Vol 18 No 8, August 2009
1674-1056/2009/18(08)/3265-06

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Entanglement sudden death induced by the Dzialoshinskii–Moriya interaction*

Zeng Hong-Fang(曾红芳), Shao Bin(邵 彬)†,
Yang Lin-Guang(杨林广), Li Jian(李 建), and Zou Jian(邹 健)

Department of Physics, School of Sciences, Beijing Institute of Technology, Beijing 100081, *China*

(Received 14 October 2008; revised manuscript received 2 December 2008)

In this paper, we study the entanglement dynamics of two-spin Heisenberg XYZ model with the Dzialoshinskii–Moriya (DM) interaction. The system is initially prepared in the Werner state. The effects of purity of the initial state and DM coupling parameter on the evolution of entanglement are investigated. The necessary and sufficient condition for the appearance of the entanglement sudden death (ESD) phenomenon has been deduced. The result shows that the ESD always occurs if the initial state is sufficiently impure for the given coupling parameter or the DM interaction is sufficiently strong for the given initial state. Moreover, the critical values of them are calculated.

Keywords: entanglement sudden death, concurrence, purity, Dzialoshinskii-Moriya interaction
PACC: 0367, 7510J

1. Introduction

Quantum entanglement is one of the fascinating features of quantum mechanics which is extremely different from classical physics, and has been widely used as an essential resource in quantum information processing (QIP)[1] such as quantum cryptography,[2] teleportation[3] and superdense coding,[4] etc. In recent years, efforts to characterize qualitatively and quantitatively entanglement properties and applied them in quantum communication and information have been ongoing. However, entanglement in multiparty quantum systems may have deteriorated due to the interaction with the environment or the interaction between the sub-systems. It is one of the main obstacles for realizing QIP. Besides the application to QIP realizations, a deeper understanding of disentanglement is also significant to quantum fundamentals. Therefore, the loss of entanglement is an important subject to research.

Recently, Yu and Eberly[5−8] have discovered that the entanglement of a pair of qubits exposed to local noisy environments can vanish completely in a finite time, in striking contrast to the decoherence of an individual qubit which decays only asymptotically. Such a surprising phenomenon was termed entanglement sudden death (ESD). Because of its intrinsic and practical interest, ESD has attracted much attention from both theoretical[9−19] and experimental[20,21] points of view. This surprising phenomenon can appear in many scenarios, such as: cavity-QED,[9−13] trapped ions,[14] and spin chain[15−17] systems, etc. Almeida *et al*[20] have shown that, using an all-optical experimental setup, even though the environment-induced decay of each system is asymptotic, quantum entanglement may suddenly disappear.

As a typical model, the interacting spin model can be used to understand the evolution of entanglement in its different bipartite partitions. A thorough understanding of the dynamical evolution of entanglement of the interacting spin model has obvious implications for the performance of quantum information processing, as well as for our understanding of fundamental quantum mechanics. Therefore, there are numerous studies on the entanglement dynamics of the Heisenberg spin chain.[22−25] In those studies the spin-spin interaction was considered, but the spin-orbit coupling was rarely included. Different from the previous work, in our paper the influence of the Dzialoshinskii–Moriya (DM) interaction (arising from the spin-orbit coupling) on the entanglement of the two-spin model is investigated. The DM interaction is an anisotropic antisymmetric exchange interaction.[26−30] It is important not only for weak ferromagnetism but also

*Project supported by National Natural Science Foundation of China (Grant No 10374007).
†Corresponding author. E-mail: sbin610@bit.edu.cn

for the spin arrangement in antiferromagnets of low symmetry. It is shown that the DM interaction can generate a phase shift with a significant entanglement-enhancing effect, which turns out to be equivalent to the Aharonov–Casher effect as a physical mechanism which causes a phase shift in a spin chain.[31]

In this paper, we investigate the time evolution of entanglement for two-spin XYZ chain with DM interaction. The ESD phenomenon would appear for some given initial state. The result shows that the ESD always occurs if the initial state is sufficiently impure or the DM interaction is sufficiently strong. The critical values are calculated. The paper is organized as follows. In Section 2, we first review the description of the XYZ spin model with DM interaction and calculate the time evolution operator of the system by the form of the Hamiltonian. In Section 3, the dynamic evolution of entanglement is discussed in detail. Lastly, a conclusion is in Section 4.

