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Circuitry implementation of a novel nonautonomous hyperchaotic Liu system based on sine input*

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Based on the three-dimensional Liu system with a nonlinear term of square, this paper appends a state variable to the system, and further adds a driving signal of the sine signal to construct a novel 4-demensional nonautonomous hyperchaotic Liu system. The appended variable is formed by the product of the nonlinear quadratic term of the original state variables and the driving signal. Through adjusting the frequency of the driving signal, the system can be controlled to show some different dynamic behaviors. By numerical simulations, the Lyapunov exponent spectrums, bifurcation diagrams and phase diagrams of the novel systems are analyzed. Furthermore, the corresponding hardware circuits are implemented. Both the experimental results and the simulation results confirm that the method is feasible. The method, which adjusts the frequency of the input sine signal to control the system to show different dynamic behaviors, can make the dynamic property of the system become more complex, but easier to be controlled accurately as well.

Keywords: chaos, hyperchaos, Liu system, frequency control
PACC: 0545, 2960

1. Introduction

Since Lorenz found the first chaos model in 1963,[1] owing to the simple structure of the Lorenz's system, people have improved on it ceaselessly, and have found some new three-dimensional chaotic systems. There are some famous systems such as the Chen system,[2] the Lü system,[3,4] the Liu system[5] and the Qi system.[6] As the members of the generalized Lorenz system family, they are widely studied by researchers for their rich dynamic behaviors.[7−14]

Recently, basing on the systems in the generalized Lorenz system family, and using the method of system variable expansion, authors have constructed many types of hyperchaotic systems.[15−20] The literature [21] has introduced a sine signal, and changed the frequency of the sine signal to realize the change of the system's dynamic behaviors.

The Liu chaotic system in Ref.[22] is a kind of chaotic system including nonlinear terms of square. Based on it and using the method of system variable expansion, we construct a novel four-dimensional nonautonomous hyperchaotic Liu system. Via the Lyapunov exponent spectrums, bifurcation diagrams and phase diagrams, we study the property of the novel systems. The corresponding hardware circuits are implemented. Simulation results and circuitry experimental results show that the novel nonautonomous hyperchaotic Liu system has rich dynamic behaviors.

2. Nonautonomous hyperchaotic Liu system and analysis on its property

2.1. Construction of the nonautonomous hyperchaotic Liu system

The Liu chaotic system is a kind of chaotic system including nonlinear terms of square, which can be expressed as follows:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = bx - hxz, \\ \dot{z} = cx^2 - dz. \end{cases} \tag{1}$$

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In this paper, we introduce a controller w, which has a driving signal of $\sin(\omega t)$, into the second equation of system (1), and let the changing rate of w equal $\dot{w} = \sin(\omega t)yz$, and then we can construct a novel system as follows:

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = bx - hxz + w, \\ \dot{z} = cx^2 - dz, \\ \dot{w} = \sin(\omega t)yz. \end{cases} \tag{2}$$

Let $a = 10$, $b = 50$, $c = 4$, $d = 4$ and $h = 2$, where ω is the angular frequency of the input signal. The relation between ω and the frequency f of the input signal satisfies $\omega = 2\pi f$.

For convenience, we express system (2) as a five-dimensional autonomous system

$$\begin{cases} \dot{x} = a(y - x), \\ \dot{y} = bx - hxz + w, \\ \dot{z} = cx^2 - dz, \\ \dot{w} = \sin(v)yz, \\ \dot{v} = \omega. \end{cases} \tag{3}$$

2.2. Analysis on the property of the nonautonomous hyperchaotic Liu system

The main dynamic property of the system can be analyzed through its Lyapunov exponent (LE) spectrums and bifurcation diagrams. For $\omega \in (0, 5)$, the Lyapunov exponent spectrum and the bifurcation diagram about variable x are shown in Fig.1.

Figure 1 shows that, when $\omega \in [3.02, 3.13]$, [3.47, 3.915] or [4.04, 5], there are two positive Lyapunov exponents, so the system is in the hyperchaotic state; when $\omega \in [0, 1.3]$, there is only one positive Lyapunov exponent, the system is in the chaotic state; when $\omega \in [2.66, 2.695]$, [3.135, 3.225] or [3.92, 4.035], the largest Lyapunov exponent of the system is zero, the system is in the periodic state; when $\omega \in [1.3, 2.655]$, [2.7, 3.015] or [3.23, 3.465], the system has no positive Lyapunov exponent, and the largest two Lyapunov exponents are zero, so the system is in the quasi-periodic state. The results shown in the bifurcation diagram are consistent with that of the Lyapunov exponent spectrum.

