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The Onsager reciprocity relation and generalized efficiency of a thermal Brownian motor*

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Based on a general model of Brownian motors, the Onsager coefficients and generalized efficiency of a thermal Brownian motor are calculated analytically. It is found that the Onsager reciprocity relation holds and the Onsager coefficients are not affected by the kinetic energy change due to the particle's motion. Only when the heat leak in the system is negligible can the determinant of the Onsager matrix vanish. Moreover, the influence of the main parameters characterizing the model on the generalized efficiency of the Brownian motor is discussed in detail. The characteristic curves of the generalized efficiency varying with these parameters are presented, and the maximum generalized efficiency and the corresponding optimum parameters are determined. The results obtained here are of general significance. They are used to analyze the performance characteristics of the Brownian motors operating in the three interesting cases with zero heat leak, zero average drift velocity or a linear response relation, so that some important conclusions in current references are directly included in some limit cases of the present paper.

Keywords: Brownian motor, Onsager coefficient, generalized efficiency, performance characteristic
PACC: 0540, 0250

1. Introduction

The phenomenon of noise-induced directed transport has attracted much interest in recent years, as it has widespread applications in physics, chemistry and biology, where it can be put to work for shuttling reliably and efficiently particles on the microscale, or even on the nanoscale.[1−3] One particular system that has been intensively studied both theoretically and experimentally is the motion of motor proteins like kinesins or dyneins. These motor proteins are responsible for the transport of cell organelles along the cytoskeleton. They move randomly but directly on average and have been modeled by Brownian ratchets. And then, the concept of Brownian motors[1−3] has been used to illuminate the directed, noise-induced transport.

The Brownian motor, which includes different types of ratchets, aims at the unidirectional movement of particles in situations where the scale of the system is so small that the thermal noise cannot be ignored. Because of such an interesting aim, different models of Brownian motors have been used not only by researchers of thermodynamics and statistical mechanics but also by those who try to explain the principle of the movement of real motor proteins[4,5] and those who try to invent the nanomachine.[6,7]

When one evaluates how a system works effectively, the efficiency is an important measure. A strict thermodynamic definition of this quantity implies that a motor performs useful work against an external load force. The conventional efficiency is just specified by the ratio of the useful work performed to the total energy expended.[2,4,8] In order to ascertain the necessary conditions for maximizing the conventional efficiency value,[9−13] the conventional efficiency has been extensively and thoroughly studied for the various flashing and rocking ratchet models. For this working definition, the load force is inevitably included. In particular, this yields the result that the efficiency assumes a zero value when no load force is acting. Motivated by this point, Derenyi *et al* [14] proposed a new efficiency that does not involve the load. That is, a task is first specified, and the generalized efficiency is defined as the ratio of the minimum energy necessary for the task to the input energy. At the same time, there has been an increasing interest in the generalized efficiency of nanodevices, which calls for a systematic comparative analysis for the various Brownian motor

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models. This viewpoint is addressed in our investigation.

In this paper, we extend the previous works[15−18] and establish a more general model of Brownian motors. The model is used to calculate the Onsager coefficients and generalized efficiency of the Brownian motor affected by the main irreversibilities existing in practical systems. Moreover, the influence of the heat leak in the system, the asymmetry of the external potential, the height of the potential barrier, the temperature ratio of two heat reservoirs, the viscous drag force, the kinetic energy change due to the particle's motion and the external load on the generalized efficiency of the Brownian motor are discussed in detail. Some significant results are obtained.

2. A thermal Brownian motor

The model of a thermal Brownian motor consists of Brownian particles moving in a sawtooth potential with an external load where the viscous medium is alternately in contact with the hot and cold heat reservoirs along the space (or reaction) coordinate.[14,16,17] The shape of a single sawtooth potential, $U_{\rm s}(x)$, located around $x=0$ is described by

$$U_{\rm s}(x)=\begin{cases} U_0\left(\dfrac{x}{L_1}+1\right), & (-L_1\leq x<0), \\ U_0\left(\dfrac{-x}{L_2}+1\right), & (0\leq x<L_2), \end{cases} \tag{1}$$

where L_1 and L_2 are the lengths of the left and right sides of a sawtooth potential and U_0 is the height of the potential barrier. The temperature profile, $T(x)$, within the interval $-L_1\leq x<L_2$ is described by