2. Model

The Hamiltonian for an anisotropic N-qubit XYZ Heisenberg chain with Dzialoshinskii–Moriya interaction can be written as

$$H = \frac{1}{2}\sum_{i=0}^{N-1}[J_x\sigma_i^x\sigma_{i+1}^x + J_y\sigma_i^y\sigma_{i+1}^y + J_z\sigma_i^z\sigma_{i+1}^z + \boldsymbol{D}\cdot(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_{i+1})], \quad (1)$$

where $\boldsymbol{\sigma}_i = (\sigma_i^x, \sigma_i^y, \sigma_i^z)$ is the vector Pauli operator of the i-th spin. The parameter J_α $(\alpha = x, y, z)$ characterizes the strength of the exchange interaction. It is well known that the XYZ chain is the most generalized Heisenberg interaction chain, and it encompasses the other well-known spin models. For example: it turns into an Ising model for $J_x = J_y = 0$, anm XY model for $J_z = 0$, an XXZ model for $J_x = J_y$ and an isotropic Heisenberg model for $J_x = J_y = J_z$ and so on. The vector $\boldsymbol{D}$ is the DM vector measuring the strength of the DM interaction.

Now, we analyze the two-spin case. The two-spin system is not only essential to the construction of the universal quantum gate, but also the most fundamental case on which further extensions can be developed. For simplicity, we shall choose a DM vector aligned along the z-direction, i.e. $\boldsymbol{D} = D\boldsymbol{z}$. By introducing the spin raising and lowering operators $\sigma_i^{\pm} = \frac{1}{2}(\sigma_i^x \pm \mathrm{i}\sigma_i^y)$, the Hamiltonian of the system can be written as

$$H = (J+\mathrm{i}D)\sigma_1^+\sigma_2^- + (J-\mathrm{i}D)\sigma_1^-\sigma_2^+ + \Delta(\sigma_1^+\sigma_2^+ + \sigma_1^-\sigma_2^-) + J_z\sigma_1^z\sigma_2^z. \quad (2)$$

Here the parameter $J = (J_x + J_y)/2$ characterizes the average strength of exchange interaction in the XY plane, and $\Delta = (J_x - J_y)/2$ measures the anisotropy in the XY plane.

In the space spanned by $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$ ($|\downarrow\rangle$ and $|\uparrow\rangle$ denote the states of spin-down and spin-up respectively), the Hamiltonian Eq.(2) can be expressed as

$$H = \begin{pmatrix} \frac{J_z}{2} & 0 & 0 & \Delta \\ 0 & -\frac{J_z}{2} & J+\mathrm{i}D & 0 \\ 0 & J-\mathrm{i}D & -\frac{J_z}{2} & 0 \\ \Delta & 0 & 0 & \frac{J_z}{2} \end{pmatrix}. \quad (3)$$

The time evolution operator $U(t) = \exp(-\mathrm{i}Ht)$ is expressed as follows:

$$U(t) = \begin{pmatrix} K & 0 & 0 & L \\ 0 & M & N\mathrm{e}^{\mathrm{i}\theta} & 0 \\ 0 & N\mathrm{e}^{-\mathrm{i}\theta} & M & 0 \\ L & 0 & 0 & K \end{pmatrix}, \quad (4)$$

where

$$K = \cos\Delta t\ \mathrm{e}^{-\mathrm{i}J_z t/2}, \qquad L = -\mathrm{i}\sin\Delta t\ \mathrm{e}^{-\mathrm{i}J_z t/2},$$
$$M = \cos\nu t\ \mathrm{e}^{\mathrm{i}J_z t/2}, \qquad N = -\mathrm{i}\sin\nu t\ \mathrm{e}^{\mathrm{i}J_z t/2}$$

with $\nu = \sqrt{J^2 + D^2}$, and $\mathrm{e}^{\mathrm{i}\theta} = \dfrac{J+\mathrm{i}D}{\nu}$.

