Vol 18 No 8, August 2009
1674-1056/2009/18(08)/3325-12
Chinese Physics B
© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Robust stability analysis for Markovian jumping stochastic neural networks with mode-dependent time-varying interval delay and multiplicative noise*

Zhang Hua-Guang(张化光)[a)b)†], Fu Jie(浮 洁)[b)],
Ma Tie-Dong(马铁东)[b)], and Tong Shao-Cheng(佟绍成)[c)]

[a)] *Key Laboratory of Integrated Automation for the Process Industry, Ministry of Education, Northeastern University, Shenyang* 110004, *China*
[b)] *School of Information Science and Engineering, Northeastern University, Shenyang* 110004, *China*
[c)] *Department of Mathematics and Physics, Liaoning University of Technology, Jinzhou* 121001, *China*

(Received 23 November 2008; revised manuscript received 13 February 2009)

This paper is concerned with the problem of robust stability for a class of Markovian jumping stochastic neural networks (MJSNNs) subject to mode-dependent time-varying interval delay and state-multiplicative noise. Based on the Lyapunov–Krasovskii functional and a stochastic analysis approach, some new delay-dependent sufficient conditions are obtained in the linear matrix inequality (LMI) format such that delayed MJSNNs are globally asymptotically stable in the mean-square sense for all admissible uncertainties. An important feature of the results is that the stability criteria are dependent on not only the lower bound and upper bound of delay for all modes but also the covariance matrix consisting of the correlation coefficient. Numerical examples are given to illustrate the effectiveness.

Keywords: mode-dependent time-varying interval delay, multiplicative noise, covariance matrix, correlation coefficient, Markovian jumping stochastic neural networks

PACC: 0545

1. Introduction

Neural networks (NNs) have attracted considerable attention in the past few decades for their applications such as pattern recognition, image processing and associative memory.[1] In the process of information storage and transmission in NNs, since time delays may cause undesirable dynamic network behaviors such as oscillation and instability, the stability and stabilization for delayed NNs have also received considerable attention from researchers,[2−12] where the stability criteria can be delay-dependent or delay-independent. Since delay-independent criteria tend to be conservative, especially when the delay is small or varies in an interval, much attention has been paid to the delay-dependent type.

On the other hand, in the real nervous system, synaptic transmission is a noisy process brought on by random fluctuation from the release of neurotransmitters.[13] Therefore, it is of practical importance and significance to study the stochastic effects on the stability of delayed NNs; some results related to this problem have recently been published,[14−19] where the stochastic disturbance is considered as a Gaussian white noise. Actually, the systems of Itô's differential equations with multiplicative white noise, which have been investigated in Refs.[20]–[22], offer a more general mathematical model to describe physical phenomena. The noises in Refs.[20] and [21] are considered as mutually independent. However, in a real system the noises are mutually dependent,[22] that is to say, the correlation coefficient between different noises should be considered.

Markovian jumping systems — hybrid systems with two components in the state — were first introduced by Krasovskii and Lidskii in Ref.[23]. The jumping systems have the advantage of modeling different types of dynamic systems subjected to abrupt

*Project supported by the National Natural Science Foundation of China (Grant Nos 60534010, 60774048, 60728307, 60804006, 60521003), the National High Technology Research and Development Program of China (863 Program) (Grant No 2006AA04Z183), the Natural Science Foundation of Liaoning Province of China (Grant No 20062018), 973 Project (Grant No 2009CB320601) and 111 Project (Grant No B08015).
†Corresponding author. E-mail: zhanghuaguang@mail.neu.edu.cn, hg_zhang@21cn.com

changes in their structure, such as failure-prone manufacturing systems, power systems and economic systems.[24] This advantage has also attracted increasing interest in the research of delayed MJSNNs,[25−27] However, the time delay investigated in the above literature is only independent of the system modes, which are generally more conservative than mode-dependent ones.[28,29] Up to now, the robust stability analysis problem for MJSNNs with mode-dependent time-varying interval delay and state-multiplicative noise has received very little attention from researchers. It remains important and challenging. This motivates our research.

In this paper, we study the robust stability problem of MJSNNs with mode-dependent time-varying interval delay and state-multiplicative noise. Unlike previous works, in this paper the noises are mutually dependent and the time-varying delay is also dependent on the mode of the system, so the considered MJSNNs are more general than those in the literature. Based on the new Lyapunov functional, the new stability criteria are derived to guarantee that MJSNNs are robustly globally asymptotically stable in the mean-square sense. All results are established in the form of a linear matrix inequality (LMI) and can be solved easily by using the interior algorithms.[30]

2. Problem formulation and preliminaries

Throughout the paper, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual condition (i.e., the filtration contains all $\mathcal{P}$–null sets and is right continuous). Let $L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ be the family of $\mathcal{F}_0$ measurable $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0} \mathbb{E}\,|\xi(\theta)|^2 < \infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. The symbol $\otimes$ denotes the notation of Kronecker product. The symbol * denotes the transposed elements in the symmetric positions of the matrix. Let $\{r(t), t \geq 0\}$ be a right continuous Markov process on the probability space which takes values in a finite state space $S = \{1, 2, \ldots, N\}$ with generator $\Pi = (\pi_{ij})_{N\times N}$ given by

$$P\{r(t+h) = j \,|r(t) = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j, \\ 1 + \pi_{ij}h + o(h), & i = j, \end{cases}$$

where $h > 0$ and $\lim_{h\to 0} o(h)/h = 0$, $\pi_{ij} \geq 0$ is the transition rate from i to j for $i \neq j$, and $\pi_{ii} = -\sum_{j\neq i}^{N} \pi_{ij}$.

Consider MJSNNs with mode-dependent time-varying interval delay and state-multiplicative noise as follows:

$$\begin{aligned} \mathrm{d}x(t) = {} & \big[-A(r(t),t)x(t) + W_0(r(t),t)f(x(t)) \\ & + W_1(r(t),t)f(x(t-\tau(r(t),t)))\big]\mathrm{d}t \\ & + C(r(t),t)x(t)\mathrm{d}w_1(t) \\ & + D(r(t),t)x(t-\tau(r(t),t))\mathrm{d}w_2(t) \\ & + W_2(r(t),t)f(x(t))\mathrm{d}w_3(t) \\ & + W_3(r(t),t)f(x(t-\tau(r(t),t)))\mathrm{d}w_4(t), \\ x(t) = {} & \phi(t), t \in [-\tau_M, 0], \end{aligned} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the neural state vector associated with the neurons. $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ is the neural activation function. $A(r(t),t) = A(r(t)) + \Delta A(r(t),t)$, $W_j(r(t),t) = W_j(r(t)) + \Delta W_j(r(t),t)$ $(j = 0,1,2,3)$, $C(r(t),t) = C(r(t)) + \Delta C(r(t),t)$, $D(r(t),t) = D(r(t)) + \Delta D(r(t),t)$, the diagonal matrix $A(r(t)) = \mathrm{diag}(a_1(r(t)), a_2(r(t)), \ldots, a_n(r(t)))$ with positive entries $a_j(r(t)) > 0$, the matrices $W_0(r(t)) \in \mathbb{R}^{n\times n}$ and $W_1(r(t)) \in \mathbb{R}^{n\times n}$ are the connection weight matrix and the delayed connection weight matrix, respectively. The quantities $\Delta A(r(t),t)$, $\Delta W_j(r(t),t)$ $(j = 0,1,2,3)$, $\Delta C(r(t),t)$, $\Delta D(r(t),t)$ represent the parameter uncertainties. The quantity $w(t) = [w_1(t), w_2(t), w_3(t), w_4(t)]^T$, independent of the Markov process, is a four-dimensional Brownian motion defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ and satisfies $\mathbb{E}\{\mathrm{d}w_j(t)\} = 0$, $\mathbb{E}\{\mathrm{d}w_j(t)\mathrm{d}w_k(t)\} = \bar{\alpha}_{jk}\mathrm{d}t$ $(j \neq k)$, $|\bar{\alpha}_{jk}| < 1$, $\mathbb{E}\{\mathrm{d}w_j(t)^2\} = \mathrm{d}t$ $(j,k = 1,2,3,4)$. The quantity $\tau_M = \max\{\tau_M(r(t)), r(t) \in S\}$, where $\tau_M(r(t))$ is defined in Assumption 1 below.

For simplicity, we write $r(t) = i \in S$ and

$$\begin{aligned} \mathrm{d}x(t) = {} & \big[-A_i(t)x(t) + W_{0i}(t)f(x(t)) \\ & + W_{1i}(t)f(x(t-\tau_i(t)))\big]\mathrm{d}t \\ & + C_i(t)x(t)\mathrm{d}w_1(t) \\ & + D_i(t)x(t-\tau_i(t))\mathrm{d}w_2(t) \\ & + W_{2i}(t)f(x(t))\mathrm{d}w_3(t) \\ & + W_{3i}(t)f(x(t-\tau_i(t)))\mathrm{d}w_4(t), \\ x(t) = {} & \phi(t), t \in [-\tau_M, 0]. \end{aligned} \tag{2}$$

