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Chaos in fractional-order generalized Lorenz system and its synchronization circuit simulation*

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The chaotic behaviours of a fractional-order generalized Lorenz system and its synchronization are studied in this paper. A new electronic circuit unit to realize fractional-order operator is proposed. According to the circuit unit, an electronic circuit is designed to realize a 3.8-order generalized Lorenz chaotic system. Furthermore, synchronization between two fractional-order systems is achieved by utilizing a single-variable feedback method. Circuit experiment simulation results verify the effectiveness of the proposed scheme.

Keywords: chaos, fractional-order generalized Lorenz chaotic system, circuit simulation, synchronization

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1. Introduction

In recent years, the study of the dynamics of fractional-order differential systems has attracted great interest from many researchers. Many systems are known to display fractional-order dynamics, such as viscoelastic systems,[1] dielectric polarization,[2] electrode-electrolyte polarization,[3] electromagnetic waves,[4] quantitative finance[5] and quantum evolution of complex systems.[6] Most recently, many investigations have been devoted to the dynamics of fractional-order chaotic dynamical systems, such as the Chua system,[7] Rössler system,[8] Chen system,[9,10] Liu system,[11] Arneodo system,[12] and so on. Meanwhile, chaotic applications in physics and engineering have attracted much attention. Since the idea of synchronizing two identical chaotic systems from different initial conditions was first introduced by Pecora and Carroll in 1990,[13] chaotic synchronization has been intensively studied.[13−18] Different types of chaotic synchronization methods have been achieved, e.g. complete synchronization, generalized projective synchronization, phase synchronization, lag synchronization, etc. Now, chaos synchronization problems in fractional-order are being widely investigated.[19−23]

The so-called generalized Lorenz system is a nonlinear system of ordinary differential equations in $\boldsymbol{R}^3$ of the following form:[24]

$$\dot{x} = \begin{pmatrix} A & 0 \\ 0 & \lambda_3 \end{pmatrix} x + x_1 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} x, \qquad (1)$$

where $x = [x_1\ x_2\ x_3]^{\mathrm{T}}$, $\lambda_3 \in \boldsymbol{R}$, and A is a 2×2 real matrix:

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

with eigenvalues $\lambda_1, \lambda_2 \in \boldsymbol{R}$ such that

$$-\lambda_2 > \lambda_1 > -\lambda_3 > 0.$$

It includes the classical Lorenz system with $a_{12}a_{21} > 0$,[24] the Lü system with $a_{12}a_{21} = 0$,[25] and the Chen system with $-1 < a_{12}a_{21} < 0$.[26] As two special cases of the generalized Lorenz system, the Liu system[27] and the Shimidzu–Morioka system[28] have $\tau = -1$. The unified system was constructed to encompass the Lorenz and the Chen systems,[29] and it is completely contained in the generalized Lorenz system (1). In 1996, with the effects of external rotation included, Stenflo[30] showed that low-frequency short-wavelength acoustic gravity waves could be described

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by using a system of four coupled nonlinear ordinary differential equations. The system is similar to the Lorenz system but with a new control parameter and a new variable. It is called the generalized Lorenz system (fourth order).[31] In Refs.[32]–[37], chaos in the fractional-order Chen system, the Lorenz system, the Lü system, the Liu system and the unified system have been studied respectively. In the present paper, the chaotic behaviours and the synchronization of the fractional-order generalized Lorenz chaotic system (fourth order) are studied. Based on the approximation method in frequency domain, a new electronic circuit model is designed to realize the fractional-order operator. The chaotic behaviours of a 3.8-order generalized Lorenz chaotic system (fourth order) and its synchronization are studied by numerical simulations and circuit experiments. This is the first report of a fractional-order generalized Lorenz system in electronic circuit and numerical simulations.

2. Fractional-order generalized Lorenz chaotic system and its circuit realization

At present, there are several definitions of a fractional-order differential system. Of them, the Riemann–Liouville definition is the most common one. The Riemann–Liouville definition is described by:

$$\frac{\mathrm{d}^\alpha f(t)}{\mathrm{d}t^\alpha} = \frac{1}{\Gamma(n-\alpha)}\frac{\mathrm{d}^n}{\mathrm{d}t^n}\int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha-n+1}}\mathrm{d}\tau, \quad n-1 \le \alpha < n, \quad (2)$$

where $\Gamma(\cdot)$ is the gamma function. Assume that the initial conditions are zero, the Laplace transform of the Riemann–Liouville fractional derivative will be

$$L\left\{\frac{\mathrm{d}^\alpha f(t)}{\mathrm{d}t^\alpha}\right\} = s^\alpha L\left\{f(t)\right\}.$$

Thus, the fractional integral operator of order "α" can be represented by the transfer function $F(s) = 1/s^\alpha$ in the frequency domain.

