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Three-qubit Fredkin gate based on cavity quantum electrodynamics*

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This paper presents a scheme for implementing a Fredkin gate on three modes of a cavity. The scheme is based on the dispersive atom-cavity interaction. By modulating the cavity frequency and the atomic transition frequency appropriately, it obtains the effective form of nonlinear interaction between photons in the three-mode cavity. This availability is testified via numerical analysis. It also considers both the situations with and without dissipation.

Keywords: Fredkin gate, cavity quantum electrodynamics
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The quantum computer first proposed by Deutsch in 1985[1] is a kind of information processor which is based on the principle of quantum mechanics. It would perform certain tasks faster than a classical computer, such as factoring a number and searching for data in an array.[2,3] This has triggered in the past years many studies on the theoretical and the practical aspects of quantum computing. A quantum logic gate, as a basic element of a quantum computer, has been simulated with different physical models in recent years such as an ion-trap system,[4,5] cavity quantum electrodynamics (QED),[6−8] the nuclear magnetic resonance (NMR) system[9] and linear optics.[10]

In many physical carriers of qubit, photon is best for quantum information processes due to its weak interaction with the environment. Thus, linear optics is applied to operate photons directly through a series of optical devices. However, if a multiqubit gate is needed, we must use the weak nonlinear Kerr medium to achieve the goal. The developing technology of microwaves and optical cavities provides a way to coherently exchange quantum information between atoms (or ions) and photons, so cavity QED can be considered as an ideal system to implement quantum information processing. Up to now, many schemes for realizing a two-qubit quantum gate with photon as the qubit have been advanced.[11−15] A multiqubit gate can be decomposed into a conditional z gate and a one-qubit gate,[16] but the procedure will get more complicated as the number of qubits increases. Hence, researchers are trying to find effective methods to directly implement a multiqubit gate. There are two important kinds of three-qubit gate known as the Toffoli gate (controlled-controlled-not gate)[17] and the Fredkin gate (controlled-swap gate)[18] which have given a simple implementation of a quantum computer to solve Grover's algorithm[19] and Deutsch's problem,[20] respectively. The three-atom-qubit Toffoli gate and the multi-atom-qubit Toffoli gate have been realized in cavity QED.[21,22] The Fredkin gate has also been realized in an ion trap system[23] and a linear optical system.[24−26] Very recently, Wang *et al* have proposed a scheme for implementing a controlled SWAP gate with N target qubits in cavity QED.[27] In their scheme, two sequences of photon pulses have been swapped after transferring a single atom cavity. Nevertheless, the experimental realization of a typical quantum algorithm may require each qubit to be treated equally. In 2008, Chang *et al* put forward an efficient scheme for realizing a three-qubit phase gate which is implemented by passing a four-level atom in a cascade configuration initially in its ground state through a three-mode optical cavity.[28] As far as we know, no scheme has been presented for implementing the three-photon-qubit Fredkin gate in a cavity QED system. In 2005, Serra *et al* proposed a scheme for realizing frequency up- and down-conversions in two-mode cavity QED via atom-field large detuning

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interaction.[29] After one year, Zou *et al* improved the scheme by atom-field resonant interaction,[30] which reduced the operation time greatly. Enlightened by the above works, we construct a theoretical model for implementing a three-qubit Fredkin gate in a three-mode cavity QED system.

This paper is organized as follows. We first propose a scheme to implement a three-qubit Fredkin gate, then we investigate the effect of decoherence on the fidelity of the three-qubit Fredkin gate. Finally, we discuss the experimental feasibility of the scheme and give a conclusion.

A three-qubit quantum Fredkin gate can be defined by the operator

$$\begin{aligned} U_{\text{Fredkin}} = & |0_c,0_a,0_b\rangle\langle 0_c,0_a,0_b| \\ & + |0_c,0_a,1_b\rangle\langle 0_c,0_a,1_b| \\ & + |0_c,1_a,0_b\rangle\langle 0_c,1_a,0_b| \\ & + |0_c,1_a,1_b\rangle\langle 0_c,1_a,1_b| \\ & + |1_c,0_a,0_b\rangle\langle 1_c,0_a,0_b| \\ & + |1_c,1_a,0_b\rangle\langle 1_c,0_a,1_b| \\ & + |1_c,0_a,1_b\rangle\langle 1_c,1_a,0_b| \\ & + |1_c,1_a,1_b\rangle\langle 1_c,1_a,1_b|. \end{aligned} \tag{1}$$

This quantum gate can be explained as a controlled-swap gate because the target qubits will exchange their information when the control qubit is setted at logic $|1_c\rangle$. In the following we mainly discuss a cavity QED implementation of the case.

