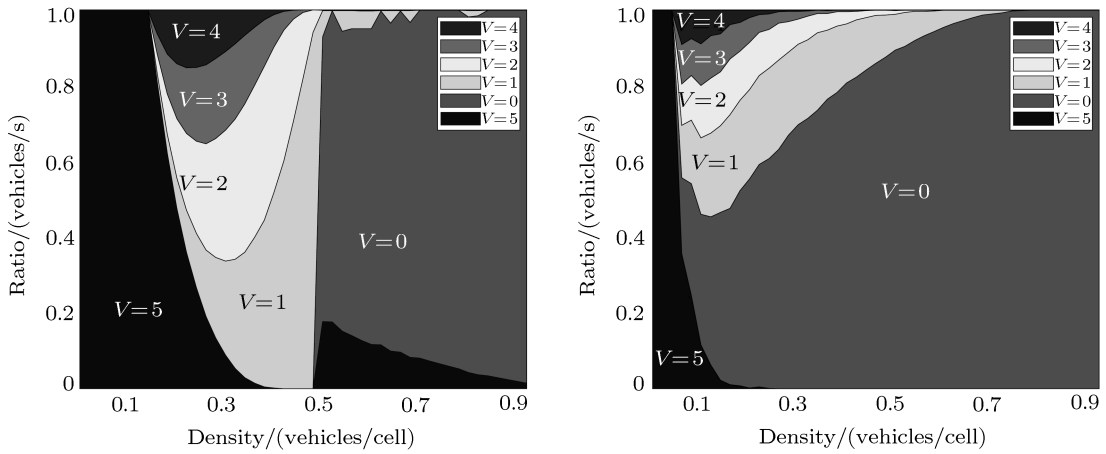
(d) The new model

Fig.3. Space–time plot of different models: set $\rho = 0.35$, $f_{\text{slow}} = 0.5$, $f_{\text{fast}} = 0.5$, other parameters are the same as Refs.[10], [12] and [13].



(a) The new model

(b) The Chen noise-first NaSch model



(c) The noise-first NaSch model (d) The KKW-1 model

Fig.4. The velocity distribution ratio and density relation: set $\rho = 0.35$, $f_{\text{slow}} = 0.5$, $f_{\text{fast}} = 0.5$, other parameters are the same as those in Refs.[10], [12] and [13].

The velocity of the KKW-1 model tends to the maximum when $0.14 \leq \rho$, and the ratio of 5 is close to zero when $\rho > 0.62$. Next, we mainly investigate the difference of velocity distribution in synchronized flow. From Fig.4, the velocity ratio of 1 and 5 in the new model is larger than other values, while the ratio in the noise-first NaSch model is less than others, and the ratio in the Chen noise-first NaSch model is between them. Thus it means that the diversity of velocity in the new model is most evident. Besides, unlike the new models and the noise-first model, in the KKW-1 model, the velocity ratio of 5 is close to zero with the increase of density, while the velocity ratio of 0 increases. Obviously, it is suggested that the velocity of the KKW model tends to a medium value. According to Figs.1 and 3, this is apparently in accord with the difference of the fundamental diagram and space–time plots.

4. Discussion about long-range correlation of traffic flow

Neubert and Jiang each proved that traffic parameters have long-range correlation in synchronized flow by using the autocorrelation function,[11,14] and Wu and Gao proposed a similar conclusion from DFA.[15] Based on our simulation results above, it is believed that the stochastic acceleration and delay should affect the velocity and flow. So it is suggested that they also influence the correlation of traffic flow. In addition, we define two stochastic actions as follows: synchronized action, $f_{\text{slow}} = f_{\text{fast}}$, means that acceleration and delay are synchronized; similarly for asynchronous action, $f_{\text{slow}} = 1 - f_{\text{fast}}$. In our simulation, we set $f_r = 0.002$, $\rho = 0.2$ (the density is in the critical area between free flow and synchronized flow). Time series are obtained through a fixed virtual loop detector mentioned above.

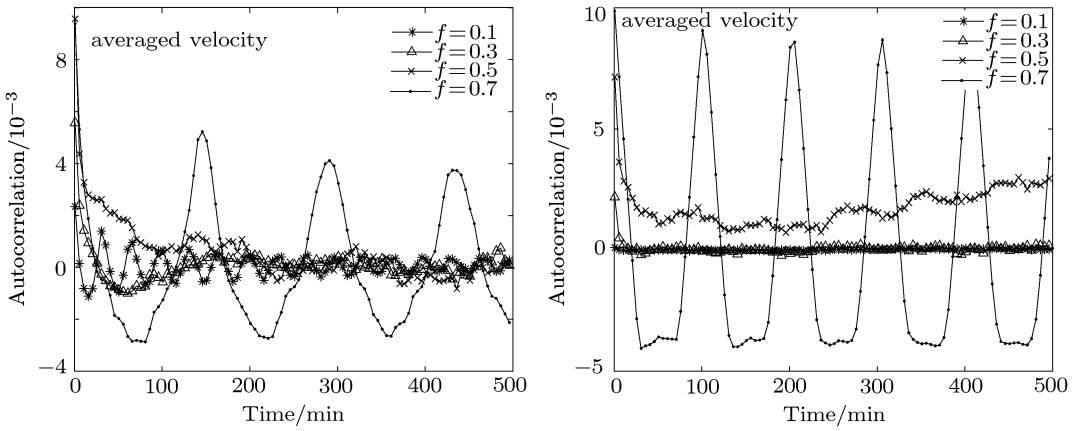
4.1. Autocorrelation function

$$\alpha_x(\tau) = \frac{\langle x(t)x(t+\tau)\rangle - \langle x(t)\rangle^2}{\langle x^2(t)\rangle - \langle x(t)\rangle^2}. \tag{9}$$

In which, $x(t)$ is the aggregated quantities of velocity, and τ is the time interval, and the brackets $\langle\ \rangle$ indicate the average over the whole series of x.