The standard definition of fractional differintegral does not allow the direct engineering applications and circuit designs of the fractional operators. In Ref.[16], an efficient method to solve this problem was to approximate the fractional operators by using the standard integer-order operators. In Table 1 of Ref.[38], the approximations of $1/s^\alpha$ with $\alpha = 0.1-0.9$ in steps of 0.1 were given to have an error of around 2 dB, and in Ref.[8] the approximation of $1/s^{0.95}$ was given to have an error of around 1 dB. We will utilize these approximations in the following simulations and electronic circuit experiments.

2.1. Fractional-order generalized Lorenz chaotic system

The generalized Lorenz chaotic system (fourth order) is described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1) + cx_4,\\
\dot{x}_2 &= dx_1 - rx_2 - x_1x_3,\\
\dot{x}_3 &= x_1x_2 - bx_3,\\
\dot{x}_4 &= -x_1 - ax_4,
\end{aligned} \quad (3)$$

where a, b, c, d and r are constant parameters. When $a = 1, b = 0.7, c = 1.5, d = 26$ and $r = 1$, system (3) has the chaotic attractors shown in Fig.1.

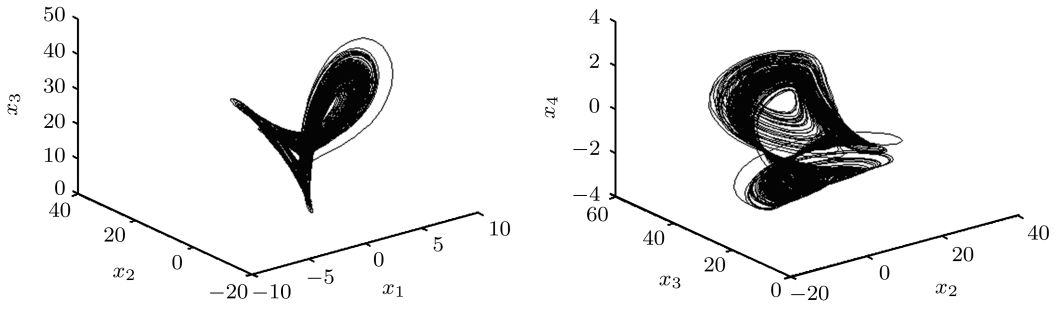


Fig.1. Chaotic attractors of generalized Lorenz system.

Now, we consider the fractional-order generalized Lorenz system given as

$$\begin{aligned}\frac{\mathrm{d}^\alpha x_1}{\mathrm{d}t^\alpha} &= a(x_2 - x_1) + cx_4,\\ \frac{\mathrm{d}^\alpha x_2}{\mathrm{d}t^\alpha} &= dx_1 - rx_2 - x_1x_3,\\ \frac{\mathrm{d}^\alpha x_3}{\mathrm{d}t^\alpha} &= x_1x_2 - bx_3,\\ \frac{\mathrm{d}^\alpha x_4}{\mathrm{d}t^\alpha} &= -x_1 - ax_4,\end{aligned} \tag{4}$$

where α is the fractional-order satisfying $0 < \alpha \leq 1$. When $(a,\ b,\ c,\ d,\ r) = (1,\ 0.7,\ 1.5,\ 20,\ 0.5)$, the fixed points and their corresponding eigenvalues are

$$S_0(0,0,0,0) : \lambda_1 = 3.5774,\ \lambda_2 = -5.0368,\ \lambda_3 = -1.0406,\ \lambda_4 = -0.7;$$

$$S_{1,2}(\pm 2.2913, \pm 5.7282, 18.75, \mp 2.2913) : \lambda_{1,2} = -1.8278 \pm 0.4855\mathrm{i} \ \text{ and } \ \lambda_{3,4} = 0.2278 \pm 2.6995\mathrm{i}.$$

Hence, $S_{1,2}$ are saddle points of index 2. According to Ref.[39], the necessary condition for system (4) to remain chaotic is

$$\alpha > \frac{2}{\pi}\arctan\frac{2.6995}{0.2278} \approx 0.9464.$$

When $\alpha = 0.95$, the numerical simulation phase portraits of the 3.8-order generalized Lorenz chaotic system are shown in Fig.2, and the largest Lyapunov exponent is 0.1719.