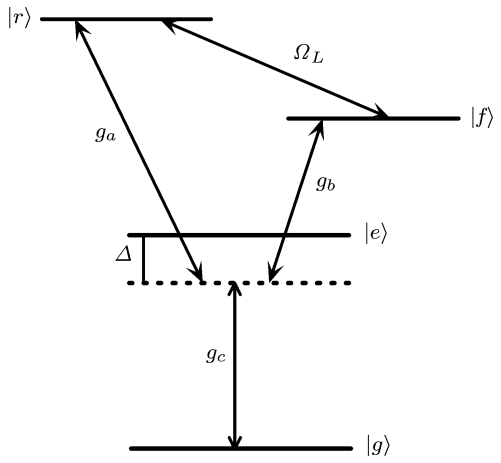


Fig.1. Atomic level structure of a four-level atom for obtaining effective interaction.

We consider a Y type four-level atom interaction with a three-mode cavity field. The atomic states are denoted by $|g\rangle$, $|e\rangle$, $|f\rangle$ and $|r\rangle$. As illustrated in Fig.1, the transitions $|g\rangle \leftrightarrow |e\rangle$, $|e\rangle \leftrightarrow |r\rangle$, and $|e\rangle \leftrightarrow |f\rangle$ are coupled to the cavity c mode, a mode and b mode, respectively. The dipole-forbidden transition $|f\rangle \leftrightarrow |r\rangle$ is driven by a classical field with frequency ω_L and amplitude Ω_L. The Hamiltonian (assuming $\hbar = 1$) of the system is

$$\begin{aligned} H &= H_1 + H_2 + H_3, \\ H_1 &= \omega_g|g\rangle\langle g| + \omega_e|e\rangle\langle e| + \omega_f|f\rangle\langle f| + \omega_r|r\rangle\langle r| \\ &\quad + \omega_a a^\dagger a + \omega_b b^\dagger b + \omega_c c^\dagger c, \\ H_2 &= g_a(a|r\rangle\langle e| + |e\rangle\langle r|a^\dagger) \\ &\quad + g_b(b|f\rangle\langle e| + |e\rangle\langle f|b^\dagger) \\ &\quad + g_c(c|e\rangle\langle g| + |g\rangle\langle e|c^\dagger), \\ H_3 &= \Omega(\mathrm{e}^{\mathrm{i}\,\omega_L t}|f\rangle\langle r| + |r\rangle\langle f|\mathrm{e}^{-\mathrm{i}\,\omega_L t}), \end{aligned} \tag{2}$$

where a (b, c) and $a^\dagger$ $(b^\dagger,\ c^\dagger)$ are the annihilation and creation operators of cavity a $(b,\ c)$ mode, respectively. g_a, g_b and g_c are the coupling constants of the interaction of an atom with the cavity a mode, b mode and c mode, respectively. In this paper, we take the values of g_a, g_b, g_c and Ω_L to be real. In the interaction picture, the interaction Hamiltonian can be written as

$$\begin{aligned} H_{\text{I}} &= g_a(a|r\rangle\langle e|\mathrm{e}^{-\mathrm{i}\,\Delta t} + \mathrm{e}^{\mathrm{i}\,\Delta t}|e\rangle\langle r|a^\dagger) \\ &\quad + g_b(b|f\rangle\langle e|\mathrm{e}^{-\mathrm{i}\,\Delta t} + \mathrm{e}^{\mathrm{i}\,\Delta t}|e\rangle\langle f|b^\dagger) \\ &\quad + g_c(c|e\rangle\langle g|\mathrm{e}^{\mathrm{i}\,\Delta t} + \mathrm{e}^{-\mathrm{i}\,\Delta t}|g\rangle\langle e|c^\dagger) \\ &\quad + \Omega(|f\rangle\langle r| + |r\rangle\langle f|), \end{aligned} \tag{3}$$

where $\Delta = \omega_e - \omega_g - \omega_c = \omega_a - (\omega_r - \omega_e) = \omega_b - (\omega_f - \omega_e)$. In order to eliminate the time-dependent term in Eq.(3), we move into a rotating frame (through the unitary transformation $\exp\left(\mathrm{i}\,\Delta|e\rangle\langle e|t\right)$) and the new Hamiltonian reads

$$\begin{aligned} H'_{\text{I}} &= g_a(a|r\rangle\langle e| + |e\rangle\langle r|a^\dagger) \\ &\quad + g_b(b|f\rangle\langle e| + |e\rangle\langle f|b^\dagger) \\ &\quad + g_c(c|e\rangle\langle g| + |g\rangle\langle e|c^\dagger) \\ &\quad + \Omega(|f\rangle\langle r| + |r\rangle\langle f|) + \Delta|e\rangle\langle e|. \end{aligned} \tag{4}$$

The quantum information is encoded in a subspace spanned by the states $\{|0_a\rangle, |1_a\rangle, |0_b\rangle, |1_b\rangle, |0_c\rangle, |1_c\rangle\}$. In our system, if the atom is in state $|g\rangle$, the states including $|0_c\rangle|0_a\rangle|0_b\rangle$, $|0_c\rangle|0_a\rangle|1_b\rangle$, $|0_c\rangle|1_a\rangle|0_b\rangle$ and $|0_c\rangle|1_a\rangle|1_b\rangle$ remain unchanged in the evolution since $H'_{\text{I}}|0_c, i_a, j_b\rangle = 0$ $(i, j = 0, 1)$. Then our object is to calculate the evolutions of the other four states containing the logic quatum state $|1_c\rangle$.