As proposed by Refs.[11] and [14], there is no long-range correlation when the value of the autocorrelation function is close to zero. From Fig.5(a), when acceleration and delay are synchronized, the effect of the preceding vehicle will become more obvious than before. When $f_{\text{slow}} = f_{\text{fast}} = 0.7$, it is in accord with the phenomenon of car-following in real traffic that the value fluctuates periodically. From Fig.5(b), when $f_{\text{slow}} = 0.1$ or $f_{\text{slow}} = 0.3$ (namely $f_{\text{fast}} = 0.9$ or $f_{\text{fast}} = 0.7$), the vehicles with asynchronous action can maintain a high speed, and so the traffic tends to flow freely. Similarly, when f_{slow} increases (that is, f_{fast} decreases), the vehicles tend to

run at a low speed, and so the periodic fluctuation will be obvious. For this reason, in the case of equal f_{slow} ($f_{\text{slow}} \leq 0.5$), the value of the function of the synchronized action is usually larger than the asynchronous action, which means that the synchronized action easily results in synchronized flow.



(a) Synchronized action (b) Asynchronous action

Fig.5. Autocorrelation functions of one minute aggregates of average velocity.

4.2. The analysis of long-range correlation by DFA

The DFA is a well-established method for determining data scaling behavior in the presence of possible trends without knowing their origin and shape. Therefore, it is one of the methods used for analyzing a nonstationary time series with power-law properties. Shang showed that highway traffic data exhibit multifractal behaviour by geometrical approaches, and Feng studied the threshold of extreme weather events by DFA.[16,17] In the method of DFA, we can calculate the value of the Hurst exponent α to determine long-range correlation of time series.[15] When $0 < \alpha < 0.5$, the series is long-range anticorrelated; when $0.5 < \alpha < 1$, it is called long-range correlated; $\alpha = 0.5$ corresponds to Gaussian white noise; $\alpha = 1$ indicates the $1/f$ noise. Wu and Gao showed that the exponent α of the KKW-1 model exhibits approximately a Possion distribution with increasing density by DFA.[15] The details of the DFA method are omitted in the paper. In the simulation, we employ the same method used in Ref.[15], and the linear least square method is built for the local trend function, and the exponent α is averaged over different segment lengths of time series.

To study the correlated characteristics of f_{slow} at the critical density, we give the relationship between the scaling exponent α and f_{slow} in Fig.6. Next, we show some critical results of the simulations. This diagram describes the fluctuation properties of the traffic flow as a function of f_{slow}. The asterisk marked line

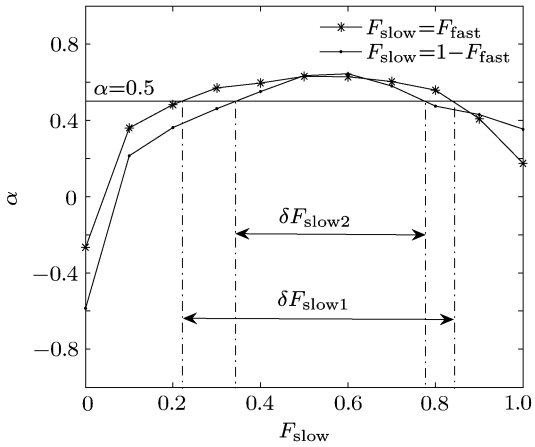


Fig.6. The calculated exponent α as a function of f_{slow}, $\rho = 0.2$.

and the point marked line represent synchronized action and asynchronous action respectively. It clearly shows that the diagram is divided into two parts by the real line (marked: $\alpha = 0.5$), with the long-range anticorrelated area under the line and long-range correlated area above the line. Additionally, two characteristic regions of f_{slow} (δF_{slow1} and δF_{slow2}) are found where traffic flow has a long-range correlation. As

shown in Fig.6, the exponent α exhibits approximately a Possion distribution with the increase of $f_{\rm slow}$, and it reaches a maximum when $f_{\rm slow}$ is between 0.5 and 0.6 (namely $f_{\rm slow} \approx f_{\rm fast}$). Besides, the value of the function for synchronized action is apparently larger than that for asynchronous action at the same $f_{\rm slow}$, and $\delta F_{\rm slow1}$ is larger than $\delta F_{\rm slow2}$. For this reason, we can conclude that synchronized action easily results in synchronized flow. It is apparent that this agrees with the description of Fig.5. In addition, the exponent α is less than 0 when $f_{\rm slow}$ is 0, which means that there is no correlation, so we omit it.

5. Conclusions

In this paper, we have studied the effect of stochastic acceleration and delay on traffic flow. Based on the noise-first NaSch model, we introduced the delay probability varying with the gap, added the anticipation headway and increased the acceleration with a certain probability. As demonstrated by our simulation, it has been found that not only can the metastable state and start–stop wave be obtained but also the synchronized flow that a wide moving jam results in. Thereby it indicats that the noise-first NaSch model can explain some complex traffic phenomena if stochastic acceleration and delay noise similar to the KKW model are introduced. Furthermore, we have investigated the effect of stochastic acceleration and delay on traffic flow when the density is in the critical area between free flow and synchronized flow. As shown in the experimental results, it is suggested that synchronized flow easily emerges at the critical density when acceleration and delay are synchronized or their probability is close to 0.5.

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