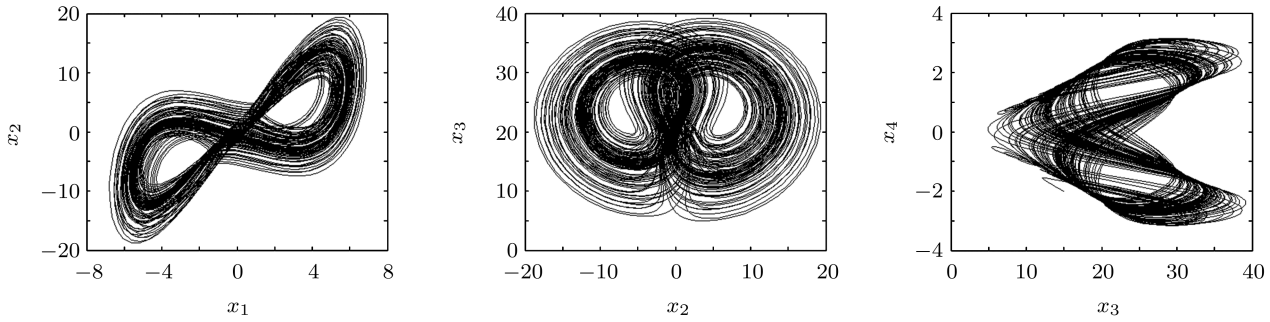


Fig.2. Numerical simulation phase portraits of 3.8-order generalized Lorenz chaotic system with $a = 1, b = 0.7, c = 1.5, d = 20$ and $r = 0.5$.

2.2. Circuit realization of fractional-order generalized Lorenz system

In this section, a new circuit unit to realize fractional-order operator and an electronic circuit to achieve a 3.8-order generalized Lorenz chaotic system are designed. According to the circuitry theory in the Laplace domain, the corresponding circuit unit between A and B in Fig.3 can be used to realize the approximations of $1/s^\alpha$ with $\alpha = 0.1$–0.9 in steps of 0.1 in Table 1 of Ref.[38] and $\alpha = 0.95$. Take $\alpha = 0.95$ as an example to show the design process of the circuit unit of the transfer function. Here we select the following transfer function approximation method presented in Ref.[8]:

$$\frac{1}{s^{0.95}} = \frac{1.281s^2 + 18.6004s + 2.0833}{s^3 + 18.4738s^2 + 2.6574s + 0.003} \tag{5}$$

and the corresponding circuit unit is displayed in Fig.4.

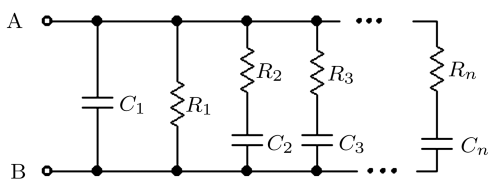


Fig.3. Circuit unit of $1/s^\alpha$.

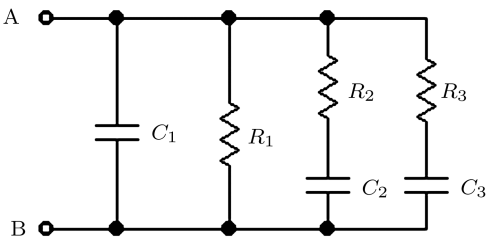


Fig.4. Circuit unit of $1/s^{0.95}$.

Using the circuitry theory in the Laplace domain, we can obtain the transfer function $H(s)$ between A and B in Fig.4 as follows:

$$
\begin{aligned}
& H(s) \\
= & R_1 // \frac{1}{sC_1} // \left(R_2 + \frac{1}{sC_2}\right) // \left(R_3 + \frac{1}{sC_3}\right) \\
= & \frac{\dfrac{1}{C_0} \cdot \dfrac{C_0}{C_1} \left(s + \dfrac{1}{R_2C_2}\right)\left(s + \dfrac{1}{R_3C_3}\right)}{s^3 + \dfrac{(R_1C_1 + R_2C_2 + R_1C_2)R_3C_3 + R_1R_2C_2(C_1 + C_3)}{R_1R_2R_3C_1C_2C_3} s^2 + \dfrac{R_1C_1 + R_2C_2 + R_1C_2 + R_1C_3 + R_3C_3}{R_1R_2R_3C_1C_2C_3} s + \dfrac{1}{R_1R_2R_3C_1C_2C_3}},
\end{aligned}
\tag{6}
$$