3. The dynamics evolution of entanglement

Before discussing the dynamics of the entanglement, it is necessary to review the measurement of the entanglement. We regard that the concurrence defined by William K. Wootters[32] is by far the most convenient method to measure the entanglement for the two-particle system. The concurrence is defined as: $C = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, and the numerical values λ_1, λ_2, λ_3, λ_4 $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4)$ are the square roots of the eigenvalues of the matrix product $R = \rho(\sigma_1^y \otimes \sigma_2^y)\rho^*(\sigma_1^y \otimes \sigma_2^y)$, ρ is the density operator of the initial state. The concurrence $C = 0$ corresponds to a separable state while $C = 1$ corresponds to maximally entangled state.

The density operator for the total system at time t is $\rho(t) = U(t)\rho(0)U^{\dagger}(t)$. Here we shall choose the initial state as the well-known Werner state:

$$\rho(0) = \frac{1-r}{4}I + r|\varphi\rangle\langle\varphi|,$$

where $|\varphi\rangle = \frac{1}{\sqrt{2}}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)$ is the Bell state. The entanglement in the initial state is measured by the purity r, $r > \frac{1}{3}$ corresponds to the entangled state and $r \leq \frac{1}{3}$ corresponds to the separable state. Then the density matrix for the system at time t takes the form

$$\rho(t) = \begin{pmatrix} x & 0 & 0 & 0 \\ 0 & y & u & 0 \\ 0 & u^* & z & 0 \\ 0 & 0 & 0 & x \end{pmatrix}, \tag{5}$$

where

$$\begin{aligned} x &= \frac{1-r}{4}, \\ y &= \frac{1-r}{4} + \frac{r}{2}(1 + \sin\theta \sin 2\nu t), \\ z &= \frac{1-r}{4} + \frac{r}{2}(1 - \sin\theta \sin 2\nu t), \\ u &= \frac{r}{2}(\cos^2 \nu t - \sin^2\theta \sin^2 \nu t + \cos^2\theta \sin^2 \nu t) \\ &\quad + \mathrm{i}r \cos\theta \sin^2 \nu t \sin\theta. \end{aligned}$$

The state of the system belongs to the so-called X-class states[33] whose density matrix is of the form

$$\rho(t) = \begin{pmatrix} x_1 & 0 & 0 & x_2 \\ 0 & x_3 & x_4 & 0 \\ 0 & x_4^* & x_5 & 0 \\ x_2^* & 0 & 0 & x_6 \end{pmatrix}. \tag{6}$$

The X-class states have the property that the corresponding two-spin density matrix preserves the X-form during the system evolution. For X-states (6), the concurrence can be derived as $C(\rho(t)) = 2\max\{0, |x_4| - \sqrt{x_1 x_6}, |x_2| - \sqrt{x_3 x_5}\}$. Therefore, the analytic expression of the concurrence of the considered system is

$$\begin{aligned} &C(\rho(t)) \\ &= \max\left\{0, r\sqrt{1 - (\sin 2\nu t \sin\theta)^2} - \frac{1-r}{2}\right\}. \end{aligned} \tag{7}$$

From the above equation, it is easy to see that the concurrence of $\rho(t)$ is periodically fluctuating with the time t. The fluctuating period $T = \pi/\sqrt{J^2 + D^2}$.

We plot the concurrence as a function of D and t for $r = 0.5$ and $J = 1$ in Fig.1. Figure 1 shows that entanglement can fall abruptly to zero, and will remain zero for a period of time before entanglement recovers. The length of the time interval for zero entanglement depends on the strength of the DM interaction.

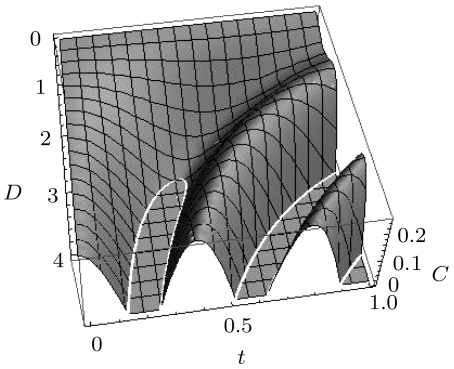


Fig.1. The concurrence for $\rho(t)$ versus the DM interaction parameter D and t. The other parameters for the plot are $r = 0.5$ and $J = 1$.

Another interesting feature of Fig.1 is that there exists a critical value D_c, under which the ESD does not happen, and above which we could observe the ESD phenomenon obviously.