Let $V(x(t),t,i) \in \mathcal{C}^{2,1}(\mathbb{R}^n \times \mathbb{R}^+ \times S; \mathbb{R}^+)$ be a positive function which is continuously twice differentiable in x and once differentiable in t. Thus, the stochastic

derivative operator $\mathcal{L}$ acting on $V(x(t),t,i)$ is defined by

$$\begin{aligned}\mathcal{L}V(x(t),t,i) &= V_t(x(t),t,i) + V_x(x(t),t,i)g_{1i}(t)\\ &\quad + \frac{1}{2}\mathrm{tr}[g_{2i}^T(t)V_{xx}(x(t),t,i)g_{2i}(t)\bar{P}]\\ &\quad + \sum_{j=1}^{N}\pi_{ij}V(x(t),t,j), \end{aligned} \tag{3}$$

where

$$V_t(x(t),t,i) = \frac{\partial V(x(t),t,i)}{\partial t},$$

$$V_x(x(t),t,i) = \left(\frac{\partial V(x(t),t,i)}{\partial x_1},\ldots,\frac{\partial V(x(t),t,i)}{\partial x_n}\right),$$

$$V_{xx}(x(t),t,i) = \left(\frac{\partial^2 V(x(t),t,i)}{\partial x_j \partial x_k}\right)_{n\times n},$$

$$\bar{P} = \begin{bmatrix} 1 & \bar{\alpha}_{12} & \bar{\alpha}_{13} & \bar{\alpha}_{14}\\ * & 1 & \bar{\alpha}_{23} & \bar{\alpha}_{24}\\ * & * & 1 & \bar{\alpha}_{34}\\ * & * & * & 1 \end{bmatrix}$$

is the covariance matrix of the augmented Wiener process vector $\mathrm{d}w(t)$, that is

$$\mathrm{d}w(t)(\mathrm{d}w(t))^T = \bar{P}\mathrm{d}t,$$

$$\begin{aligned} g_{1i}(t) &= -A_i(t)x(t) + W_{0i}(t)f(x(t))\\ &\quad + W_{1i}(t)f(x(t-\tau_i(t))), \end{aligned}$$

$$\begin{aligned} g_{2i}(t) = [&C_i(t)x(t) \;\; D_i(t)x(t-\tau_i(t)) \; W_{2i}(t)f(x(t))\\ &W_{3i}(t)f(x(t-\tau_i(t)))]. \end{aligned}$$

Assumption 1 The time-varying delays $\tau_i(t)$ satisfy

$$0 \le \tau_{im} \le \tau_i(t) \le \tau_{iM},\ \dot{\tau}_i(t) \le \mu_i, \tag{4}$$

where τ_{im}, τ_{iM} and μ_i are constants for any $i \in S$.

Assumption 2 The neural activation function $f_j(x_j)$ satisfies

$$0 \le \frac{f_j(x_j) - f_j(y_j)}{x_j - y_j} \le l_j,\ \forall x_j, y_j \in \mathbb{R}, j = 1,\ldots,n, \tag{5}$$

which implies that

$$f_j(x_j(t))[f_j(x_j(t)) - l_j x_j(t)] \le 0,\ f_j(0) = 0. \tag{6}$$

Assumption 3 The parameter uncertainties $\Delta A_i(t)$, $\Delta W_{0i}(t)$, $\Delta W_{1i}(t)$, $\Delta C_i(t)$, $\Delta D_i(t)$, $\Delta W_{2i}(t)$ and $\Delta W_{3i}(t)$ satisfy

$$\begin{aligned} &[\Delta A_i(t)\ \Delta W_{0i}(t)\ \Delta W_{1i}(t)\ \Delta C_i(t)\ \Delta D_i(t)\ \Delta W_{2i}(t)\\ &\Delta W_{3i}(t)] = H_i F_i(t)[E_{1i}\ E_{2i}\ E_{3i}\ E_{4i}\ E_{5i}\ E_{6i}\ E_{7i}], \end{aligned} \tag{7}$$

where H_i, E_{ji} $(j = 1,2,\ldots,7)$ are known constant matrices and $F_i(t)$ are unknown time-varying matrices and satisfy $F_i^T(t)F_i(t) \le I$, for $\forall t \in \mathbb{R}, i \in S$.

Definition 1 For any $\xi \in L^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, $r(0) = i_0 \in S$, the equilibrium solution of system (1) is said to be robustly, globally and asymptotically stable in the mean-square sense if

$$\lim_{t\to\infty} \mathbb{E}\left|x(t,\xi,i_0)\right|^2 = 0, \tag{8}$$

where $x(t,\xi,i_0)$ is the solution of system (1) at time t under the initial state ξ and initial mode i_0.

Lemma 1 For any vector $x, y \in \mathbb{R}^n$, matrices $A, P \in \mathbb{R}^{n\times n}$, $D \in \mathbb{R}^{n\times m}$ and $F(t) \in \mathbb{R}^{m\times m}$ with $P > 0$, $F^T(t)F(t) \le I$ and scalar $\varepsilon > 0$, the following inequalities hold:[31]

(i) $2x^T y \le x^T P^{-1} x + y^T P y$.

(ii) $DF(t)N + N^T F^T(t) D^T \le \varepsilon^{-1} DD^T + \varepsilon N^T N$.

(iii) If $P - \varepsilon DD^T > 0$, then $(A + DF(t)N)^T P^{-1}(A + DF(t)N) \le A^T(P - \varepsilon DD^T)^{-1}A + \varepsilon N^T N$.

Lemma 2 For matrix $g(t) = [g_1(t), g_2(t), g_3(t), g_4(t)] \in \mathbb{R}^{n\times 4}$, positive matrix $G \in \mathbb{R}^{n\times n}$, vector $w(t)$, matrix $\bar{P}$, where $w(t)$ and $\bar{P}$ are defined in the above description of $\mathcal{L}V(x_t,t,i)$, the following equation holds:

$$\begin{aligned} &\mathbb{E}\left\{\left[\int_{t-\tau}^{t} g(s)\mathrm{d}w(s)\right]^T G \int_{t-\tau}^{t} g(s)\mathrm{d}w(s)\right\}\\ &= \mathbb{E}\left\{\int_{t-\tau}^{t} \mathrm{tr}[g^T(s)Gg(s)\bar{P}]\mathrm{d}s\right\}. \end{aligned} \tag{9}$$

The proof can be found in Appendix A.

3. Main results

For the convenience of presentation, in the following, we denote $\tau_m = \min\{\tau_{im}, i \in S\}$, $\tau_o = \tau_M - \tau_m$, $\delta = \max\{-\pi_{ii}, i \in S\}$.

The main results of this paper are presented as follows:

Theorem 1 For given scalars $\tau_m \ge 0$, $\tau_M > 0$, μ_i, $\bar{\alpha}_{jk}$ $(j,k = 1,2,3,4, j < k)$ and $l = \max_{1\le j\le n}\{l_j\}$, the MJSNNs described by system (1) are robustly, globally and asymptotically stable in the mean-square sense, if there exist real matrices $P_i > 0$, $Q_j > 0$ $(j = 1,2,3,4)$, $R_1 > 0$, $R_2 > 0$, $S_1 > 0$, $S_2 > 0$, M_{ji}, N_{ji}, T_{ji}, V_{ji}, X^i_{jk}, Y^i_{jk} $(j,k = 1,2,3)$, $\Lambda = \mathrm{diag}(\lambda_1,\lambda_2,\ldots,\lambda_n) \ge 0$, diagonal matrices $K_1 \ge 0$,

$K_2 \geq 0$ of appropriate dimensions and positive scalars ε_{1i}, ε_{2i} such that the following LMIs hold for $i \in S$:

$$\Phi_i = \begin{bmatrix} \Phi_1^i & M_i & N_i & T_i & V_i H_i & \hat{Z}_i & 0 \\ * & -S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -S_2 & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{S} & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_{1i} I & 0 & 0 \\ * & * & * & * & * & -\tilde{Z}_i & \tilde{Z}_i \tilde{H}_i \\ * & * & * & * & * & * & -\varepsilon_{2i} I \end{bmatrix} < 0, \tag{10}$$

$$\Omega_1^i = \begin{bmatrix} X_i & \bar{M}_i \\ * & R_1 \end{bmatrix} \geq 0, \tag{11}$$

$$\Omega_2^i = \begin{bmatrix} Y_i & \bar{N}_i \\ * & R_2 \end{bmatrix} \geq 0, \tag{12}$$

$$\Omega_3^i = \begin{bmatrix} X_i + Y_i & \bar{T}_i \\ * & R_1 + R_2 \end{bmatrix} \geq 0, \tag{13}$$

where

$$\Phi_1^i = \begin{bmatrix} \Phi_{1,1}^i & \Phi_{1,2}^i & \Phi_{1,3}^i & \Phi_{1,4}^i & \Phi_{1,5}^i & N_{1i} & -T_{1i} \\ * & \Phi_{2,2}^i & \Phi_{2,3}^i & \Phi_{2,4}^i & \Phi_{2,5}^i & N_{2i} & -T_{2i} \\ * & * & \Phi_{3,3}^i & \Phi_{3,4}^i & \Phi_{3,5}^i & 0 & 0 \\ * & * & * & \Phi_{4,4}^i & \Phi_{4,5}^i & 0 & 0 \\ * & * & * & * & \Phi_{5,5}^i & N_{3i} & -T_{3i} \\ * & * & * & * & * & -Q_3 & 0 \\ * & * & * & * & * & * & -Q_4 \end{bmatrix},$$