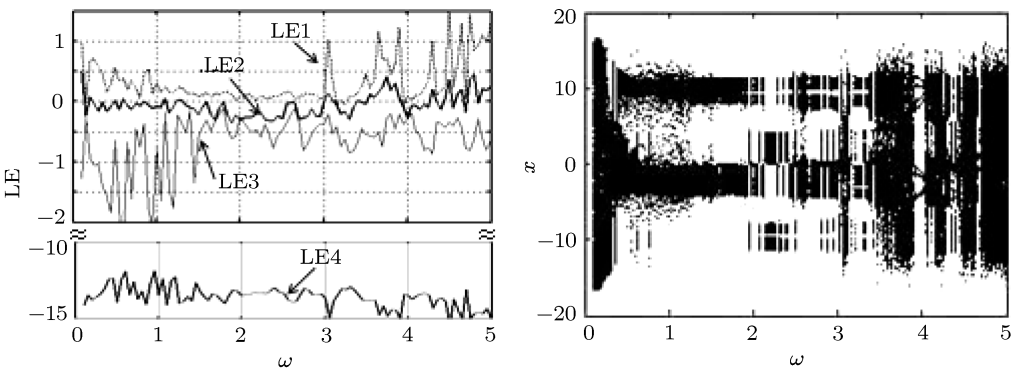


Fig.1. The Lyapunov exponent spectrum of system (3) and its bifurcation diagram about x.

3. Numerical simulations and the chaotic diagrams

To simulate system (3) numerically, we choose typical angular frequency ω to determine the Lyapunov exponents, as is shown in Table 1. The corresponding phase diagrams are shown as Fig.2. From Table 1 and Fig.2, we can see that, while increasing the angular frequency ω, the state of system (3) will evolve from chaotic to quasi-periodic, periodic, quasi-periodic, hyperchaotic, periodic, quasi-periodic, hyperchaotic, periodic, hyperchaotic states, successively.

Table 1. Then Lyapinov exponents, attractor types and motion types for different parameter ω.

ω	LE1	LE2	LE3	LE4	LE5	attractor type	motion type
0.63	0.11544	–0.064654	–1.9004	–12.15	0	chaotic attractor	chaotic motion
1.89	0.037777	–0.04238	–0.38127	–13.787	0	quasi-periodic orbit	quasi-periodic motion
2.68	0.07181	–0.091474	–0.55994	–13.42	0	periodic orbit	periodic motion
2.9	0.07847	–0.069986	–0.35235	–13.656	0	quasi-periodic orbit	quasi-periodic motion
3.1	0.50015	0.021891	–0.5674	–13.72	0	hyperchaotic attractor	hyperchaotic motion
3.165	–0.0954	–0.14131	–0.76066	–13.089	0	periodic orbit	periodic motion
3.43	0.06715	–0.033226	–0.03226	–13.41	0	quasi-periodic orbit	quasi-periodic motion
3.68	1.1839	0.32265	–0.53107	–14.975	0	hyperchaotic attractor	hyperchaotic motion
3.98	0.018828	–0.17816	–0.6083	–13.232	0	periodic orbit	periodic motion
4.6	0.41178	0.090152	–0.33356	–14.162	0	hyperchaotic attractor	hyperchaotic motion