The concurrence C versus t for the different values of D is illustrated in Fig.2. From Fig.2, we can see that the concurrence remains constant when $D = 0$. When $0 < D < D_c$, the concurrence would fluctuating with time t but there is no ESD phenomenon. When $D \geq D_c$, we can observe the distinct ESD.

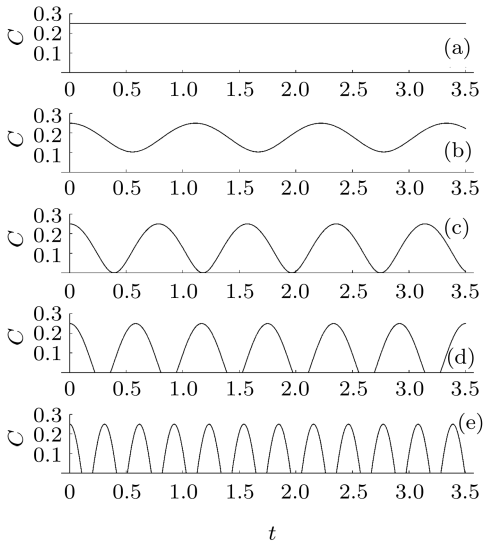


Fig.2. The concurrence versus t for different values of D. The figure (a) corresponds to $D = 0$, (b) corresponds to $D = 1$, (c) corresponds to $D = 1.732$, (d) corresponds to $D = 2.5$, and (e) corresponds to $D = 5$ respectively. The other parameters are the same as those in Fig.1.

As a fact, the density matrix for the system can

be expressed as the form

$$\rho(t) = \frac{1-r}{4} I + r|\varphi'\rangle\langle\varphi'|, \tag{8}$$

where

$$\begin{aligned} &|\varphi'\rangle \\ &= \frac{1}{\sqrt{2}}[(\cos\nu t - \sin\theta \sin\nu t - \mathrm{i}\cos\theta\sin\nu t)|\downarrow\uparrow\rangle \\ &+ (\cos\nu t + \sin\theta\sin\nu t - \mathrm{i}\cos\theta\sin\nu t)|\uparrow\downarrow\rangle]. \end{aligned} \tag{9}$$

If $D = 0$, i.e. $\theta = 0$,

$$|\varphi'\rangle = \frac{1}{\sqrt{2}}\mathrm{e}^{-\mathrm{i}\nu t}(|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle).$$

There is no difference except the phase factor between the $|\varphi'\rangle$ and the Bell-state $|\varphi\rangle$. So the density matrix would not bring up any changes with the time evolution. Therefore, the entanglement is maintained at a steady value all the time in this case.

When $D \neq 0$, the maximal entangled state $|\varphi\rangle$ would evolve to the no-maximal entangled state $|\varphi'\rangle$. So the concurrence would change with the time t. The result indicates that the DM interaction would alter the degree of the entanglement of the two-spin with the time evolution. With the increase of the interaction, the concurrence fluctuates with the evolution of the time t, but it cannot decrease to zero until the $D \geq D_c$. By analyzing the analytic expression of the concurrence, the necessary and sufficient condition for the ESD is

$$r(1 + 2\cos\theta) \leq 1. \tag{10}$$

For the given purity r of the initial state, the ESD occurs if and only if

$$D \geq D_c = \frac{J\sqrt{3r^2 + 2r - 1}}{1 - r}. \tag{11}$$

From the above equation, we can see that the critical value D_c is proportional to the exchange coupling parameter J. The result shows that there are competitions between the DM interaction and the exchange interaction in the processing of disentanglement. The DM interaction tries to decrease the entanglement, but the exchange interaction counteracts this processing.

According to Eq.(10), we plot the D_c as a function of r in Fig.3. Obviously, as r increases, the DM interaction should be enhanced if there is an ESD phenomenon. We know that the larger the value of r, the bigger the degree of the entanglement of the initial state. In other words, the more entanglement at the beginning, the stronger the coupling of the DM interaction inducing the ESD.