$$\begin{aligned}
\Phi_{1,1}^i &= (1+\delta\tau_o)Q_1 + Q_3 + Q_4 + M_{1i} + M_{1i}^T - V_{1i}A_i \\
&\quad - A_i^T V_{1i}^T + \tau_M X_{1,1}^i + \tau_o Y_{1,1}^i + \sum_{j=1}^{N} \pi_{ij} P_j \\
&\quad + \varepsilon_{1i} E_{1i}^T E_{1i} + \varepsilon_{2i} E_{4i}^T E_{4i}, \\
\Phi_{1,2}^i &= M_{2i}^T - M_{1i} - N_{1i} + T_{1i} - A_i^T V_{2i}^T \\
&\quad + \tau_M X_{1,2}^i + \tau_o Y_{1,2}^i, \\
\Phi_{1,3}^i &= K_1 L + V_{1i} W_{0i} - \varepsilon_{1i} E_{1i}^T E_{2i}, \\
\Phi_{1,4}^i &= V_{1i} W_{1i} - \varepsilon_{1i} E_{1i}^T E_{3i}, \\
\Phi_{1,5}^i &= P_i - V_{1i} - A_i^T V_{3i}^T + M_{3i}^T + \tau_M X_{1,3}^i + \tau_o Y_{1,3}^i, \\
\Phi_{2,2}^i &= -(1-\mu_i)Q_1 - M_{2i} - M_{2i}^T - N_{2i} - N_{2i}^T + T_{2i} \\
&\quad + T_{2i}^T + \tau_M X_{2,2}^i + \tau_o Y_{2,2}^i + \varepsilon_{2i} E_{5i}^T E_{5i}, \\
\Phi_{2,3}^i &= V_{2i} W_{0i}, \\
\Phi_{2,4}^i &= V_{2i} W_{1i} + K_2 L, \\
\Phi_{2,5}^i &= -V_{2i} - M_{3i}^T - N_{3i}^T + T_{3i}^T + \tau_M X_{2,3}^i + \tau_o Y_{2,3}^i, \\
\Phi_{3,3}^i &= -K_1^T - K_1 + (1+\delta\tau_o)Q_2 + \varepsilon_{1i} E_{2i}^T E_{2i} \\
&\quad + \varepsilon_{2i} E_{6i}^T E_{6i}, \\
\Phi_{3,4}^i &= \varepsilon_{1i} E_{2i}^T E_{3i}, \\
\Phi_{3,5}^i &= \Lambda + W_{0i}^T V_{3i}^T, \\
\Phi_{4,4}^i &= -(1-\mu_i)Q_2 - K_2 - K_2^T + \varepsilon_{1i} E_{3i}^T E_{3i} \\
&\quad + \varepsilon_{2i} E_{7i}^T E_{7i}, \\
\Phi_{4,5}^i &= W_{1i}^T V_{3i}^T, \\
\Phi_{5,5}^i &= -V_{3i} - V_{3i}^T + \tau_M R_1 + \tau_o R_2 + \tau_M X_{3,3}^i + \tau_o Y_{3,3}^i, \\
\tilde{S} &= S_1 + S_2, \\
Z_i &= P_i + l\Lambda + \tau_M S_1 + \tau_o S_2, \\
\tilde{\mathrm{Z}}_i &= \bar{P} \otimes Z_i, \\
\tilde{H}_i &= \mathrm{diag}\left(H_i, H_i, H_i, H_i\right),
\end{aligned}$$

$$\hat{Z}_i^T = \begin{bmatrix} Z_i C_i & \bar{\alpha}_{12} Z_i D_i & \bar{\alpha}_{13} Z_i W_{2i} & \bar{\alpha}_{14} Z_i W_{3i} & 0\ 0\ 0 \\ \bar{\alpha}_{12} Z_i C_i & Z_i D_i & \bar{\alpha}_{23} Z_i W_{2i} & \bar{\alpha}_{24} Z_i W_{3i} & 0\ 0\ 0 \\ \bar{\alpha}_{13} Z_i C_i & \bar{\alpha}_{23} Z_i D_i & Z_i W_{2i} & \bar{\alpha}_{34} Z_i W_{3i} & 0\ 0\ 0 \\ \bar{\alpha}_{14} Z_i C_i & \bar{\alpha}_{24} Z_i D_i & \bar{\alpha}_{34} Z_i W_{2i} & Z_i W_{3i} & 0\ 0\ 0 \end{bmatrix},$$

$$X_i = \begin{bmatrix} X_{1,1}^i & X_{1,2}^i & X_{1,3}^i \\ * & X_{2,2}^i & X_{2,3}^i \\ * & * & X_{3,3}^i \end{bmatrix},$$

$$Y_i = \begin{bmatrix} Y_{1,1}^i & Y_{1,2}^i & Y_{1,3}^i \\ * & Y_{2,2}^i & Y_{2,3}^i \\ * & * & Y_{3,3}^i \end{bmatrix},$$

$$\begin{aligned}
M_i^T &= [M_{1i}^T\ M_{2i}^T\ 0\ 0\ M_{3i}^T\ 0\ 0], \\
\bar{M}_i^T &= [M_{1i}^T\ M_{2i}^T\ M_{3i}^T], \\
N_i^T &= [N_{1i}^T\ N_{2i}^T\ 0\ 0\ N_{3i}^T\ 0\ 0], \\
\bar{N}_i^T &= [N_{1i}^T\ N_{2i}^T\ N_{3i}^T], \\
T_i^T &= [T_{1i}^T\ T_{2i}^T\ 0\ 0\ T_{3i}^T\ 0\ 0], \\
\bar{T}_i^T &= [T_{1i}^T\ T_{2i}^T\ T_{3i}^T], \\
V_i^T &= [V_{1i}^T\ V_{2i}^T\ 0\ 0\ V_{3i}^T\ 0\ 0].
\end{aligned}$$

Proof Choose a Lyapunov–Krasovskii functional candidate $V_i(t) = V(x(t), t, i)$ as

$$V_i(t) = V_{1i}(t) + V_{2i}(t) + V_{3i}(t) + V_{4i}(t)$$

with

$$V_{1i}(t) = x^T(t)P_ix(t) + 2\sum_{k=1}^{n}\lambda_k\int_0^{x_k} f_k(s)\mathrm{d}s,$$

$$\begin{aligned}V_{2i}(t) &= \int_{t-\tau_i(t)}^{t}[x^T(s)Q_1x(s) + f^T(x(s))Q_2f(x(s))]\mathrm{d}s\\ &\quad+\int_{t-\tau_m}^{t}x^T(s)Q_3x(s)\mathrm{d}s\\ &\quad+\int_{t-\tau_M}^{t}x^T(s)Q_4x(s)\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_{3i}(t) &= \delta\int_{-\tau_M}^{-\tau_m}\int_{t+s}^{t}[x^T(\theta)Q_1x(\theta)+f^T(x(\theta))Q_2\\ &\quad\times f(x(\theta))]\mathrm{d}\theta\mathrm{d}s,\end{aligned}$$

$$\begin{aligned}V_{4i}(t) &= \int_{-\tau_M}^{0}\int_{t+s}^{t}g_{1i}^T(\theta)R_1g_{1i}(\theta)\mathrm{d}\theta\mathrm{d}s\\ &\quad+\int_{-\tau_M}^{0}\int_{t+s}^{t}\mathrm{tr}[g_{2i}^T(\theta)S_1g_{2i}(\theta)\bar{P}]\mathrm{d}\theta\mathrm{d}s\\ &\quad+\int_{-\tau_M}^{-\tau_m}\int_{t+s}^{t}g_{1i}^T(\theta)R_2g_{1i}(\theta)\mathrm{d}\theta\mathrm{d}s\\ &\quad+\int_{-\tau_M}^{-\tau_m}\int_{t+s}^{t}\mathrm{tr}[g_{2i}^T(\theta)S_2g_{2i}(\theta)\bar{P}]\mathrm{d}\theta\mathrm{d}s.\end{aligned}$$

Then, by formula (3), the stochastic differential of $V_i(t)$ along with system (1) can be obtained as follows:

$$\begin{aligned}\mathcal{L}V_i(t) &\le 2x^T(t)P_ig_{1i}(t) + \mathrm{tr}[g_{2i}^T(t)Z_ig_{2i}(t)\bar{P}]\\ &\quad+\sum_{j=1}^{N}\pi_{ij}x^T(t)P_jx(t) + 2f^T(x(t))\Lambda g_{1i}(t)\\ &\quad+x^T(t)[(1+\delta\tau_o)Q_1+Q_3+Q_4]x(t)\\ &\quad+f^T(x(t))(1+\delta\tau_o)Q_2f(x(t))\\ &\quad-(1-\mu_i)[x^T(t-\tau_i(t))Q_1x(t-\tau_i(t))\\ &\quad+f^T(x(t-\tau_i(t)))Q_2f(x(t-\tau_i(t)))]\\ &\quad-x^T(t-\tau_m)Q_3x(t-\tau_m)\\ &\quad-x^T(t-\tau_M)Q_4x(t-\tau_M)\\ &\quad+g_{1i}^T(t)(\tau_MR_1+\tau_oR_2)g_{1i}(t)\\ &\quad-\int_{t-\tau_i(t)}^{t}g_{1i}^T(s)R_1g_{1i}(s)\mathrm{d}s\\ &\quad-\int_{t-\tau_i(t)}^{t-\tau_m}g_{1i}^T(s)R_2g_{1i}(s)\mathrm{d}s\\ &\quad-\int_{t-\tau_M}^{t-\tau_i(t)}g_{1i}^T(s)(R_1+R_2)g_{1i}(s)\mathrm{d}s\\ &\quad-\int_{t-\tau_i(t)}^{t}\mathrm{tr}[g_{2i}^T(s)S_1g_{2i}(s)\bar{P}]\mathrm{d}s\\ &\quad-\int_{t-\tau_i(t)}^{t-\tau_m}\mathrm{tr}[g_{2i}^T(s)S_2g_{2i}(s)\bar{P}]\mathrm{d}s\\ &\quad-\int_{t-\tau_M}^{t-\tau_i(t)}\mathrm{tr}[g_{2i}^T(s)\tilde{S}g_{2i}(s)\bar{P}]\mathrm{d}s.\end{aligned}\tag{14}$$

The proof of formula (14) can be found in Appendix B.