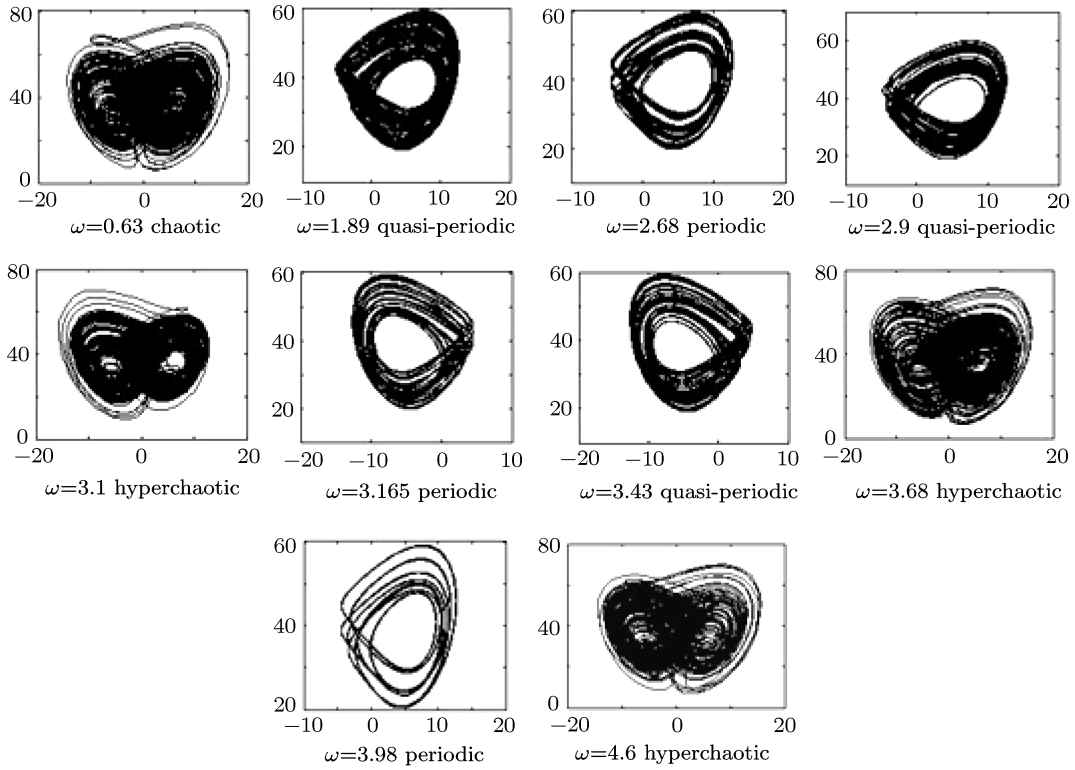


Fig.2. The x–z phase diagrams of system (3) for different ω.

4. Circuitry implementation of the system and its experimental results

The most direct and simplest ways to realize a chaotic system physically are through circuits, and the dynamic behaviors of many chaotic systems are often confirmed by circuits.[23−25]

For circuit design, some problems should be noticed. The type of operating amplifiers we chose is LM741, and the type of the multiplier is AD633JN. From the phase diagrams in Fig.2 obtained by Matlab simulations, it is shown that the highest level of the signal in the chaotic state is about 80 volts. In order to ensure the circuit working normally, we use the transform method of proportional compression to re-adjust the equations of the original system, and change ex-

pression (3) as follows:

$$\begin{cases} 0.1\dot{x} = y - x, \\ 0.1\dot{y} = 5x - 2xz - 0.1w, \\ 0.1\dot{z} = 4x^2 - 0.4z, \\ 0.1\dot{w} = \sin(\omega t)yz. \end{cases} \tag{4}$$

The adjusted system (4) is equivalent to the original system (3), and the physical property of the systems is not changed.

Because the gain of the multiplier AD633JN is 0.1, from Fig.3 we can obtain the circuit equations as follows:

$$\begin{cases} (R_0C)\dfrac{\mathrm{d}x}{\mathrm{d}t} = -\dfrac{R}{R_1}x + \dfrac{R}{R_2}y, \\ (R_0C)\dfrac{\mathrm{d}y}{\mathrm{d}t} = \dfrac{R}{R_5}x - \dfrac{R}{10R_3}xz + \dfrac{R}{R_4}w, \\ (R_0C)\dfrac{\mathrm{d}z}{\mathrm{d}t} = \dfrac{R}{10R_7}x^2 - \dfrac{R}{R_6}z, \\ (R_0C)\dfrac{\mathrm{d}w}{\mathrm{d}t} = -\dfrac{R}{R_8}\sin(\omega t)yz. \end{cases} \tag{5}$$

According to Eqs.(4) and (5) we have

$$\frac{10R}{R_1} = \frac{10R}{R_2} = 10, \quad \frac{10R}{R_5} = 50, \quad \frac{R}{10R_3} = 2,$$
$$\frac{10R}{R_4} = 0.1, \quad \frac{R}{10R_7} = 4, \quad \frac{10R}{R_6} = 0.4,$$
$$n = \frac{10R}{0.01R_8}, \quad R_0C = 0.1.$$

Then we design the circuits of system (3), as is shown in Fig.3.

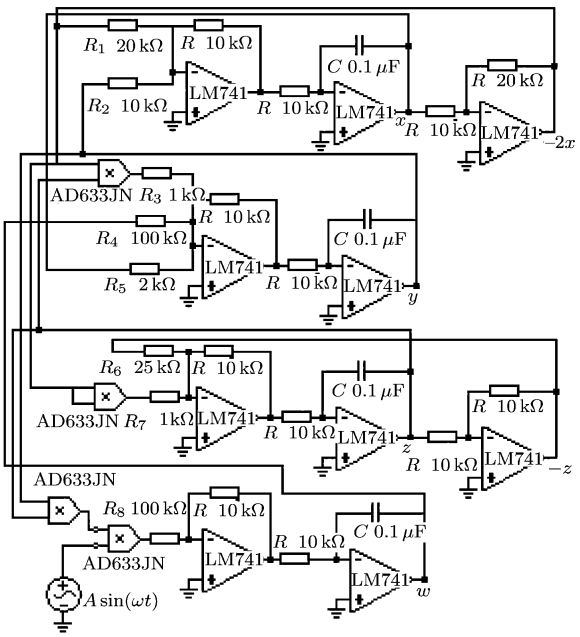


Fig.3. The circuit implementation of system (3).