where C_0 is the unit parameter. Letting $C_0 = 1\ \mu F$ and $F(s) = H(s)C_0 = 1/s^{0.95}$ and comparing Eq.(6) with Eq.(5), we can obtain the values of resistances and capacitances in Fig.4 as follows: $R_1 = 694.6\ \text{M}\Omega$, $R_2 = 62.82\ \text{M}\Omega$, $R_3 = 0.3260\ \text{M}\Omega$, $C_1 = 0.7794\ \mu\text{F}$, $C_2 = 0.2699\ \mu\text{F}$, and $C_3 = 0.2133\ \mu\text{F}$. In a similar way, we can have the corresponding values of resistances and capacitances of circuit unit to realize $1/s^\alpha$ in Fig.3, which are listed in Table 1 and Table 2 respectively.

Table 1. Values of resistances in circuit unit of $1/s^\alpha$ (2 dB).

α	n	$R_1/\text{M}\Omega$	$R_2/\text{M}\Omega$	$R_3/\text{M}\Omega$	$R_4/\text{M}\Omega$	$R_5/\text{M}\Omega$	$R_6/\text{M}\Omega$
0.95 (1 dB)	3	694.6	32.82	0.3260			
0.9	3	63.10	1.598	0.01581			
0.8	5	39.80	9.839	0.9330	0.09319	0.009555	
0.7	6	25.13	9.793	2.860	0.5600	0.1215	0.02776
0.6	6	15.85	12.32	3.500	1.091	0.3486	0.1203
0.5	6	9.997	9.694	3.467	1.363	0.5494	0.2434
0.4	6	6.310	7.026	2.990	1.375	0.6461	0.3351
0.3	5	3.983	4.908	2.401	1.248	0.7100	
0.2	4	2.512	3.394	1.865	1.104		
0.1	3	0.5849	2.382	1.432			

Table 2. Values of capacitances in circuit unit of $1/s^\alpha$ (2 dB).

α	n	C_1/μ F	$C_2/\mu\text{F}$	$C_3/\mu\text{F}$	$C_4/\mu\text{F}$	$C_5/\mu\text{F}$	$C_6/\mu\text{F}$
0.95 (1 dB)	3	0.7794	0.2699	0.2133			
0.9	3	0.4410	0.4870	0.2937			
0.8	5	0.1884	0.7619	0.4520	0.2545	0.1396	
0.7	6	0.1068	1.5834	0.6048	0.3448	0.1770	0.08660
0.6	6	0.09261	1.749	0.9036	0.4254	0.1954	0.08311
0.5	6	0.06310	2.591	1.148	0.4631	0.1820	0.06510
0.4	6	0.02818	3.715	1.281	0.4091	0.1277	0.03616
0.3	5	0.02512	4.898	1.117	0.2399	0.04704	
0.2	4	0.01259	5.239	0.5362	0.05094		
0.1	3	0.0006310	2.517	0.02509			

According to the circuit unit of $1/s^\alpha$, using the circuitry theory and the circuitry design method in Refs.[11] and [40], we design a circuit to realize the fractional-order generalized Lorenz chaotic system (4) with $\alpha = 0.95$ in Fig.5, where AD633 is a multiplier with an output coefficient of 0.1, LM741 is an operational amplifier, $R_1 = 694.6\ \text{M}\Omega$, $R_2 = 62.82\ \text{M}\Omega$, $R_3 = 0.3260\ \text{M}\Omega$, $C_1 = 0.7794\ \mu\text{F}$, $C_2 = 0.2699\ \mu\text{F}$, $C_3 = 0.2133\ \mu\text{F}$, $R_4 = R_5 = R_8 = R_{15} = R_{21} = R_{24} = R_{26} = R_{27} = 100\ \text{k}\Omega$, $R_6 = 66.7\ \text{k}\Omega$, $R_7 = R_9 = R_{10} = R_{14} = R_{16} = R_{17} = R_{19}$, $R_{22} = R_{23} = R_{25} = R_{28} = R_{29} = 10\ \text{k}\Omega$, $R_{11} = 5\ \text{k}\Omega$, $R_{12} = R_{18} = 1\ \text{k}\Omega$, $R_{13} = 200\ \text{k}\Omega$, and $R_{20} = 143\ \text{k}\Omega$. Utilizing electronic workbench Multisim 2001 to simulate system (4), we obtain its phase portraits as shown in Fig.6. The circuit experiment simulation shows that the experimental results are in agreement with numerical simulations.