For arbitrary matrices $M_{ji}, N_{ji}, T_{ji}, V_{ji}$ $(j=1,2,3)$ with compatible dimensions, we have

$$\begin{aligned}&2\zeta^T(t)M_i\big[x(t)-x(t-\tau_i(t))\\ &-\int_{t-\tau_i(t)}^{t}g_{1i}(s)\mathrm{d}s-\int_{t-\tau_i(t)}^{t}g_{2i}(s)\mathrm{d}w(s)\big]=0,\end{aligned}\tag{15}$$

$$\begin{aligned}&2\zeta^T(t)N_i\big[x(t-\tau_m)-x(t-\tau_i(t))\\ &-\int_{t-\tau_i(t)}^{t-\tau_m}g_{1i}(s)\mathrm{d}s-\int_{t-\tau_i(t)}^{t-\tau_m}g_{2i}(s)\mathrm{d}w(s)\big]=0,\end{aligned}\tag{16}$$

$$\begin{aligned}&2\zeta^T(t)T_i\big[x(t-\tau_i(t))-x(t-\tau_M)\\ &-\int_{t-\tau_M}^{t-\tau_i(t)}g_{1i}(s)\mathrm{d}s-\int_{t-\tau_M}^{t-\tau_i(t)}g_{2i}(s)\mathrm{d}w(s)\big]=0,\end{aligned}\tag{17}$$

$$\begin{aligned}&2\zeta^T(t)V_i[-A_i(t)x(t)-g_{1i}(t)\\ &+W_{0i}(t)f(x(t))+W_{1i}(t)f(x(t-\tau_i(t)))]=0,\end{aligned}\tag{18}$$

where

$$\begin{aligned}\zeta^T(t) &= [x^T(t)\ x^T(t-\tau_i(t))\ f^T(x(t))\ f^T(x(t-\tau_i(t)))\\ &\quad g_{1i}^T(t)\ x^T(t-\tau_m)\ x^T(t-\tau_M)].\end{aligned}$$

For formulas (15)–(17), by (i) of Lemma 1, we have

$$\begin{aligned}&-2\zeta^T(t)M_i\int_{t-\tau_i(t)}^{t}g_{2i}(s)\mathrm{d}w(s)\\ &\le\zeta^T(t)M_iS_1^{-1}M_i^T\zeta(t)\\ &\quad+\Big(\int_{t-\tau_i(t)}^{t}g_{2i}(s)\mathrm{d}w(s)\Big)^TS_1\int_{t-\tau_i(t)}^{t}g_{2i}(s)\mathrm{d}w(s),\end{aligned}\tag{19}$$

$$\begin{aligned}&-2\zeta^T(t)N_i\int_{t-\tau_i(t)}^{t-\tau_m}g_{2i}(s)\mathrm{d}w(s)\\ &\le\zeta^T(t)N_iS_2^{-1}N_i^T\zeta(t)\\ &\quad+\Big(\int_{t-\tau_i(t)}^{t-\tau_m}g_{2i}(s)\mathrm{d}w(s)\Big)^TS_2\int_{t-\tau_i(t)}^{t-\tau_m}g_{2i}(s)\mathrm{d}w(s),\end{aligned}\tag{20}$$

$$\begin{aligned}&-2\zeta^T(t)T_i\int_{t-\tau_M}^{t-\tau_i(t)}g_{2i}(s)\mathrm{d}w(s)\\ &\le\zeta^T(t)T_i\tilde{S}^{-1}T_i^T\zeta(t)\\ &\quad+\Big(\int_{t-\tau_M}^{t-\tau_i(t)}g_{2i}(s)\mathrm{d}w(s)\Big)^T\tilde{S}\int_{t-\tau_M}^{t-\tau_i(t)}g_{2i}(s)\mathrm{d}w(s).\end{aligned}\tag{21}$$

For formula (18), by (ii) of Lemma 1, we have

$$\begin{aligned}&2\zeta^T(t)V_i(\tilde{A}_i+\Delta\tilde{A}_i(t))\zeta(t)\\&=2\zeta^T(t)V_i(\tilde{A}_i+H_iF_i(t)\tilde{E}_i)\zeta(t)\\&\le\zeta^T(t)(V_i\tilde{A}_i+\tilde{A}_i^TV_i^T+\varepsilon_{1i}^{-1}V_iH_iH_i^TV_i^T\\&\quad+\varepsilon_{1i}\tilde{E}_i^T\tilde{E}_i)\zeta(t),\end{aligned}\tag{22}$$

where

$$\begin{aligned}&\tilde{A}_i=[-A_i\ 0\ W_{0i}\ W_{1i}\ -I\ 0\ 0],\\&\Delta\tilde{A}_i(t)=[-\Delta A_i(t)\ 0\ \Delta W_{0i}(t)\ \Delta W_{1i}(t)\ 0\ 0\ 0],\\&\tilde{E}_i=[-E_{1i}\ 0\ E_{2i}\ E_{3i}\ 0\ 0\ 0].\end{aligned}$$

Based on (iii) of Lemma 1, it is easy to know

$$\begin{aligned}&\mathrm{tr}[g_{2i}^T(t)Z_ig_{2i}(t)\bar{P}]\\&\le\zeta^T(t)\Phi_2^i\zeta(t)+\varepsilon_{2i}\tilde{\zeta}^T(t)\\&\quad\times\mathrm{diag}\left(E_{4i}^TE_{4i},E_{5i}^TE_{5i},E_{6i}^TE_{6i},E_{7i}^TE_{7i}\right)\tilde{\zeta}(t),\end{aligned}\tag{23}$$

where

$$\begin{aligned}&\tilde{\zeta}^T(t)=[x^T(t)\ x^T(t-\tau_i(t))\ f^T(x(t))\ f^T(x(t-\tau_i(t)))],\\&\Phi_2^i=\tilde{C}_i^T(\tilde{Z}_i^{-1}-\varepsilon_{2i}\mathrm{diag}(H_iH_i^T,H_iH_i^T,H_iH_i^T,\\&\quad H_iH_i^T)^{-1}\tilde{C}_i,\end{aligned}$$

$$\tilde{C}_i=\begin{bmatrix}C_i&0&0&0&0\,0\,0\\0&D_i&0&0&0\,0\,0\\0&0&W_{2i}&0&0\,0\,0\\0&0&0&W_{3i}&0\,0\,0\end{bmatrix}.$$

The proof of formula (23) can be found in Appendix C.

On the other hand, the following equations hold:

$$\begin{aligned}0&=\int_{t-\tau_M}^t\hat{\zeta}^T(t)X_i\hat{\zeta}(t)\mathrm{d}s-\int_{t-\tau_M}^t\hat{\zeta}^T(t)X_i\hat{\zeta}(t)\mathrm{d}s\\&=\tau_M\hat{\zeta}^T(t)X_i\hat{\zeta}(t)-\int_{t-\tau_i(t)}^t\hat{\zeta}^T(t)X_i\hat{\zeta}(t)\mathrm{d}s\\&\quad-\int_{t-\tau_M}^{t-\tau_i(t)}\hat{\zeta}^T(t)X_i\hat{\zeta}(t)\mathrm{d}s,\end{aligned}\tag{24}$$

$$\begin{aligned}0&=\int_{t-\tau_M}^{t-\tau_m}\hat{\zeta}^T(t)Y_i\hat{\zeta}(t)\mathrm{d}s-\int_{t-\tau_M}^{t-\tau_m}\hat{\zeta}^T(t)Y_i\hat{\zeta}(t)\mathrm{d}s\\&=\tau_o\hat{\zeta}^T(t)Y_i\hat{\zeta}(t)-\int_{t-\tau_i(t)}^{t-\tau_m}\hat{\zeta}^T(t)Y_i\hat{\zeta}(t)\mathrm{d}s\\&\quad-\int_{t-\tau_M}^{t-\tau_i(t)}\hat{\zeta}^T(t)Y_i\hat{\zeta}(t)\mathrm{d}s,\end{aligned}\tag{25}$$

where

$$\hat{\zeta}^T(t)=[x^T(t)\ x^T(t-\tau_i(t))\ g_{1i}^T(t)].$$

From Assumption 2, for any matrices $K_m=\mathrm{diag}(k_{m1},k_{m2},\dots,k_{mn})\ge0$ $(m=1,2)$, we can obtain

$$\begin{aligned}0&\le2\sum_{j=1}^nk_{1j}f_j(x_j(t))[l_jx_j(t)-f_j(x_j(t))]\\&\quad+2\sum_{j=1}^nk_{2j}f_j(x_j(t-\tau_i(t)))[l_jx_j(t-\tau_i(t))\\&\quad-f_j(x_j(t-\tau_i(t)))]\\&=2x^T(t)K_1Lf(x(t))-2f^T(x(t))K_1f(x(t))\\&\quad+2x^T(t-\tau_i(t))K_2Lf(x(t-\tau_i(t)))\\&\quad-2f^T(x(t-\tau_i(t)))K_2f(x(t-\tau_i(t))).\end{aligned}\tag{26}$$

By Lemma 2, we can easily get

$$\begin{aligned}&\mathbb{E}\left\{\left[\int_{t-\tau_i(t)}^tg_{2i}(s)\mathrm{d}w(s)\right]^TS_1\int_{t-\tau_i(t)}^tg_{2i}(s)\mathrm{d}w(s)\right\}\\&=\mathbb{E}\left\{\int_{t-\tau_i(t)}^t\mathrm{tr}[\,g_{2i}^T(s)S_1g_{2i}(s)\bar{P}]\mathrm{d}s\right\},\end{aligned}\tag{27}$$

$$\begin{aligned}&\mathbb{E}\left\{\left[\int_{t-\tau_i(t)}^{t-\tau_m}g_{2i}(s)\mathrm{d}w(s)\right]^TS_2\int_{t-\tau_i(t)}^{t-\tau_m}g_{2i}(s)\mathrm{d}w(s)\right\}\\&=\mathbb{E}\left\{\int_{t-\tau_i(t)}^{t-\tau_m}\mathrm{tr}[\,g_{2i}^T(s)S_2g_{2i}(s)\bar{P}]\mathrm{d}s\right\},\end{aligned}\tag{28}$$

$$\begin{aligned}&\mathbb{E}\left\{\left[\int_{t-\tau_M}^{t-\tau_i(t)}g_{2i}(s)\mathrm{d}w(s)\right]^T\tilde{S}\int_{t-\tau_M}^{t-\tau_i(t)}g_{2i}(s)\mathrm{d}w(s)\right\}\\&=\mathbb{E}\left\{\int_{t-\tau_M}^{t-\tau_i(t)}\mathrm{tr}[\,g_{2i}^T(s)\tilde{S}g_{2i}(s)\bar{P}]\mathrm{d}s\right\}.\end{aligned}\tag{29}$$

Adding Eqs.(15)–(18) and (24)–(26) to expression (14), and taking expectation on both sides of expression (14), then using Eqs.(19)–(23) and (27)–(29), we can get

$$\begin{aligned}&\mathbb{E}\mathcal{L}V_i(t)\\&\le\mathbb{E}\zeta^{\mathrm{T}}(t)\big[\Phi_1^i+M_iS_1^{-1}M_i^T+N_iS_2^{-1}N_i^T\\&\quad+T_i\tilde{S}^{-1}T_i^T+\varepsilon_{1i}^{-1}V_iH_iH_i^TV_i^T+\Phi_2^i\big]\zeta(t)\\&\quad-\mathbb{E}\int_{t-\tau_i(t)}^t\bar{\zeta}^T(t,s)\Omega_{i1}\bar{\zeta}(t,s)\mathrm{d}s\\&\quad-\mathbb{E}\int_{t-\tau_i(t)}^{t-\tau_m}\bar{\zeta}^T(t,s)\Omega_{i2}\bar{\zeta}(t,s)\mathrm{d}s\\&\quad-\mathbb{E}\int_{t-\tau_M}^{t-\tau_i(t)}\bar{\zeta}^T(t,s)\Omega_{i3}\bar{\zeta}(t,s)\mathrm{d}s,\end{aligned}\tag{30}$$

where $\bar{\zeta}^T(t,s) = [\hat{\zeta}^T(t)\ g_{1i}^T(s)]$, Φ_2^i is defined in expression (23).