$$T(x)=\begin{cases} T_{\rm h}, & (-L_1\leq x<0); \\ T_{\rm c}, & (0\leq x<L_2), \end{cases} \tag{2}$$

where $T_{\rm h}$ and $T_{\rm c}$ are the temperatures of the hot and cold heat reservoirs, respectively. Both $U_{\rm s}(x)$ and $T(x)$ are taken to have the same period such that $U_{\rm s}(x+L)=U_{\rm s}(x)$ and $T(x+L)=T(x)$ as shown in Fig.1, where $L=L_1+L_2$ is the period of the sawtooth potential. When the external load, f, is considered, the equivalent potential corresponding to the load is linear, fx, so that the equivalent sawtooth potential with the load becomes $U(x)=U_{\rm s}+fx$.

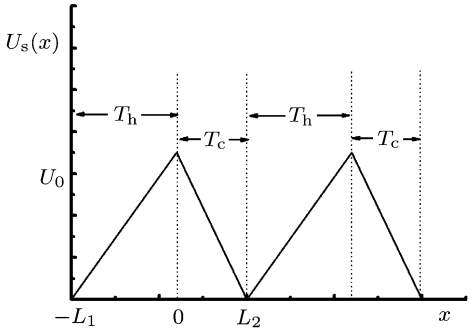


Fig.1. The schematic diagram of the ratchet potential $U_{\rm s}(x)$. The temperature profile is shown above the potential profile.

Because the dynamic equation governing the Brownian particle in inhomogeneous media depends on the specific environment to which the particle is exposed,[19,20] the inhomogeneous medium in the system is assumed to be highly viscous and its corresponding dynamic equation takes a specific form of Smoluchowski equation first derived by Sancho *et al*[21] and widely applied by other researchers:[2,12,16,22,23]

$$\begin{aligned} &\frac{\partial}{\partial t}\left(P(x,t)\right) \\ =&\frac{\partial}{\partial x}\left[\frac{1}{\gamma(x)}\left(U'(x)P+\frac{\partial}{\partial x}\left(T(x)P\right)\right)\right], \end{aligned} \tag{3}$$

where Boltzmann's constant, $k_{\rm B}$, is taken to be unity, $P=P(x,t)$ is the probability density of finding the particle at position x at time $t, U'(x)={\rm d}U(x)/{\rm d}x$, $\gamma(x)$ is the friction coefficient at position x. Starting from the above model and Eq.(3), one can find that the constant current at a steady state is given by[16]

$$J=\frac{-F}{G_1G_2+(A_1+A_2+A_3)F}, \tag{4}$$

where

$$\begin{aligned} F&={\rm e}^{a-b}-1, \\ G_1&=\frac{L_1}{aT_{\rm h}}\left(1-{\rm e}^{-a}\right)+\frac{L_2}{bT_{\rm c}}{\rm e}^{-a}\left({\rm e}^{b}-1\right), \\ G_2&=\frac{\gamma L_1}{a}\left({\rm e}^{a}-1\right)+\frac{\gamma L_2}{b}{\rm e}^{a}\left(1-{\rm e}^{-b}\right), \\ A_1&=\frac{\gamma}{T_{\rm h}}\left(\frac{L_1}{a}\right)^2\left(a+{\rm e}^{-a}-1\right), \\ A_2&=\frac{\gamma L_1L_2}{abT_{\rm c}}\left(1-{\rm e}^{-a}\right)\left({\rm e}^{b}-1\right), \\ A_3&=\frac{\gamma}{T_{\rm c}}\left(\frac{L_2}{b}\right)^2\left({\rm e}^{b}-1-b\right), \end{aligned} \tag{5}$$

$a = \dfrac{U_0 + fL_1}{T_\mathrm{h}}$, and $b = \dfrac{U_0 - fL_2}{T_\mathrm{c}}$.