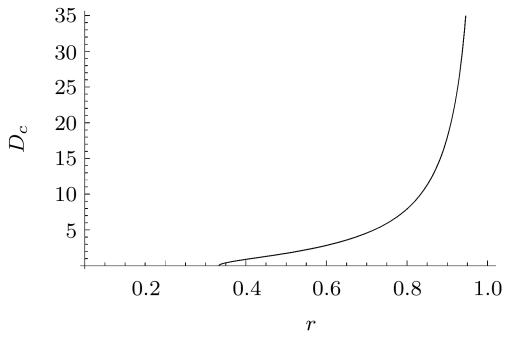


Fig.3. The critical value D_c versus the initial state parameter r. For this plot the other parameters are the same as those in Fig.1.

In a similar way, we could present the schematic curves of C as a function of r and t. As seen in Fig.4 ($D = 5$), it is interesting that the concurrence fluctuates standardly periodically with the time evolution. The ESD appears only in the region $r_0 < r \leq r_c$, where $r_0 = 1/3$ is the boundary of the entangled initial state and unentangled state. The critical value r_c describes the upper limit of entanglement of the initial state which ESD could appear for the given D. Under the same D, the duration of the ESD would be reduced with the increase in r within the considered region.

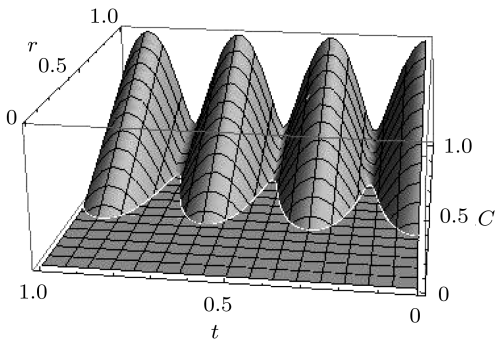


Fig.4. The concurrence for $\rho(t)$ versus the purity r of the initial state and t. The other parameters for the plot are $D = 5$ and $J = 1$.

In order to know the evolution of entanglement for a given value of D, we plot the schematic curves of the concurrence C versus t for the different values of r in Fig.5 ($D = 5$). Figures 5(a), 5(b) and 5(c) illustrate the process from the ESD appearing to vanishing with the increase of r.

We can also obtain the condition for the ESD for

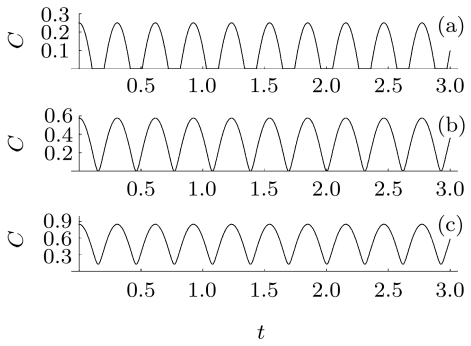


Fig.5. The concurrence versus t for different purity r of the initial state. The figure (a) corresponds to $r = 0.5$, (b) corresponds to $r = 0.7179$, (c) corresponds to $r = 0.9$. The other parameter values for the plot are the same as those in Fig.4.

the given parameter of the DM interaction

$$r \leq r_c = 1 - \frac{2}{2 + \sqrt{1 + \left(\frac{D}{J}\right)^2}}. \qquad (12)$$

The relation between D/J and r_c is plotted in Fig.6.

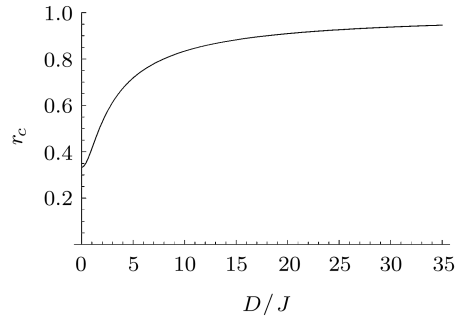


Fig.6. The critical value r_c versus the ratio of the strength of DM interaction D to the exchange interaction parameter J. For this plot we have chosen the other parameters as the same as those in Fig.4.

It is seen that r_c monotonously increases with the parameter D/J. That is to say, the range of the initial state at which the ESD happens would be larger with the increase of the D/J.