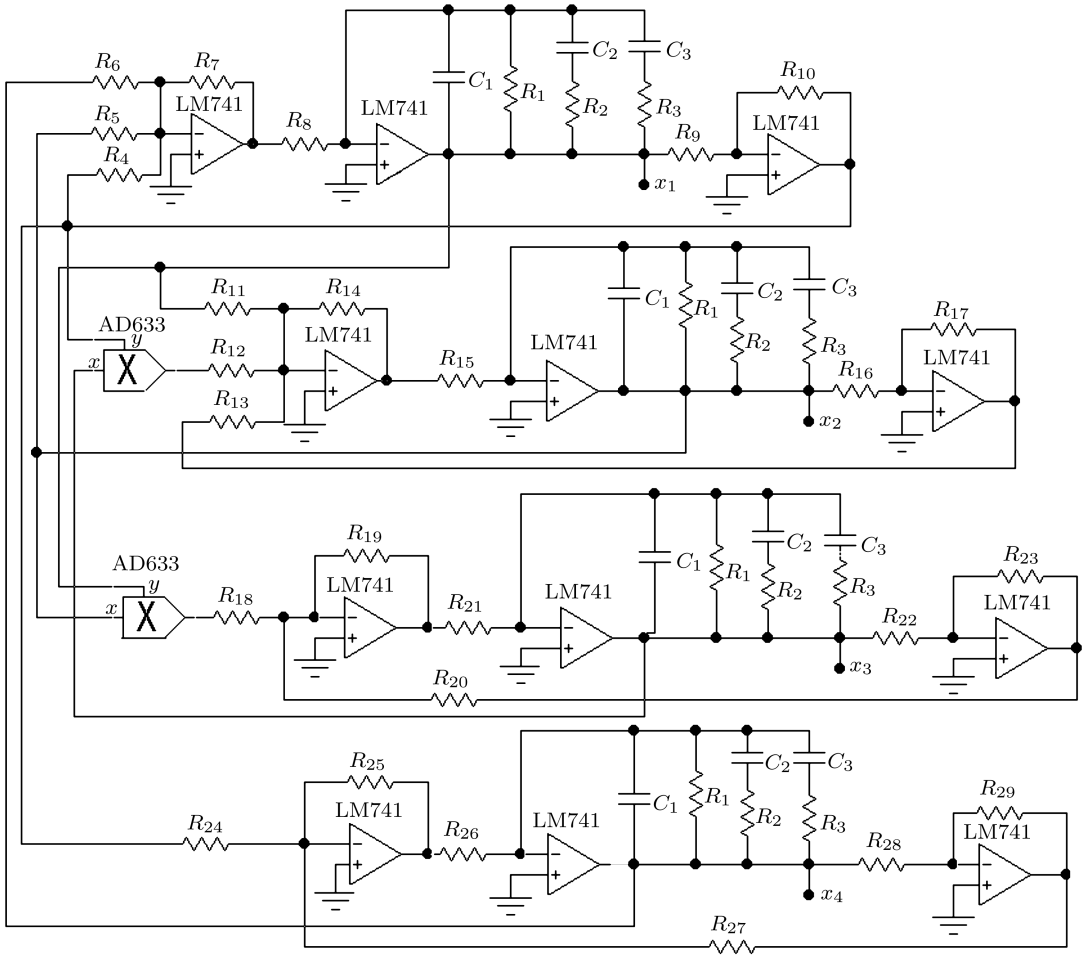


Fig.5. Circuitry realization of 3.8-order generalized Lorenz chaotic system.

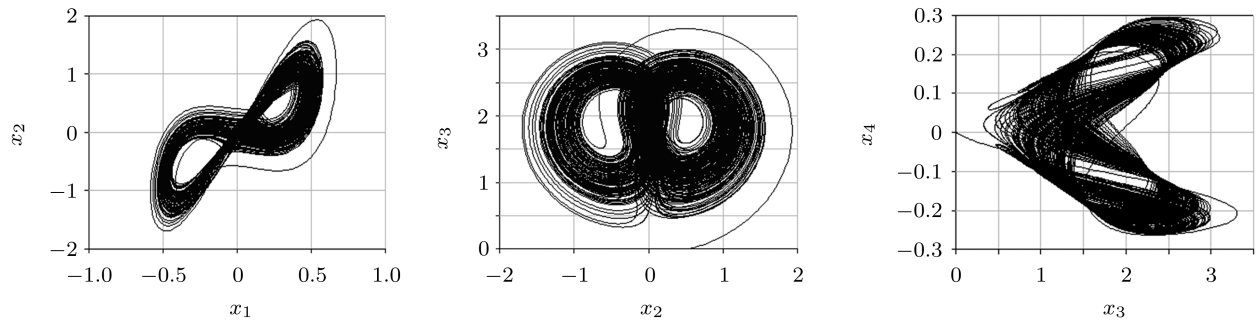


Fig.6. Circuit simulation phase portraits of 3.8-order generalized Lorenz chaotic system.