When the thermodynamic forces in the Brownian motor $X_1 = f/T_\mathrm{c}$ and

$$X_2 = \frac{T_\mathrm{h} - T_\mathrm{c}}{T_\mathrm{c}^2} = \frac{\Delta T}{T_\mathrm{c}^2}$$

are very small, the constant current may be simplified to

$$J = -C[X_1 - (U_0/L)X_2], \tag{6}$$

where

$$C = \frac{T_\mathrm{c}}{\gamma L}\left[\frac{\alpha}{\sinh(\alpha)}\right]^2$$

and $\alpha = U_0/(2T_\mathrm{c})$. By using Eq.(6), the particle's average drift velocity

$$v = JL \tag{7}$$

can be conveniently calculated.

3. The Onsager reciprocity relation

It can be clearly seen that the above model acts as a heat engine when the current flows from left to right and the particle undergoes a periodic motion. In the system, there are two heat flows. One is the heat leak between the hot and cold regions, i.e., $k\Delta T$, where k is the coefficient of the heat leak between the hot and cold regions for a particle. The other is the heat flow resulting from the particles moving through the hot and cold regions. To climb up the equivalent potential barrier and overcome the viscous drag force, the system absorbs the heat, $(U_0 + fL_1) + \gamma v L_1$, from the hot reservoir when the particle moves through the hot region. On the other hand, the kinetic energy change[17] due to the particle's recrossing the boundary between the two regions is equal to $\Delta T/2$. Thus, during the time $t = L/v$, the total heat transferred from the hot reservoir to the system is given by

$$Q_\mathrm{h} = U_0 + (\gamma v + f)\, L_1 + \Delta T/2 + tk\Delta T. \tag{8}$$

Similarly, one can calculate that during the time t, the total heat transferred from the system to the cold reservoir is given by

$$Q_\mathrm{c} = U_0 - (\gamma v + f)\, L_2 + \Delta T/2 + tk\Delta T. \tag{9}$$

It is worth pointing out that the model adopted here is more general than those in Refs.[15] and [16] and the important results in these references can be directly derived from some limit cases in the present paper.

According to the theory of nonequilibrium thermodynamics, the rate of entropy production may be expressed as

$$\begin{aligned}\dot{S} &= -\frac{\dot{Q}_\mathrm{h}}{T_\mathrm{h}} + \frac{\dot{Q}_\mathrm{c}}{T_\mathrm{c}} \\ &= \left(\frac{U_0}{L}v + k\Delta T + \frac{v}{2L}\Delta T\right)\left(\frac{1}{T_\mathrm{c}} - \frac{1}{T_\mathrm{h}}\right) \\ &\quad - \frac{(\gamma v + f)v}{L}\left(\frac{L_1}{T_\mathrm{h}} + \frac{L_2}{T_\mathrm{c}}\right).\end{aligned} \tag{10}$$

In the linear response regime, equation (10) may be rewritten as

$$\dot{S} = (X_1, X_2)\begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22}\end{pmatrix}\begin{pmatrix} X_1 \\ X_2\end{pmatrix}, \tag{11}$$

where L_{ij} are the Onsager coefficients. Substituting Eqs.(6), (7) into Eq.(10) yields

$$\begin{aligned}\dot{S} = &\left(\frac{C}{L} - \frac{\gamma C^2}{T_\mathrm{c}}\right)\left(U_0^2 X_2^2 - 2U_0 L X_1 X_2 + L^2 X_1^2\right) \\ &+ kT_\mathrm{c}^2 X_2^2.\end{aligned} \tag{12}$$

Comparing Eq.(12) with Eq.(11), one can find that the Onsager coefficients in the system are determined by

$$L_{11} = CL(1 - \gamma CL/T_\mathrm{c}), \tag{13}$$

$$L_{21} = L_{12} = CU_0(\gamma CL/T_\mathrm{c} - 1), \tag{14}$$

and

$$L_{22} = (CU_0^2/L)(1 - \gamma CL/T_\mathrm{c}) + kT_\mathrm{c}^2. \tag{15}$$

It is seen from Eqs.(13)–(15) that in the linear response regime, the Onsager coefficients are not affected by the kinetic energy change due to the particle's motion. The Onsager coefficients offer a set of information about the intrinsic nonequilibrium thermodynamic properties of the system. First, one can check that the reciprocity relation, $L_{21} = L_{12}$, is fulfilled and that the diagonal coefficients L_{11} and L_{22} are positive as they should be. It is easily found from Eqs.(13)–(15) that the determinant of the L_{ij} matrix cannot be equal to zero, i.e.,

$$L_{11}L_{22} \neq L_{12}^2. \tag{16}$$

This shows clearly that the model mentioned above is irreversible and there is an entropy production in the Brownian motor, because the influence of the heat leak in the system is taken into account. It is different from the case discussed in Ref.[15].