In order to capture the wave easily, we let $R_0C = 10^{-4}$ to increase the frequency of the output signal for 1000 times, and let $C = 10$ nF, $R_0 = R = 10$ kΩ. Experiments show that we can obtain ideal output waves in this condition. If R_0C is too small, high frequency interferences are easy to be introduced, which will cause the observed results to be distorted.

The amplitude of the input signal is 1 V, the type of functional signal generator is DG1022, and the type of oscillograph is CS-4135A. Changing the frequency of the functional signal generator, we can observe the x–z phase diagrams of system (4) with the oscillograph, which are shown in Fig.4. The horizontal scale of the oscillograph is 1 V/div, and the vertical scale is 0.5 V/div.

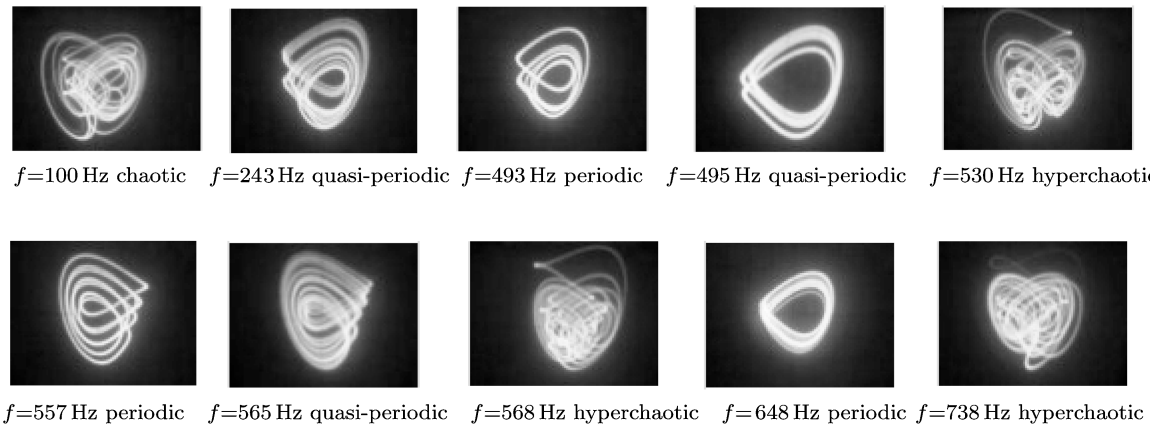


Fig.4. The x–z phase diagrams of system (3) observed by the oscillograph.

It is shown that, the x–z phase diagrams observed by the oscillograph are consistent with the x–z phase diagrams obtained by numerical simulations.

From the experimental results, we found that, while the frequency of the input signal changes from 100 Hz to 738 Hz, the phase diagrams of the periodic state and the quasi-periodic state will be reversed, mutation will occur in the system, and the system will change from one equilibrium to another, indicating that the nonautonomous hyperchaotic Liu system has richer dynamic behaviors.

5. Conclusions

The Liu system is a three-dimensional autonomous system. Based on the three-dimensional autonomous system, this paper appends a state variable, and introduces a sine signal as a driving signal. The appended state variable is formed by the product of the nonlinear quadratic term of the original state variables and the driving signal. We feed the appended variable back to the second equation of the original system, and then construct a novel 4-demensional nonautonomous hyperchaotic Liu system. Through adjusting the frequency of the driving signal, we can control the system to show different dynamic behaviors. By numerical simulations, the Lyapunov exponent spectrums, bifurcation diagrams and phase diagrams of the novel systems are analyzed. The corresponding circuits are implemented and the validating experiments are carried out. Experimental results and computer simulations confirm that the method is feasible.

Through adjusting the frequency of the input sine signal, one can easily control the system to evolve among different dynamic behaviors, such as hyperchaos, chaos, quasi-period and period. The method, which adjusts the frequency of the input sine signal to control the dynamic behaviors of the system, can make the dynamic property of the system become more complex, but easier to be controlled accurately as well, which has some practical values of reference in engineering applications.

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