3. Synchronization between two fractional-order generalized Lorenz chaotic systems

In this section, synchronization between two fractional-order generalized Lorenz chaotic systems is achieved by a single state variable feedback in electronic workbench Multisim 2001.

Consider the 3.8-order generalized Lorenz chaotic system. The drive system is

$$\frac{\mathrm{d}^{0.95}x_1}{\mathrm{d}t^{0.95}} = a(x_2 - x_1) + cx_4,$$
$$\frac{\mathrm{d}^{0.95}x_2}{\mathrm{d}t^{0.95}} = dx_1 - rx_2 - x_1x_3,$$

$$\frac{\mathrm{d}^{0.95}x_3}{\mathrm{d}t^{0.95}} = x_1x_2 - bx_3,$$
$$\frac{\mathrm{d}^{0.95}x_4}{\mathrm{d}t^{0.95}} = -x_1 - ax_4, \tag{7}$$

and the response system with a single state variable feedback controller is

$$\frac{\mathrm{d}^{0.95}y_1}{\mathrm{d}t^{0.95}} = a(y_2 - y_1) + cy_4,$$
$$\frac{\mathrm{d}^{0.95}y_2}{\mathrm{d}t^{0.95}} = dy_1 - ry_2 - y_1y_3 + u,$$
$$\frac{\mathrm{d}^{0.95}y_3}{\mathrm{d}t^{0.95}} = y_1y_2 - by_3,$$
$$\frac{\mathrm{d}^{0.95}y_4}{\mathrm{d}t^{0.95}} = -y_1 - ay_4, \tag{8}$$

where $u = d(x_1 - y_1)$, $a = 1, b = 0.7, c = 1.5, d = 20$ and $r = 0.5$.

Now we define the state errors between the response system (8) and the drive system (7) as $e_i = y_i - x_i$, $(i = 1, 2, 3, 4)$. Subtracting Eq.(7) from Eq.(8) yields the following error dynamical system:

$$\frac{\mathrm{d}^{0.95}e_1}{\mathrm{d}t^{0.95}} = a(e_2 - e_1) + ce_4,$$
$$\frac{\mathrm{d}^{0.95}e_2}{\mathrm{d}t^{0.95}} = -re_2 - y_1e_3 - x_3e_1,$$
$$\frac{\mathrm{d}^{0.95}e_3}{\mathrm{d}t^{0.95}} = y_1e_2 + x_2e_1 - be_3,$$
$$\frac{\mathrm{d}^{0.95}e_4}{\mathrm{d}t^{0.95}} = -e_1 - ae_4. \tag{9}$$

Taking the Laplace transform on both sides of Eq.(9), letting $E_i(s) = L\{e_i(t)\}$ $(i = 1, 2, 3, 4)$ and using $L(\mathrm{d}^\alpha e_i/\mathrm{d}t^\alpha) = s^\alpha E_i(s) - s^{\alpha-1}e_i(0)$ $(i = 1, 2, 3, 4)$, lead to

$$\begin{aligned}
&s^{0.95}E_1(s) - s^{0.95-1}e_1(0)\\
&= aE_2(s) - aE_1(s) + cE_4(s)\,,\\
&s^{0.95}E_2(s) - s^{0.95-1}e_2(0)\\
&= -rE_2(s) - L(y_1e_3) - L(x_3e_1)\,,\\
&s^{0.95}E_3(s) - s^{0.95-1}e_3(0)\\
&= L(y_1e_2) + L(x_2e_1) - bE_3(s)\,,\\
&s^{0.95}E_4(s) - s^{0.95-1}e_4(0)\\
&= -E_1(s) - aE_4(s)\,.
\end{aligned} \tag{10}$$

Equation (10) can be rewritten as follows:

$$E_1(s) = \frac{aE_2(s) + cE_4(s) + s^{0.95-1}e_1(0)}{s^{0.95} + a}\,,$$
$$E_2(s) = \frac{s^{0.95-1}e_2(0) - L(y_1e_3) - L(x_3e_1)}{s^{0.95} + r}\,,$$
$$E_3(s) = \frac{L(y_1e_2) + L(x_2e_1) + s^{0.95-1}e_3(0)}{s^{0.95} + b}\,,$$
$$E_4(s) = \frac{s^{0.95-1}e_4(0) - E_1(s)}{s^{0.95} + a}\,. \tag{11}$$