When the influence of the heat leak is negligible, $k = 0$. The determinant of the L_{ij} matrix is equal

to zero, i.e., $L_{11}L_{22} = L_{12}^2$, so that the engine can be operated in a reversible regime at zero entropy production. More importantly, the relation implies that the dimensionless coupling strength[24]

$$q = \frac{L_{12}}{\sqrt{L_{11}L_{22}}} = -1. \tag{17}$$

This means that in such an approximate case, the Brownian motor is built under the condition of tight coupling.[15,25]

4. The generalized efficiency

From Eqs.(8) and (9), one can find that the net work, W, done by the Brownian motor in one cycle is

$$W = (\gamma v + f)\, L, \tag{18}$$

where the term $(\gamma v + f)$ can be taken as a generalized load or external force.[8] Note that the relation resulting from Eqs.(8), (9) and (18) is consistent with the first law, $Q_{\rm h} = Q_{\rm c} + W$, as shown in Fig.2. Based

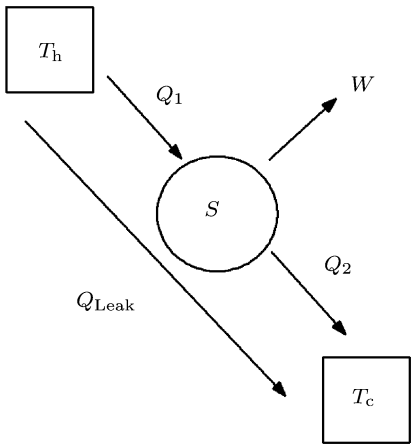


Fig.2. The equivalent cycle diagram of a thermal Brownian motor. $T_{\rm h}$ and $T_{\rm c}$ are the temperatures of the hot and cold heat reservoirs; $Q_1 = U_0 + (\gamma v + f)\, L_1 + \Delta T/2$ and $Q_2 = U_0 - (\gamma v + f)\, L_2 + \Delta T/2$ are the heats exchanged between the two reservoirs and the system S due to the particle's motion; $Q_{\rm Leak}$ is the heat leak and W is the work.

on Eqs.(8) and (18), the generalized efficiency of the Brownian motor, as suggested in Refs.[14] and [16], may be defined as

$$\eta = \frac{W}{Q_{\rm h}} = \frac{(\gamma v + f)\, L}{U_0 + (\gamma v + f)\, L_1 + \frac{\Delta T}{2} + k(L/v)\Delta T}, \tag{19}$$

where both the work against the load f and the dissipation energy associated with the directional movement against the frictional force are regarded as meaningful from a "rectification" viewpoint.[26,27] The generalized efficiency[28] emphasizes its more general behavior. It differs from the conventional efficiency by an additional addend in the numerator $\gamma v L$, which accounts for the work done by friction forces on a particle, and lays more emphasis on how the input energy is utilized for the directional movement than on how much energy is stored inside the whole system.[26]

In order to explore how the generalized efficiency of the system depends on some quantities characterizing the model, we introduce some new parameters $u = U_0/T_{\rm c}$, $l = L_2/L_1$, $\beta = T_{\rm h}/T_{\rm c}$, $\lambda = fL_1/T_{\rm c}$ and $\Gamma = \gamma L_1^2/T_{\rm c}$. Using Eqs.(4), (7) and (19), we can plot the characteristic curves of the Brownian motor, as shown in Figs.3–8.

Figure 3 gives the curves of η varying with k for some given values of the parameters u, β, l, Γ and λ. It is seen from Fig.3 that the generalized efficiency of the system is a monotonically decreasing function of k. The reason is quite clear. The smaller the heat leak coefficient k is, the smaller the heat leak loss, and consequently, the larger the generalized efficiency. When $k = 0$, the efficiency attains its maximum.