Let $\tilde{\Phi}_i = \Phi_1^i + M_i S_1^{-1} M_i^T + N_i S_2^{-1} N_i^T + T_i \tilde{S}^{-1} T_i^T + \varepsilon_{1i}^{-1} V_i H_i H_i^T V_i^T + \Phi_2^i$, applying the Schur complement techniques[30] to the inequality $\tilde{\Phi}_i < 0$, then pre-multiplying and post-multiplying the resulting matrix by $\mathrm{diag}(I, I, I, \ldots, I, \tilde{Z}_i)$, we can easily obtain the LMI (10). Therefore, if inequalities (10)–(13) are satisfied, it implies $\mathbb{E}\mathcal{L}V_i(t) \leq 0$ for $i \in S$ and $\zeta(t) \neq 0$.

It indicates that system (1) is robustly, globally and asymptotically stable in the mean-square sense. This completes the proof.

Remark 1 By setting $Q_j = 0$ ($j = 1, 2, 3, 4$) in Lyapunov–Krasovskii functional of Theorem 1, the delay/interval-dependent and derivative-independent criteria can be obtained easily for unknown μ_i and the corresponding proof is similar to Theorem 1, which is omitted.

Remark 2 The main results given in Theorem 1 are general enough to cover many existing results[15−18] with Markovian jumping mode "1" and $\mathrm{d}w_1(t) = \mathrm{d}w_2(t) = \mathrm{d}w_3(t) = \mathrm{d}w_4(t)$.

As a special case of system (1), we discuss the stability of an uncertain stochastic neural network (SNN) with time-varying interval delay, which is described as follows:

$$\begin{aligned}\mathrm{d}x(t) = {} & [-A(t)x(t) + W_0(t)f(x(t)) \\ & + W_1(t)f(x(t-\tau(t)))]\mathrm{d}t + [C(t)x(t) \\ & + D(t)x(t-\tau(t)) + W_2(t)f(x(t)) \\ & + W_3(t)f(x(t-\tau(t)))]\mathrm{d}w_0(t), \\ x(t) = {} & \phi(t),\ t \in [-\tau_M, 0], \end{aligned} \tag{31}$$

where $0 \leq \tau_m \leq \tau(t) \leq \tau_M$, $\dot{\tau}(t) \leq \mu$, $w_0(t)$ is a one-dimensional Brownian motion.

Corollary 1 For given scalars $\tau_m \geq 0$, $\tau_M > 0$, μ and $l = \max_{1\leq j\leq n}\{l_j\}$, the SNNs described by Eq.(31) are robustly, globally and asymptotically stable in the mean-square sense, if there exist real matrices $P > 0$, $Q_j > 0$ ($j = 1, 2, 3, 4$), $R_1 > 0$, $R_2 > 0$, $S_1 > 0$, $S_2 > 0$, M_j, N_j, T_j, V_j, X_{jk}, Y_{jk} ($j, k = 1, 2, 3$), $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n) \geq 0$, diagonal matrices $K_1 \geq 0$, $K_2 \geq 0$ of appropriate dimensions and positive scalars ε_1, ε_2 such that the following LMIs hold:

$$\Xi = \begin{bmatrix} \Xi_1 & M & N & T & VH & \hat{Z} & 0 \\ * & -S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -S_2 & 0 & 0 & 0 & 0 \\ * & * & * & -\tilde{S} & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & -\tilde{Z} & \tilde{Z}\tilde{H} \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0, \tag{32}$$

$$\Omega_1 = \begin{bmatrix} X & \bar{M} \\ * & R_1 \end{bmatrix} \geq 0, \tag{33}$$

$$\Omega_2 = \begin{bmatrix} Y & \bar{N} \\ * & R_2 \end{bmatrix} \geq 0, \tag{34}$$

$$\Omega_3 = \begin{bmatrix} X+Y & \bar{T} \\ * & R_1+R_2 \end{bmatrix} \geq 0, \tag{35}$$

where

$$\Xi_1 = \begin{bmatrix} \Xi_{1,1} & \Xi_{1,2} & \Xi_{1,3} & \Xi_{1,4} & \Xi_{1,5} & N_1 & -T_1 \\ * & \Xi_{2,2} & \Xi_{2,3} & \Xi_{2,4} & \Xi_{2,5} & N_2 & -T_2 \\ * & * & \Xi_{3,3} & \Xi_{3,4} & \Xi_{3,5} & 0 & 0 \\ * & * & * & \Xi_{4,4} & \Xi_{4,5} & 0 & 0 \\ * & * & * & * & \Xi_{5,5} & N_3 & -T_3 \\ * & * & * & * & * & -Q_3 & 0 \\ * & * & * & * & * & * & -Q_4 \end{bmatrix},$$

$$\begin{aligned}\Xi_{1,1} &= Q_1 + Q_3 + Q_4 + M_1 + M_1^T - V_1 A - A^T V_1^T \\ &\quad + \tau_M X_{1,1} + \tau_o Y_{1,1} + \varepsilon_1 E_1^T E_1 + \varepsilon_2 E_4^T E_4, \\ \Xi_{1,2} &= M_2^T - M_1 - N_1 + T_1 - A^T V_2^T \\ &\quad + \tau_M X_{1,2} + \tau_o Y_{1,2}, \\ \Xi_{1,3} &= LK_1 + V_1 W_0 - \varepsilon_1 E_1^T E_2, \\ \Xi_{1,4} &= V_1 W_1 - \varepsilon_1 E_1^T E_3, \\ \Xi_{1,5} &= P - V_1 - A^T V_3^T + M_3^T + \tau_M X_{1,3} + \tau_o Y_{1,3}, \\ \Xi_{2,2} &= -(1-\mu)Q_1 - M_2 - M_2^T - N_2 - N_2^T + T_2 \\ &\quad + T_2^T + \tau_M X_{2,2} + \tau_o Y_{2,2} + \varepsilon_2 E_5^T E_5, \\ \Xi_{2,3} &= V_2 W_0, \quad \Xi_{2,4} = V_2 W_1 + LK_2, \\ \Xi_{2,5} &= -V_2 + T_3^T - M_3^T - N_3^T + \tau_M X_{2,3} + \tau_o Y_{2,3}, \\ \Xi_{3,3} &= -K_1^T - K_1 + Q_2 + \varepsilon_1 E_2^T E_2 + \varepsilon_2 E_6^T E_6, \\ \Xi_{3,4} &= \varepsilon_1 E_2^T E_3, \Xi_{3,5} = \Lambda + W_0^T V_3^T, \\ \Xi_{4,4} &= -(1-\mu)Q_2 - K_2 - K_2^T + \varepsilon_1 E_3^T E_3 + \varepsilon_2 E_7^T E_7, \\ \Xi_{4,5} &= W_1^T V_3^T, \\ \Xi_{5,5} &= -V_3 - V_3^T + \tau_M R_1 + \tau_o R_2 + \tau_M X_{3,3} + \tau_o Y_{3,3}, \end{aligned}$$

$Z = \tau_M S_1 + \tau_o S_2 + P + l\Lambda, \tilde{Z} = \mathrm{diag}\,(Z, Z, Z, Z)\,,$
$\tilde{H} = \mathrm{diag}\,(H, H, H, H)\,,$

$$\hat{Z}^T = \begin{bmatrix} ZC & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & ZD & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & ZW_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & ZW_3 & 0 & 0 & 0 \end{bmatrix},$$

$$X = \begin{bmatrix} X_{1,1} & X_{1,2} & X_{1,3} \\ * & X_{2,2} & X_{2,3} \\ * & * & X_{3,3} \end{bmatrix},$$

$$Y = \begin{bmatrix} Y_{1,1} & Y_{1,2} & Y_{1,3} \\ * & Y_{2,2} & Y_{2,3} \\ * & * & Y_{3,3} \end{bmatrix},$$

$$M^T = [M_1^T\ M_2^T\ 0\ 0\ M_3^T\ 0\ 0],$$
$$\bar{M}^T = [M_1^T\ M_2^T\ M_3^T],$$
$$N^T = [N_1^T\ N_2^T\ 0\ 0\ N_3^T\ 0\ 0],$$
$$\bar{N}^T = [N_1^T\ N_2^T\ N_3^T],$$
$$T^T = [T_1^T\ T_2^T\ 0\ 0\ T_3^T\ 0\ 0],$$
$$\bar{T}^T = [T_1^T\ T_2^T\ T_3^T],$$
$$V^T = [V_1^T\ V_2^T\ 0\ 0\ V_3^T\ 0\ 0].$$

Proof By setting jumping mode $S = 1$ and $\bar{P} = I$ in Lyapunov–Krasovskii functional of Theorem 1, robust stability criteria can be obtained and the proof is similar to Theorem 1, which is omitted.