By using the final-value theorem of the Laplace transformation, we obtain

$$\begin{aligned}
\lim_{x\to\infty} e_1(t) &= \lim_{s\to 0^+} sE_1(s)\\
&= \lim_{s\to 0^+} \frac{saE_2(s) + scE_4(s) + s^{0.95}e_1(0)}{s^{0.95} + a}\,,\\
\lim_{x\to\infty} e_2(t) &= \lim_{s\to 0^+} sE_2(s)\\
&= \lim_{s\to 0^+} \frac{s^{0.95}e_2(0) - sL(y_1e_3) - sL(x_3e_1)}{s^{0.95} + r}\,,\\
\lim_{x\to\infty} e_3(t) &= \lim_{s\to 0^+} sE_3(s)\\
&= \lim_{s\to 0^+} \frac{sL(y_1e_2) + sL(x_2e_1) + s^{0.95}e_3(0)}{s^{0.95} + b}\,,\\
\lim_{x\to\infty} e_4(s) &= \lim_{s\to 0^+} sE_4(s)\\
&= \lim_{s\to 0^+} \frac{s^{0.95}e_4(0) - sE_1(s)}{s^{0.95} + a}\,.
\end{aligned} \tag{12}$$

If we assume that $E_1(s)$ and $E_2(s)$ are bounded, i.e. $E_1(s) \le M$ and $E_2(s) \le M$, it follows that $\lim\limits_{t\to\infty} e_1(t) = \lim\limits_{t\to\infty} e_2(t) = 0$, and $\lim\limits_{t\to\infty} e_4(t) = 0$. Because of the attractiveness of the attractor, there exists $P > 0$ such that $\max(|x_i|, |y_i|) \le P$, $(i = 1, 2)$, and so $L(y_1e_2) \le PE_2(s), L(x_2e_1) \le PE_1(s)$. Thus, according to the circuit unit of $1/s^\alpha$ and the circuit of controller u (Fig.7), a synchronization circuit is designed to synchronize two generalized Lorenz chaotic systems (7) and (8) in Fig.8. When we set delay-time $t = 20$ s, the circuit simulation results are depicted in Fig.9. From it, one can find that the state variables y_1, y_2, y_3 and y_4 quickly converge to the state variables x_1, x_2, x_3 and x_4 respectively, as switch S is turned on. It is clear that the two fractional systems (7) and (8) can achieve synchronization quickly.

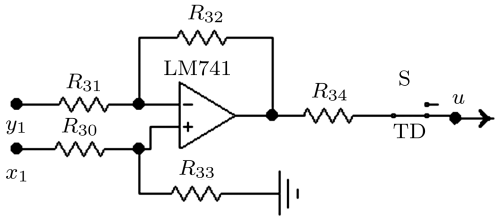


Fig.7. Circuitry of controller u.