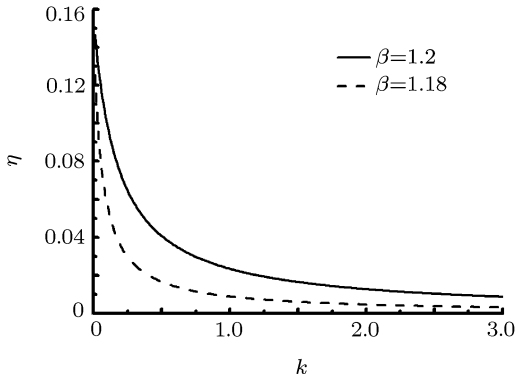


Fig.3. The variation of η for different β with k for $l = 0.6$, $u = 2$, $\Gamma = 1$ and $\lambda = 0.2$.

Figure 4 shows the dependence of the generalized efficiency on the asymmetry parameter l for some given values of the parameters u, β, k, Γ and λ. It is clearly seen that the relation between the efficiency and the asymmetry parameter l is complicated. When the coefficient of the heat leak $k = 0$, the efficiency increases linearly with increasing asymmetry parameter l. When k is not equal to zero but very small, the efficiency is not a monotonic function of l so that there is an optimized value of l at which the efficiency attains its maximum. When k is large, the efficiency decreases with increasing l. When $k > 0$, there is a maximum of l at which the efficiency is equal to zero.

When $l > l_{\max}$, the efficiency is smaller than zero and the Brownian heat engine loses its role. The maximum of l is independent of k and Γ but dependent on the choice of the parameters u, λ and β.

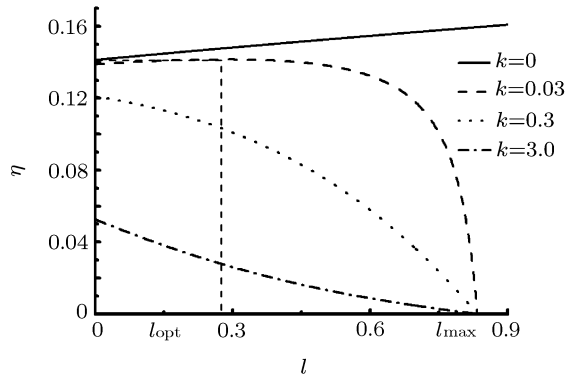


Fig.4. The variation of η with l for different k, at $\beta = 1.2$, $u = 2$, $\Gamma = 1$ and $\lambda = 0.2$, where $l_{\rm opt}$ and $l_{\max}$ are the values of l at the maximum efficiency and zero efficiency, respectively.

Figure 5 indicates the influence of λ on the generalized efficiency for given values of the parameters u, β, l, Γ with different k. It shows that when $k = 0$, the efficiency increases linearly with increasing λ. The result is compatible with that obtained in Ref.[16]. When $k > 0$, the efficiency is not a monotonic function of λ. There exists an optimized value of λ at which the efficiency attains its maximum. With increasing k, the maximum efficiency and the corresponding optimized λ will decrease. When λ is equal to $\lambda_{\min}$ or $\lambda_{\max}$, which is called the stall force,[29] the current is equal to zero, and consequently, the efficiency is always equal to zero. This implies that for a Brownian

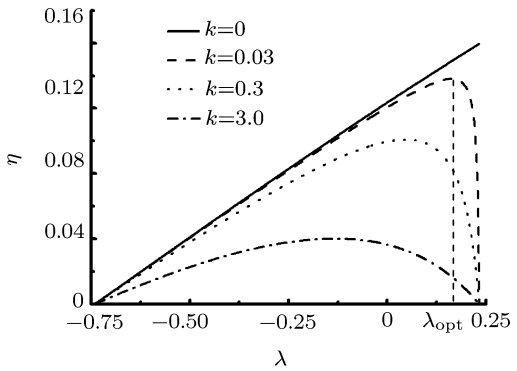


Fig.5. The curves of η varying with λ for $\beta = 1.2$, $l = 0.6$, $u = 2$ and $\Gamma = 1$, where $\lambda_{\rm opt}$ is the value of λ at the maximum efficiency.

motor, the range of λ lies in $\lambda_{\min} < \lambda < \lambda_{\max}$. Obviously, the parameters $\lambda_{\min}$ and $\lambda_{\max}$ are independent of k and Γ but dependent on the choice of the parameters u, l and β.