Remark 3 In Refs.[15], [16] and [18],

$$-\int_{t-\tau}^{t} g_1^T(s) R g_1(s) \mathrm{d}s$$

is estimated as

$$-\int_{t-\tau(t)}^{t} g_1^T(s) R g_1(s) \mathrm{d}s,$$

and

$$-\int_{t-\tau}^{t-\tau(t)} g_1^T(s) R g_1(s) \mathrm{d}s$$

is ignored. In Ref.[37], $-2\xi^T(t)\tilde{M}\int_{t-\tau(t)}^{t} y(s)\mathrm{d}s$ are enlarged by inequality transformation. In addition, references [15]–[18] do not consider information of the lower bound of time delay. In this paper, the problems mentioned above are all considered and overcome.

Remark 4 Let $\Xi_1 < 0$ in formula (32) and $\varepsilon_1 = \varepsilon_2 = 0$ in Ξ_1, moreover, equations (33)–(35) hold, delay/interval-dependent and derivative-dependent stability criteria for determinate NNs can be obtained easily. Furthermore, let $Q_1 = Q_2 = 0$ in Ξ_1, delay/interval-dependent and derivative-independent stability criteria for determinate NNs can be also obtained; the proof is similar to Corollary 1, which is omitted.

4. Illustrated examples

Example 1 Consider a two-mode delayed MJSNN system (1) governed by the generators: $\Pi = \begin{bmatrix} -7 & 7 \\ 6 & -6 \end{bmatrix}$, and the system parameters are given as follows: $\bar{\alpha}_{12} = \bar{\alpha}_{13} = \bar{\alpha}_{14} = \bar{\alpha}_{23} = \bar{\alpha}_{24} = \bar{\alpha}_{34} = 0.4$, $L = 0.5I$.

For mode 1, the parameters are described as $A_1 = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$, $W_{01} = W_{11} = \begin{bmatrix} -1 & 0 \\ 0 & -1.2 \end{bmatrix}$, $W_{21} = W_{31} = \begin{bmatrix} -0.2 & 0 \\ 0 & 0.3 \end{bmatrix}$, $C_1 = D_1 = 0.4I$, $H_1 = 0.1I$, $E_{11} = E_{21} = E_{31} = E_{41} = E_{51} = E_{61} = E_{71} = 0.1I$, the time-varying delay $\tau_1(t)$ satisfies Eq.(3) with $\tau_{1m} = 0.7$, $\tau_{1M} = 1.0$, $\mu_1 = 0.9$.

For mode 2, the parameters are described as $A_2 = \begin{bmatrix} 2 & 0 \\ 0 & 3.5 \end{bmatrix}$, $W_{02} = \begin{bmatrix} 1.5 & 0 \\ 0 & -2 \end{bmatrix}$, $W_{12} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{bmatrix}$, $W_{22} = W_{32} = \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.2 \end{bmatrix}$, $C_2 = D_2 = 0.6I$, $H_2 = 0.2I$, $E_{12} = E_{22} = E_{32} = E_{42} = E_{52} = E_{62} = E_{72} = 0.2I$, the time-varying delay $\tau_2(t)$ satisfies Eq.(3) with $\tau_{2m} = 0.8$, $\tau_{2M} = 1.046$, $\mu_2 = 0.8$.

By solving the LMIs in Theorem 1 via Matlab Toolbox, the feasible solutions are obtained as follows:

$$P_1 = \begin{bmatrix} 14.2707 & 3.3448 \\ 3.3448 & 15.7492 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 17.2395 & 3.6504 \\ 3.6504 & 13.1707 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.0033 & -0.008 \\ -0.008 & 0.0287 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.0906 & -0.5681 \\ -0.5681 & 89.2671 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 14.8103 & 3.5037 \\ 3.5037 & 15.8153 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 14.8188 & 3.4894 \\ 3.4894 & 15.8672 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 0.0051 & -0.0040 \\ -0.0040 & 0.0147 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 8.2663 & 3.0897 \\ 3.0897 & 4.9500 \end{bmatrix},$$

$$S_1 = \begin{bmatrix} 0.0117 & -0.0205 \\ -0.0205 & 0.0547 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 20.3286 & 4.0185 \\ 4.0185 & 18.7710 \end{bmatrix},$$

$$K_1 = \mathrm{diag}\,(45.8496,\ 53.7978)\,,$$

$$K_2 = \mathrm{diag}\,(9.0358,\ 16.4149)\,,$$

$$\varepsilon_{11} = 7.2596, \quad \varepsilon_{12} = 11.8550,$$

$$\varepsilon_{21} = 20.3706, \quad \varepsilon_{22} = 27.5860.$$

The switching signal and the state curves of the example are depicted in Figs.1 and 2, respectively. It can be seen from Fig.2 that system (1) with the given switching signal is robustly, globally and asymptotically stable in the mean-square sense.

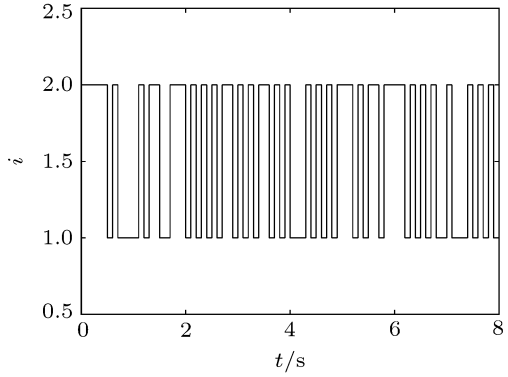


Fig.1. Switching signal of the system (1).

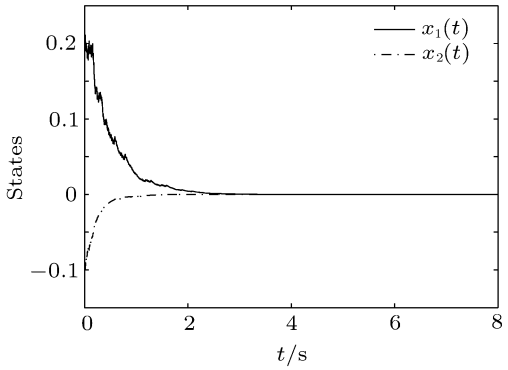


Fig.2. State curves of system (1) with two modes.

Example 2 Consider the delayed SNNs (31) with the parameters as follows (Example 2 in Ref.[18]):

$$A = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.4 & -0.7 \\ 0.1 & 0 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} -0.2 & 0.6 \\ 0.5 & -0.1 \end{bmatrix},$$

$$C = 0.5I, \quad D = \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix},$$

$$H = [0.1 - 0.1]^T, \quad E_1 = [0.2\ 0.3],$$
$$E_2 = [0.2\ -0.3], \quad E_3 = [-0.2\ -0.3],$$
$$\Delta C(t) = \Delta D(t) = 0, \quad W_2 = W_3 = 0.1I, \quad L = 0.5I.$$

When μ is unknown, by setting $Q_1 = Q_2 = 0$ in Corollary 1, we can obtain that system (31) is robustly, globally and asymptotically stable in the mean-square sense. For $\tau_m = 0$, unknown μ, the upper bound τ_M is listed in Table 1. From Table 1, it can be concluded that the obtained results in this paper are less conservative than those in Refs.[15], [17] and [18]. For $\tau_m = 1$, the upper bound $\tau_M = 2.028$.

Table 1. Allowable upper bound of τ_M for unknown μ.

methods	[15]	[17]	[18]	our result (Corollary 1)
$W_2 = W_3 = 0$	0.419	0.746	0.7795	1.049
$W_2 = W_3 = 0.1I$	–	0.649	0.7056	1.037

For the determinate NNs that are described in Remark 4, we choose the same parameters as the above SNNs. For $\tau_m = 1$ and unknown μ, the upper bound τ_M, which guarantees the stability of determinate NNs, is 6.6805. Under the same condition, the upper bound τ_M in Ref.[19] is 3.902. Thus the result in this paper is less conservative than that in Ref.[19].

5. Conclusions

The problem of robust stability for MJSNNs with mode-dependent time-varying interval delay and state-multiplicative noise is studied in this paper. A new stability criterion has been proposed to guarantee the robust, global and asymptotic stability of the MJSNNs. Moreover, free weighting matrices have been introduced into the LMI conditions to reduce the conservatism of the existing results. Numerical examples show the effectiveness of the method.

Appendix A

The proof of Lemma 2

From the left of formula (9), we have

$$\begin{aligned}&\left(\int_{t-\tau}^{t} g(s)\mathrm{d}w(s)\right)^T G\int_{t-\tau}^{t} g(s)\mathrm{d}w(s)\\ =&\sum_{i=1}^{4}\left(\int_{t-\tau}^{t} g_i(s)\mathrm{d}w_i(s)\right)^T G\int_{t-\tau}^{t} g_i(s)\mathrm{d}w_i(s)\\ &+2\sum_{i=1}^{4}\sum_{j>i}^{4}\left(\int_{t-\tau}^{t} g_i(s)\mathrm{d}w_i(s)\right)^T G\\ &\times\int_{t-\tau}^{t} g_j(s)\mathrm{d}w_j(s).\end{aligned}\tag{A1}$$

For the right of formula (9), we can get

$$\begin{aligned}\int_{t-\tau}^{t}\mathrm{tr}[g^T(s)Gg(s)\bar{P}]\mathrm{d}s=&\int_{t-\tau}^{t}\Bigg(\sum_{i=1}^{4} g_i^T(s)Gg_i(s)\\ &+2\sum_{i=1}^{4}\sum_{j>i}^{4}\bar{\alpha}_{ij}g_i^T(s)Gg_j(s)\Bigg)\mathrm{d}s.\end{aligned}\tag{A2}$$

From Ref.[32], it is easy to know that

$$\begin{aligned}&\mathbb{E}\left(\int_{t-\tau}^{t} g_i(s)\mathrm{d}w_i(s)\right)^T G\int_{t-\tau}^{t} g_i(s)\mathrm{d}w_i(s)\\ =&\,\mathbb{E}\int_{t-\tau}^{t} g_i^T(s)Gg_i(s)\mathrm{d}s,\end{aligned}\tag{A3}$$

when $i\neq j$, if the following formula is satisfied

$$\begin{aligned}&\mathbb{E}\left(\int_{t-\tau}^{t} g_i(s)\mathrm{d}w_i(s)\right)^T G\int_{t-\tau}^{t} g_j(s)\mathrm{d}w_j(s)\\ =&\,\mathbb{E}\int_{t-\tau}^{t}\bar{\alpha}_{ij}g_i^T(s)Gg_j(s)\mathrm{d}s,\end{aligned}\tag{A4}$$

then Lemma 2 will hold. Now, we will give a proof of formula (A4).