4. Conclusion

In this paper, we have studied the dynamics of the entanglement of a two-spin XYZ system under DM interaction, and have investigated the condition of the so-called ESD. Namely, the entanglement of the two-spin will suffer a sudden death if we choose a special initial state. The results demonstrate that the DM interaction may have some disentanglement effects. It is shown that there are competitions between the DM interaction and the exchange interaction in the processing of disentanglement. The DM interaction tries to decrease the entanglement, but exchange interaction counteracts this processing. For the phenomenon of the ESD we have deduced the necessary and sufficient condition for the appearance and plot the tendency of the critical value with the evolvement of the other parameters and have analyzed the mechanism of the conclusions. Therefore, in quantum information processing and quantum computing, in order to prevent the two-spin entanglement from sudden death, one could deal with proper environments by changing the initial states. It maybe useful for the study of the quantum channel and the quantum information transmission by grasping the knowledge of the control of the DM interaction.

References

[1] Nielsen M A and Chuang I L 2000 *Quantum Computation and Quantum Information* (Cambridge: Cambridge University Press) (in UK)
[2] Ekert A K 1991 *Phys. Rev. Lett.* **67** 661
[3] Bennett C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895
[4] Bennett C H and Wiesner S J 1992 *Phys. Rev. Lett.* **69** 2881
[5] Yu T and Eberly J H 2004 *Phys. Rev. Lett.* **93** 140404
[6] Yu T and Eberly J H 2006 *Phys. Rev. Lett.* **97** 140403
[7] Yu T and Eberly J H 2006 *Opt. Commun.* **264** 393
[8] Eberly J H and Yu T 2007 *Science* **316** 555
[9] Man Z X, Xia Y J and An N B 2008 *J. Phys.* B **41** 085503
[10] Man Z X, Xia Y J and An N B 2008 *J. Phys.* B **41** 155501
[11] Zheng Q, Zhang X P and Ren Z Z 2008 *Chin. Phys.* B **17** 3553
[12] Guo J L and Song H S 2008 *J. Phys.* A **41** 085302
[13] López C E, Romero G, Lastra F, Solano E and Retamal J C 2008 *Phys. Rev. Lett.* **101** 080503
[14] Abdel-Aty M and Moya-Cessa H 2007 *Phys. Lett.* A **369** 372
[15] Wang Z M, Shao B and Zou J 2007 *Chin. Phys. Lett.* **24** 855
[16] Cui H T, Li K and Yi X X 2007 *Phys. Lett.* A **365** 44
[17] Su X Q, Wang A M, Ma X S and Qiu L 2007 *J. Phys.* A **40** 11385
[18] Zhang G F 2007 *Chin. Phys.* **16** 1855
[19] Man Z X and Xia Y J 2008 *Chin. Phys.* B **17** 3198
[20] Almeida M P, de Melo F, Hor-Meyll M, Salles A, Walborn S P, Souto Ribeiro P H and Davidovich L 2007 *Science* **316** 579

[21] Salles A, de Melo F, Almeida M P, Hor-Meyll M, Walborn S P, Souto Ribeiro P H and Davidovich L 2008 *Phys. Lett.* A **78** 022322

[22] Yeo Y, Liu T, Lu Y E and Yang Q Z 2005 *J. Phys.* A **38** 3235

[23] Shao B, Zeng T H and Zou J 2005 *Commun. Theor. Phys.* **44** 255

[24] Abliz A, Gao H J, Xie X C, Wu Y S and Liu W M 2006 *Phys. Rev.* A **74** 052105

[25] Kargarian M, Jafari R, and Langari A 2008 *Phys. Rev.* A **77** 032346

[26] Moriya T 1960 *Phys. Rev. Lett.* **4** 228

[27] Dzyaloshinsky I 1958 *J. Phys. Chem. Solids* **4** 241

[28] Moriya T 1960 *Phys. Rev.* **120** 91

[29] Cai Z, Lu W B and Liu Y J 2008 *Acta Phys. Sin.* **57** 7267 (in Chinese)

[30] Shan C J, Cheng W W, Liu T K, Huang Y X and Li H 2008 *Acta Phys. Sin.* **57** 2687 (in Chinese)

[31] Maruyama K, Iitaka T and Nori F 2007 *Phys. Rev.* A **75** 012325

[32] Wootters W K 1998 *Phys. Rev. Lett.* **80** 2245

[33] Yu T and Eberly J H 2007 *Quantum Inf. Comput.* **7** 459