Figure 6 shows the influence of the coefficient of viscous force on the generalized efficiency for some given values of the parameters u, β, λ, l with different k. It is clearly seen from Fig.6 or Eqs.(4) and (19) that when $k = 0$, the efficiency is a constant because γv is equal to a constant for some given values of the parameters u, β, λ and l. When $k > 0$, the efficiency always decreases with increasing coefficient of viscous force.

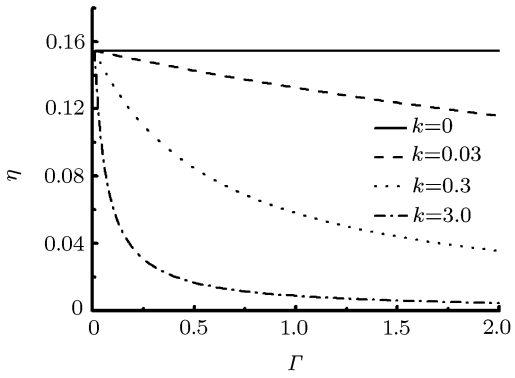


Fig.6. The curves of η varying with Γ for $\beta = 1.2$, $l = 0.6$, $u = 2$ and $\lambda = 0.2$.

Figure 7 represents the generalized efficiency as a function of the temperature ratio of two reservoirs for some given values of the parameters u, Γ, λ, l with different k. It is found that the generalized efficiency is a monotonically increasing function of β whether the heat leak in the system is considered or not. When $k > 0$, there exists a minimum of β at which the generalized efficiency is equal to zero. When $\beta < \beta_{\min}$, the efficiency is smaller than zero and the Brownian

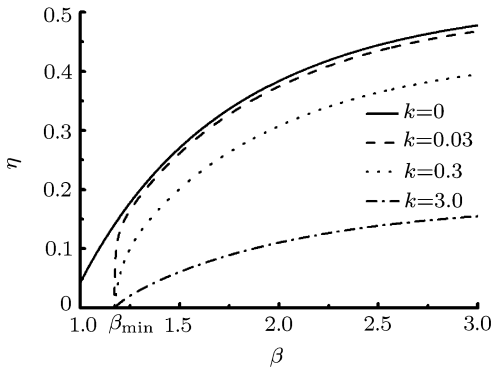


Fig.7. The curves of η versus β for $l = 0.6$, $u = 2$, $\Gamma = 1$ and $\lambda = 0.2$, where $\beta_{\min}$ is the value of β at the zero efficiency.

heat engine loses its role. The minimum of β is independent of k and Γ but dependent on the choice of the parameters u, λ and l.

Figure 8 shows the influence of u on the generalized efficiency for some given values of the parameters β, Γ, λ, l with different k. It is seen that there exists an optimum value of u at which the generalized efficiency attains its maximum. When the heat leak in the system increases, the maximum value of the generalized efficiency will decrease while the corresponding optimum value of u will increase. When u is equal to $u_{\min}$ or $u_{\max}$, we have $\eta = 0$. This implies that for a Brownian motor, the range of u lies in $u_{\min} < u < u_{\max}$. When $k = 0$, $u_{\min}$=0. When $k > 0$, the parameters $u_{\min}$ and $u_{\max}$ are independent of k and Γ but dependent on the choice of the parameters β, λ and l.

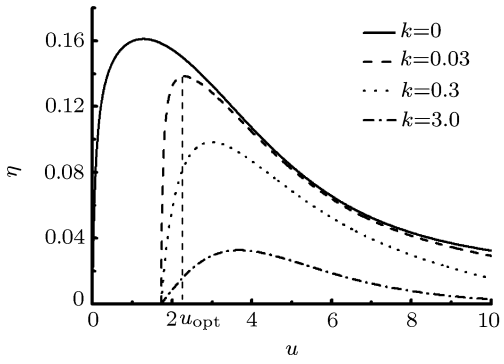


Fig.8. The curves of η versus u for $\beta = 1.2$, $l = 0.6$, $\Gamma = 1$ and $\lambda = 0.2$, where $u_{\rm opt}$ is the value of u at the maximum efficiency.