Take one-dimensional condition for example, for φ_1,φ_2 are bounded functions, $\varphi_1\in\mathbb{R}$, $\varphi_2\in\mathbb{R}$, $\varsigma(t)$ and $\eta(t)$ are one-dimensional Brownian motion and satisfy $\mathbb{E}\{\mathrm{d}\varsigma(t)\mathrm{d}\eta(t)\}=\bar{\alpha}\mathrm{d}t$, $|\bar{\alpha}|<1$. Define

$$\begin{aligned}\int_{t-\tau}^{t}\varphi_1(s)\mathrm{d}\varsigma(s)&=\lim_{\lambda\to0}\sum_{i=1}^{n} a_i(\varsigma_{t_{i+1}}-\varsigma_{t_i})\\ &=\lim_{\lambda\to0}\sum_{i=1}^{n} a_i\Delta\varsigma_i,\\ \int_{t-\tau}^{t}\varphi_2(s)\mathrm{d}\eta(s)&=\lim_{\rho\to0}\sum_{j=1}^{n} b_j(\eta_{t_{j+1}}-\eta_{t_j})\\ &=\lim_{\rho\to0}\sum_{j=1}^{n} b_j\Delta\eta_j.\end{aligned}$$

where

$$\begin{aligned}\lambda&=\max\{\Delta\varsigma_1,\Delta\varsigma_2,\dots,\Delta\varsigma_n\},\\ \rho&=\max\{\Delta\eta_1,\Delta\eta_2,\dots,\Delta\eta_n\}.\end{aligned}$$

For $a_ib_j\Delta\varsigma_i$ and $\Delta\eta_j$ are independent if $i\neq j$, then

$$\begin{aligned}&E\left(\int_{t-\tau}^{t}\varphi_1(s)\mathrm{d}\varsigma(s)S\int_{t-\tau}^{t}\varphi_2(s)\mathrm{d}\eta(s)\right)\\ =&E\Big(\lim_{\lambda\to0}\sum_{i=1}^{n} a_i\Delta\varsigma_i\lim_{\rho\to0}\sum_{j=1}^{n} b_j\Delta\eta_j\Big)\\ =&\lim_{\lambda\to0,\rho\to0}\sum_{i=1}^{n}\sum_{j=1}^{n}E(a_ib_j\Delta\varsigma_i\Delta\eta_j)\\ =&\lim_{\lambda\to0,\rho\to0}\sum_{i=1}^{n}E(a_ib_i\Delta\varsigma_i\Delta\eta_i)\\ &+\lim_{\lambda\to0,\rho\to0}\sum_{i=1}^{n}\sum_{j\neq i}^{n}E(a_ib_j\Delta\varsigma_i\Delta\eta_j)\\ =&\lim_{\Delta t\to0}E\sum_{i=1}^{n}\bar{\alpha}a_ib_i(t_{i+1}-t_i)\\ =&E\int_{t-\tau}^{t}\bar{\alpha}\varphi_1(s)\varphi_2(s)\mathrm{d}s,\end{aligned}\tag{A5}$$

where $\Delta t=\max\{\Delta t_1,\Delta t_2,\dots,\Delta t_n\}$. So

$$\begin{aligned}&\mathbb{E}\left(\int_{t-\tau}^{t}\varphi_1(s)\mathrm{d}\varsigma(s)\int_{t-\tau}^{t}\varphi_2(s)\mathrm{d}\eta(s)\right)\\ =&\,\mathbb{E}\int_{t-\tau}^{t}\bar{\alpha}\varphi_1(s)\varphi_2(s)\mathrm{d}s.\end{aligned}\tag{A6}$$

It is easy to extend the one-dimensional condition to the multi-dimensional condition, that is to say, the formula (A4) is satisfied. This completes the proof of Lemma 2.

Appendix B

The proof of formula (14)

By formula (5), it is easy to get

$$\begin{aligned}&\mathrm{tr}[\,g_{2i}(t)\bar{P}g_{2i}^T(t)\Lambda(\mathrm{diag}(\dot{f}_1(x_1(t)),\dots,\\ &\dot{f}_n(x_n(t)))-lI)]\leq0,\end{aligned}$$

that is to say,

$$\begin{aligned}&\mathrm{tr}\big[\,g_{2i}(t)\bar{P}g_{2i}^T(t)\Lambda\,\mathrm{diag}(\dot{f}_1(x_1(t)),\dots,\dot{f}_n(x_n(t)))\big]\\ \leq&\,l\,\mathrm{tr}[\,g_{2i}^T(t)\Lambda g_{2i}(t)\bar{P}].\end{aligned}\tag{B1}$$

So

$$\begin{aligned}&\mathcal{L}V_{1i}(t)\\&\le 2x^T(t)P_ig_{1i}(t)+\mathrm{tr}[\,g_{2i}^T(t)(P_i+l\Lambda)g_{2i}(t)\bar{P}]\\&\quad+\sum_{j=1}^{N}\pi_{ij}x^T(t)P_jx(t)+2f^T(x(t))\Lambda g_{1i}(t).\end{aligned}\tag{B2}$$

It is obvious that

$$\begin{aligned}&\mathcal{L}V_{2i}(t)\\&\le x^T(t)(Q_1+Q_3+Q_4)x(t)+f^T(x(t))Q_2f(x(t))\\&\quad-(1-\mu_i)[x^T(t-\tau_i(t))Q_1x(t-\tau_i(t))\\&\quad+f^T(x(t-\tau_i(t)))Q_2f(t-\tau_i(t))]\\&\quad-x^T(t-\tau_m)Q_3x(t-\tau_m)\\&\quad-x^T(t-\tau_M)Q_4x(t-\tau_M)\\&\quad+\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}[x^T(s)Q_1x(s)\\&\quad+f^T(x(s))Q_2f(x(s))]\mathrm{d}s.\end{aligned}\tag{B3}$$

Note that $\pi_{ij}\ge 0$ for $j\ne i$ and $\pi_{ii}\le 0$, then we have

$$\begin{aligned}&\sum_{j=1}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}[x^T(s)Q_1x(s)\\&\quad+f^T(x(s))Q_2f(x(s))]\mathrm{d}s\\&=\sum_{j\ne i}^{N}\pi_{ij}\int_{t-\tau_j(t)}^{t}[x^T(s)Q_1x(s)\\&\quad+f^T(x(s))Q_2f(x(s))]\mathrm{d}s\\&\quad+\pi_{ii}\int_{t-\tau_i(t)}^{t}[x^T(s)Q_1x(s)+f^T(x(s))Q_2f(x(s))]\mathrm{d}s\\&\le-\pi_{ii}\int_{t-\tau_M}^{t}[x^T(s)Q_1x(s)+f^T(x(s))Q_2f(x(s))]\mathrm{d}s\\&\quad+\pi_{ii}\int_{t-\tau_m}^{t}[x^T(s)Q_1x(s)+f^T(x(s))Q_2f(x(s))]\mathrm{d}s\\&\le\delta\int_{t-\tau_M}^{t-\tau_m}[x^T(s)Q_1x(s)+f^T(x(s))Q_2f(x(s))]\mathrm{d}s.\end{aligned}$$

Substituting the above formula into expression (B3), we can get

$$\begin{aligned}&\mathcal{L}V_{2i}(t)\\&\le x^T(t)(Q_1+Q_3+Q_4)x(t)+f^T(x(t))Q_2f(x(t))\\&\quad-(1-\mu_i)[x^T(t-\tau_i(t))Q_1x(t-\tau_i(t))\\&\quad+f^T(x(t-\tau_i(t)))Q_2f(x(t-\tau_i(t)))]\\&\quad-x^T(t-\tau_m)Q_3x(t-\tau_m)\\&\quad-x^T(t-\tau_M)Q_4x(t-\tau_M)\\&\quad+\delta\int_{t-\tau_M}^{t-\tau_m}[x^T(s)Q_1x(s)\\&\quad+f^T(x(s))Q_2f(x(s))]\mathrm{d}s.\end{aligned}\tag{B4}$$

It is also easy to know

$$\begin{aligned}&\mathcal{L}V_{3i}(t)\\&=\delta\tau_o[x^T(t)Q_1x(t)+f^T(x(t))Q_2f(x(t))]\\&\quad-\delta\int_{t-\tau_M}^{t-\tau_m}[x^T(s)Q_1x(s)\\&\quad+f^T(x(s))Q_2f(x(s))]\mathrm{d}s.\end{aligned}\tag{B5}$$

$$\begin{aligned}&\mathcal{L}V_{4i}(t)\\&=g_{1i}^T(t)(\tau_MR_1+\tau_oR_2)g_{1i}(t)\\&\quad-\int_{t-\tau_i(t)}^{t}g_{1i}^T(s)R_1g_{1i}(s)\mathrm{d}s\\&\quad-\int_{t-\tau_M}^{t-\tau_i(t)}g_{1i}^T(s)(R_1+R_2)g_{1i}(s)\mathrm{d}s\\&\quad-\int_{t-\tau_i(t)}^{t-\tau_m}g_{1i}^T(s)R_2g_{1i}(s)\mathrm{d}s\\&\quad+\mathrm{tr}[g_{2i}^T(t)(\tau_MS_1+\tau_oS_2)g_{2i}(t)\bar{P}]\\&\quad-\int_{t-\tau_i(t)}^{t}\mathrm{tr}[g_{2i}^T(s)S_1g_{2i}(s)\bar{P}]\mathrm{d}s\\&\quad-\int_{t-\tau_M}^{t-\tau_i(t)}\mathrm{tr}[g_{2i}^T(s)\tilde{S}g_{2i}(s)\bar{P}]\mathrm{d}s\\&\quad-\int_{t-\tau_i(t)}^{t-\tau_m}\mathrm{tr}[g_{2i}^T(s)S_2g_{2i}(s)\bar{P}]\mathrm{d}s.\end{aligned}\tag{B6}$$

Combining expressions (B2) and (B4)–(B6), it is easy to get formula (14). This completes the proof.