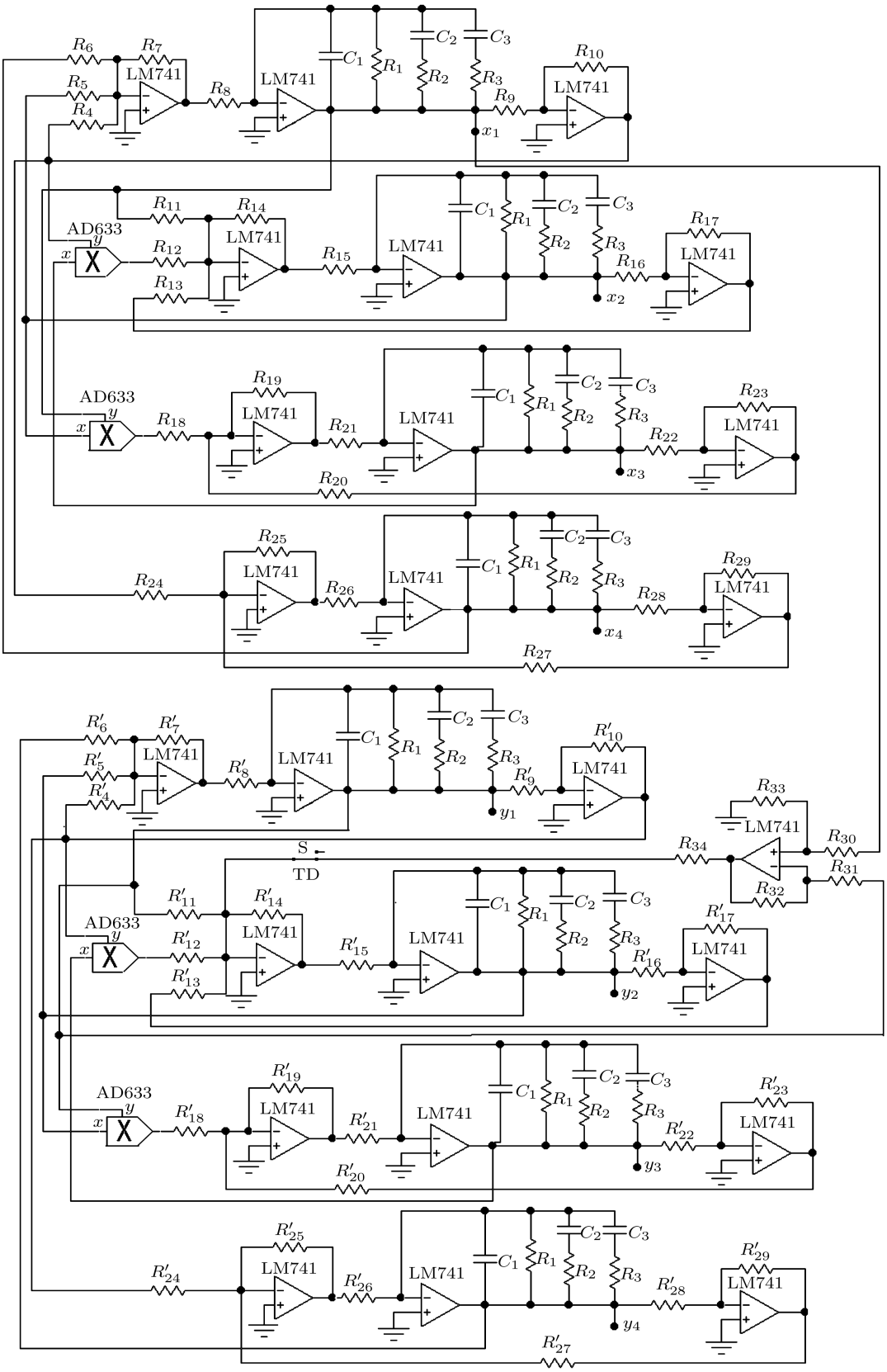


Fig.8. Synchronization circuitry between factional order chaotic systems (7) and (8).

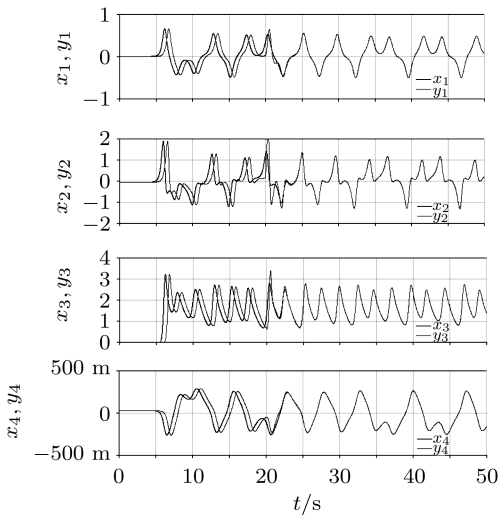


Fig.9. Observed state variable evolution with time on electronic workbench Multisim2001.

As Multisim 2001 software is the model which is based on the actual circuit components, its simulation results should be basically consistent with those of the actual circuit experiment. So the results of circuit simulation are effective.

4. Conclusions

In this paper, we have studied the chaotic behaviours of a fractional-order generalized Lorenz chaotic system and its synchronization circuit simulation. First, based on the approximation method in frequency domain, a new electronic circuit unit is designed to realize the fractional-order operator. The circuit unit of fractional-order operator, which we designed, is simple with smaller capacitance values and is easy to achieve in engineering. Then, the chaotic behaviours of the 3.8-order fractional-order generalized chaotic system have been studied not only by numerical simulations, but also by circuit simulations. Furthermore, via circuit simulation, synchronization between two fractional-order systems is achieved by utilizing a single-variable feedback method. We emphasise that these schemes can straightforwardly be extended to other similar chaotic fractional-order differential systems. It is believed that the circuit unit of fractional-order operator, fractional-order generalized Lorenz chaotic system and its synchronization scheme have potential applications in security communication.

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