5. Discussion

5.1. The case of $k = 0$

When the heat leak in the system is negligible, $k = 0$, equation (19) may be simplified to

$$\eta = \frac{(\gamma v + f)\, L}{U_0 + (\gamma v + f)\, L_1 + \dfrac{\Delta T}{2}}. \tag{20}$$

Equation (20) shows clearly that the kinetic energy change due to the particle motion always reduces the efficiency of the Brownian motor.

When the kinetic energy change due to the particle motion is further ignored, equation (20) may be written as

$$\eta = \frac{(\gamma v + f)\, L}{U_0 + (\gamma v + f)\, L_1}, \tag{21}$$

which is just the result obtained in Ref.[16]. It implies the fact that the model adopted in Ref.[16] is only a special case of the present model so that all the results in Ref.[16] can be directly derived from the model in the present paper. When choosing $L_2 = 0$ and $\gamma = 1$,[15] one has $L_1 = L$, $a = (U_0 + fL)/T_{\rm h}$ and $b = U_0/T_{\rm c}$, so equations (4) and (12)–(15) may be, respectively, reduced to

$$J = \frac{v}{L} = \frac{a^2 T_{\rm h}}{L^2} \frac{{\rm e}^{-a} - {\rm e}^{-b}}{a\left({\rm e}^{-b} - {\rm e}^{-a}\right) + \left(1 - {\rm e}^{-b}\right)\left(1 - {\rm e}^{-a}\right)}, \tag{22}$$

$$\dot{S} = \frac{T_{\rm c}}{L^2}\left(\frac{\alpha}{\sinh\alpha}\right)^2 \left[1 - (\alpha/\sinh\alpha)^2\right] \times \left(U_0^2 X_2^2 - 2U_0 L X_1 X_2 + L^2 X_1^2\right), \tag{23}$$

$$L_{11} = T_{\rm c}\left(\frac{\alpha}{\sinh\alpha}\right)^2 \left[1 - (\alpha/\sinh\alpha)^2\right], \tag{24}$$

$$L_{21} = L_{12} = -\frac{U_0 T_{\rm c}}{L}\left(\frac{\alpha}{\sinh\alpha}\right)^2 \left[1 - (\alpha/\sinh\alpha)^2\right], \tag{25}$$

and

$$L_{22} = \frac{U_0^2 T_{\rm c}}{L^2}\left(\frac{\alpha}{\sinh\alpha}\right)^2 \left[1 - (\alpha/\sinh\alpha)^2\right]. \tag{26}$$

Equation (22) is identical with Eq.(4) in Ref.[15]. However, equations (23)–(26) are different from those obtained in Ref.[15]. Only when $v \ll f$ can the mean heat flows exchanged between the system and the two heat reservoirs be simply expressed as[15] $Q_{\rm h}/t = (U_0/L + f)v$ and $Q_{\rm c}/t = (U_0/L)v$, respectively. In such a case, $1 \gg (\alpha/\sinh\alpha)^2$ and one can directly derive some results obtained in Ref.[15] from Eqs.(21) and (23)–(26), i.e.,

$$\eta = \frac{fL}{U_0 + fL}, \tag{27}$$

$$\dot{S} = \frac{T_{\rm c}}{L^2}\left[\frac{\alpha}{\sinh(\alpha)}\right]^2 \left(U_0^2 X_2^2 - 2U_0 L X_1 X_2 + L^2 X_1^2\right), \tag{28}$$

$$L_{11} = T_{\rm c}\left[\frac{\alpha}{\sinh(\alpha)}\right]^2, \tag{29}$$

$$L_{21} = L_{12} = -\frac{U_0 T_{\rm c}}{L}\left[\frac{\alpha}{\sinh(\alpha)}\right]^2, \tag{30}$$

and

$$L_{22} = \frac{U_0^2 T_{\rm c}}{L^2}\left[\frac{\alpha}{\sinh(\alpha)}\right]^2. \tag{31}$$

The above results show clearly that the model adopted in Ref.[15] is also a special case of the present model and that the conclusions obtained in Ref.[15] are only suitable for some limit cases.