Appendix C

The proof of formula (23)

$$\begin{aligned}&\mathrm{tr}[\,g_{2i}^T(t)Z_ig_{2i}(t)\bar{P}]\\&=\mathrm{tr}\left\{\begin{bmatrix}\Upsilon_{1,1}&\Upsilon_{1,2}&\Upsilon_{1,3}&\Upsilon_{1,4}*&\Upsilon_{2,2}&\Upsilon_{2,3}&\Upsilon_{2,4}*&*&\Upsilon_{3,3}&\Upsilon_{3,4}*&*&*&\Upsilon_{4,4}\end{bmatrix}\begin{bmatrix}1&\bar{\alpha}_{12}&\bar{\alpha}_{13}&\bar{\alpha}_{14}*&1&\bar{\alpha}_{23}&\bar{\alpha}_{24}*&*&1&\bar{\alpha}_{34}*&*&*&1\end{bmatrix}\right\}\\&=\Upsilon_{1,1}+2\bar{\alpha}_{12}\Upsilon_{1,2}+2\bar{\alpha}_{13}\Upsilon_{1,3}+2\bar{\alpha}_{14}\Upsilon_{1,4}+\Upsilon_{2,2}\\&\quad+2\bar{\alpha}_{23}\Upsilon_{2,3}+2\bar{\alpha}_{24}\Upsilon_{2,4}+\Upsilon_{3,3}+2\bar{\alpha}_{34}\Upsilon_{3,4}+\Upsilon_{4,4}\\&=\tilde{\zeta}^T(t)\big[\mathrm{diag}\,(C_i,D_i,W_{2i},W_{3i})\\&\quad+\mathrm{diag}\,(\Delta C_i(t),\Delta D_i(t),\Delta W_{2i}(t),\Delta W_{3i}(t))\big]^T\tilde{Z}_i\\&\quad\times\big[\mathrm{diag}\,(C_i,D_i,W_{2i},W_{3i})\\&\quad+\mathrm{diag}\,(\Delta C_i(t),\Delta D_i(t),\Delta W_{2i}(t),\Delta W_{3i}(t))\big]\tilde{\zeta}^T(t)\\&=\tilde{\zeta}^T(t)\big[\mathrm{diag}\,(C_i,D_i,W_{2i},W_{3i})\\&\quad+H_iF_i(t)\mathrm{diag}\,(E_{4i},E_{5i},E_{6i},E_{7i})\big]^T\\&\quad\times\tilde{Z}_i\big[\mathrm{diag}\,(C_i,D_i,W_{2i},W_{3i})\\&\quad+H_iF_i(t)\mathrm{diag}\,(E_{4i},E_{5i},E_{6i},E_{7i})\big]\tilde{\zeta}(t),\end{aligned}$$

where

$$\begin{aligned}
\Upsilon_{1,1} &= x^T(t)C_i^T(t)Z_iC_i(t)x(t),\\
\Upsilon_{1,2} &= x^T(t)C_i^T(t)Z_iD_i(t)x(t-\tau_i(t)),\\
\Upsilon_{1,3} &= x^T(t)C_i^T(t)Z_iW_{2i}(t)f(x(t)),\\
\Upsilon_{1,4} &= x^T(t)C_i^T(t)Z_iW_{3i}(t)f(x(t-\tau_i(t))),\\
\Upsilon_{2,2} &= x^T(t-\tau_i(t))D_i^T(t)Z_iD_i(t)x(t-\tau_i(t)),\\
\Upsilon_{2,3} &= x^T(t-\tau_i(t))D_i^T(t)Z_iW_{2i}(t)f(x(t)),\\
\Upsilon_{2,4} &= x^T(t-\tau_i(t))D_i^T(t)Z_iW_{3i}(t)f(x(t-\tau_i(t))),\\
\Upsilon_{3,3} &= f^T(x(t))W_{2i}^T(t)Z_iW_{2i}(t)f(x(t)),\\
\Upsilon_{3,4} &= f^T(x(t))W_{2i}^T(t)Z_iW_{3i}(t)f(x(t-\tau_i(t))),\\
\Upsilon_{4,4} &= f^T(x(t-\tau_i(t)))W_{3i}^T(t)Z_iW_{3i}(t)f(x(t-\tau_i(t))).
\end{aligned}$$

By (iii) of Lemma 1, we can further have

$$\begin{aligned}
&\mathrm{tr}[\,g_{2i}^T(t)Z_ig_{2i}\bar{P}]\\
&\le \tilde{\zeta}^T(t)\Big\{\big(\mathrm{diag}\,(C_i,D_i,W_{2i},W_{3i})\,\big)^T\\
&\quad\times\big[\tilde{Z}_i^{-1}-\varepsilon_{2i}\mathrm{diag}\,\big(H_iH_i^T,H_iH_i^T,H_iH_i^T,H_iH_i^T\big)\,\big]^{-1}\\
&\quad\times\mathrm{diag}\,(C_i,D_i,W_{2i},W_{3i})\\
&\quad+\varepsilon_{2i}\mathrm{diag}\,\big(E_{4i}^TE_{4i},E_{5i}^TE_{5i},E_{6i}^TE_{6i},E_{7i}^TE_{7i}\big)\Big\}\tilde{\zeta}(t)\\
&=\zeta^T(t)\tilde{C}_i^T\big[\tilde{Z}_i^{-1}\\
&\quad-\varepsilon_{2i}\mathrm{diag}\,\big(H_iH_i^T,H_iH_i^T,H_iH_i^T,H_iH_i^T\big)\,\big]^{-1}\tilde{C}_i\zeta(t).\\
&\quad+\varepsilon_{2i}\tilde{\zeta}^T(t)\mathrm{diag}\,\big(E_{4i}^TE_{4i},E_{5i}^TE_{5i},E_{6i}^TE_{6i},E_{7i}^TE_{7i}\big)\,\tilde{\zeta}(t)\\
&=\zeta^T(t)\,\Phi_2^i\zeta(t)+\varepsilon_{2i}\tilde{\zeta}^T(t)\\
&\quad\times\mathrm{diag}\,\big(E_{4i}^TE_{4i},E_{5i}^TE_{5i},E_{6i}^TE_{6i},E_{7i}^TE_{7i}\big)\,\tilde{\zeta}(t).
\end{aligned}$$

This completes the proof.

References

[1] Young S and Scott P N 1997 *IEEE Trans. Image Process* **6** 357
[2] Arik S 2003 *Phys. Lett. A* **311** 504
[3] Zhang H G and Wang Z S 2007 *IEEE Trans. Neural Netw.* **18** 947
[4] Zhang Y J, Yue D and Tian E G 2009 *Neurocomputing* **72** 1265
[5] Zhang H G, Wang Z S and Liu D R 2008 *IEEE Trans. Neural Netw.* **19** 855
[6] He Y, Liu G P, Rees D and Wu M 2007 *IEEE Trans. Neural Netw.* **18** 1850
[7] Cui B T, Chen J and Lou X Y 2008 *Chin. Phys.* B **17** 1670
[8] Li D, Wang H, Yang D, Zhang X H and Wang S L 2008 *Chin. Phys.* B **17** 4091
[9] Li C D and Feng G 2009 *Neurocomputing* **72** 1179
[10] Zhang Q 2008 *Chin. Phys.* B **17** 125
[11] Tang Y, Zhong H H and Fang J A 2008 *Chin. Phys.* B **17** 4080
[12] Chen D L and Zhang W D 2008 *Chin. Phys.* B **17** 1506
[13] Haykin S 1994 *Neural Networks* (New York: Prentice Hall)
[14] Feng W, Yang S Y, Fu W and Wu H X 2009 *Chaos, Solitons and Fractals* **41** 414
[15] Huang H and Feng G 2007 *Physica* A **381** 93
[16] Zhang J H, Shi P and Qiu J Q 2007 *Nonlinear Anal.* **8** 1349
[17] Chen W H and Lu X M 2008 *Phys. Lett. A* **372** 1061
[18] Li H Y, Chen B, Zhou Q and Fang S L 2008 *Phys. Lett.* A **372** 3385
[19] Fu J, Zhang H G and Ma T D 2009 *Progress in Natural Science* **19** in press
[20] Stoica A M and Yaesh I 2008 *Automatica* **44** 2157
[21] Gershon E and Shaked U 2008 *Automatica* **44** 574
[22] Gershon E, Shaked U and Berman N 2007 *IEEE Trans. Automat. Contr.* **52** 1773
[23] Krasovskii N N and Lidskii E A 1961 *Automation and Remote Control* **22** 1021
[24] Boukas E K, Liu Z K and Liu G X 2001 *International Journal of Control* **74** 329
[25] Wang Z D, Liu Y R, Yu L and Liu X H 2006 *Phys. Lett.* A **356** 346
[26] Zhang H G and Wang Y C 2008 *IEEE Trans. Neural Netw.* **19** 366
[27] Huang H, Ho D W C and Qu Y Z 2007 *Neural Networks* **20** 799
[28] Xu S Y, Chen T W and Lam J 2003 *IEEE Trans. Automat. Contr.* **48** 900
[29] Wang Y C and Zhang H G 2008 *Progress in Natural Science* **18** 309
[30] Boyd S, Chaoui L EI, Feron E and Balakrishnan V 1994 *Linear Matrix Inequalities in System and Control Theory* (Philadelphia PA: SIAM)
[31] Wang Y, Xie L and De Souza C E 1992 *System Control Lett.* **19** 139
[32] Mao X 1997 *Stochastic Differential Equations and Applications* (West Sussex, UK: Horwood)