5.2. The case of $v = 0$

When $v = 0$, one can obtain

$$\frac{T_c}{T_h} = \frac{U_0 - fL_2}{U_0 + fL_1} \tag{32}$$

from Eq.(4). By using Eq.(32), equations (19) and (20) may be, respectively, reduced to

$$\eta = 0, \tag{33}$$

and

$$\eta = \frac{W}{Q_h} = \frac{fL\Delta T}{fLT_h + (\Delta T)^2/2}, \tag{34}$$

while equations (21) and (27) becomes the same equation,[15,16] i.e.,

$$\eta = 1 - \frac{T_c}{T_h}. \tag{35}$$

This shows clearly that when the heat leak and/or the kinetic energy change due to the particle motion are considered, the efficiency of the Brownian motor cannot attain the Carnot efficiency even when the Brownian motor is operated under the condition of zero current.

5.3. The linear response regime

When both X_1 and X_2 are very small, the Brownian motor is operated in the linear response regime. Equation (19) may be simplified as

$$\eta = \frac{C_1[u(\beta-1)-(1+l)\lambda]\{C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda\}}{C_1[u(\beta-1)-(1+l)\lambda]\left\{u+\{C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda\}\frac{1}{1+l}+\frac{\beta-1}{2}\right\}+k\Gamma(1+l)^2(\beta-1)}, \tag{36}$$

where $C_1 = C\gamma LT_c$. It can be proved from Eq.(36) that the efficiency is a monotonically decreasing function of k and Γ but a monotonically increasing function of β, as shown by the curves in Figs.3, 6 and 7, respectively. It is also seen from Eq.(36) that when

$$u(\beta-1)-(1+l)\lambda = 0 \tag{37}$$

or

$$C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda = 0, \tag{38}$$

the efficiency is equal to zero. This implies the fact that there exist some optimum values of the parameters l, λ and u at which the efficiency will attain its maximum, as shown by the curves in Figs.4, 5 and 8, respectively. Using Eq.(36), we can calculate the maximum efficiency, η_{max}, and the corresponding parameters, u_{opt}, l_{opt} and λ_{opt}, for given values of k, β and Γ, as listed in Table 1.

Table 1. The values of the parameters u_{opt}, l_{opt}, λ_{opt} and η_{max} for $\beta = 1.2$ and some given values of k and Γ.

k	Γ	u_{opt}	l_{opt}	λ_{opt}	η_{max}
0.03	0.1	2.13	0.90	0.20	0.16
	0.5	2.10	0.67	0.20	0.15
	1.0	1.90	0.38	0.19	0.15
0.3	0.1	2.04	0.45	0.20	0.15
	0.5	1.94	0.10	0.19	0.13
	1.0	1.86	0.09	0.13	0.12

Similarly, when the Brownian motor is operated in the linear response regime, equations (20) and (21) may be, respectively, simplified as

$$\eta = \frac{C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda}{u+\{C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda\}\dfrac{1}{1+l}+\dfrac{\beta-1}{2}}, \tag{39}$$

and

$$\eta = \frac{C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda}{u+\{C_1[u(\beta-1)-(1+l)\lambda]+(1+l)\lambda\}\dfrac{1}{1+l}}. \tag{40}$$

In such a case, the efficiency is a monotonically increasing function of l and λ and independent of the friction coefficient γ, as shown by the solid lines in Figs.4, 5 and 6, respectively. Moreover, it is easily proved that there exists an optimum value of u at which the efficiency will attain its maximum, as shown by the solid line in Fig.8.

6. Conclusions

On the basis of the general model of Brownian motors restricted in a periodic asymmetric piecewise-linear potential and in contact alternately with hot and cold heat reservoirs along the space coordinate, we have systemically analyzed the influence of some main factors such as the energy changes due to particle's motion, external potential, external load, external heat reservoirs, frictional force, heat leak, and so on, on the performance of the modeling system. General expressions for the Onsager coefficients and the generalized efficiency of the system have been analytically derived. Some important curves, which can reveal the inherent characteristics of the Brownian motor, have been plotted. The results obtained here have been directly used to discuss the performance of several different Brownian motors described in current literature. It is also expected that these general results may be put to use in some diverse technological devices that pump and separate efficiently and reliably Brownian particles in the corresponding physical and biological systems.[8,30−32]

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