We first assume that the state for the system at

time t is

$$\begin{aligned}|\Psi(t)\rangle = {} & C_a(t)|g,0_a,1_b,1_c\rangle + C_b(t)|e,0_a,1_b,0_c\rangle \\ & + C_c(t)|f,0_a,0_b,0_c\rangle \\ & + C_d(t)|r,0_a,0_b,0_c\rangle + C_e(t)|e,1_a,0_b,0_c\rangle \\ & + C_f(t)|g,1_a,0_b,1_c\rangle. \end{aligned} \tag{5}$$

The probability amplitudes in Eq.(5) are governed by the Schrödinger equation:

$$\mathrm{i}\frac{\partial|\Psi(t)\rangle}{\partial t} = H_{\mathrm{I}}'|\Psi(t)\rangle, \tag{6}$$

and we obtain the following equations

$$\mathrm{i}\dot{C}_a = g_c C_b, \tag{7}$$

$$\mathrm{i}\dot{C}_b = g_c C_a + g_b C_c + \Delta C_b, \tag{8}$$

$$\mathrm{i}\dot{C}_c = g_b C_b + \Omega C_d, \tag{9}$$

$$\mathrm{i}\dot{C}_d = g_a C_e + \Omega C_c, \tag{10}$$

$$\mathrm{i}\dot{C}_e = g_a C_d + g_c C_f + \Delta C_e, \tag{11}$$

$$\mathrm{i}\dot{C}_f = g_c C_e. \tag{12}$$

We assume that

$$\dot{C}_b = \dot{C}_c = \dot{C}_d = \dot{C}_e = 0, \tag{13}$$

which can be considered that there is no transition to the states $|e,0_a,1_b,0_c\rangle$, $|f,0_a,0_b,0_c\rangle$, $|r,0_a,0_b,0_c\rangle$ and $|e,1_a,0_b,0_c\rangle$. We can then solve for C_b, C_c, C_d, and C_e from Eqs.(8)–(11) and substitute the resulting solutions into Eqs.(7) and (12), the obtained results are

$$\begin{aligned}\mathrm{i}\dot{C}_a = {} & -\frac{g_c^2}{\Delta}C_a - \frac{g_a^2 g_b^2 g_c^2}{\Delta^3\Omega^2\left(1-\dfrac{g_a^2 g_b^2}{\Delta^2\Omega^2}\right)}C_a \\ & - \frac{g_a g_b g_c^2}{\Delta^2\Omega\left(1-\dfrac{g_a^2 g_b^2}{\Delta^2\Omega^2}\right)}C_f, \\ \mathrm{i}\dot{C}_f = {} & -\frac{g_c^2}{\Delta}C_f - \frac{g_a^2 g_b^2 g_c^2}{\Delta^3\Omega^2\left(1-\dfrac{g_a^2 g_b^2}{\Delta^2\Omega^2}\right)}C_f \\ & - \frac{g_a g_b g_c^2}{\Delta^2\Omega\left(1-\dfrac{g_a^2 g_b^2}{\Delta^2\Omega^2}\right)}C_a. \end{aligned} \tag{14}$$

In the large detuning condition $\Delta \gg \{g_a, g_b, g_c, \Omega\}$, the subsystem evolution equations are equivalent to the case that we have the following Hamiltonian in the subspace $\{|g,0_a,1_b,1_c\rangle, |g,1_a,0_b,1_c\rangle\}$:

$$\begin{aligned}H_{\mathrm{eff}} = {} & -\frac{g_c^2}{\Delta}(|g,0_a,1_b,1_c\rangle\langle g,0_a,1_b,1_c| \\ & + |g,1_a,0_b,1_c\rangle\langle g,1_a,0_b,1_c|) \\ & - \frac{g_a g_b g_c^2}{\Delta^2\Omega}(|g,0_a,1_b,1_c\rangle\langle g,1_a,0_b,1_c| \\ & + |g,1_a,0_b,1_c\rangle\langle g,0_a,1_b,1_c|). \end{aligned} \tag{15}$$

We then assume that the state for the system at time t is

$$\begin{aligned}|\Psi'(t)\rangle = {} & C_a'(t)|g,1_a,1_b,1_c\rangle + C_b'(t)|e,1_a,1_b,0_c\rangle \\ & + C_c'(t)|r,0_a,1_b,0_c\rangle \\ & + C_d'(t)|f,1_a,0_b,0_c\rangle + C_e'(t)|f,0_a,1_b,0_c\rangle \\ & + C_f'(t)|r,1_a,0_b,0_c\rangle \\ & + C_g'(t)|e,0_a,2_b,0_c\rangle + C_h'(t)|e,2_a,0_b,0_c\rangle \\ & + C_i'(t)|g,0_a,2_b,1_c\rangle \\ & + C_j'(t)|g,2_a,0_b,1_c\rangle. \end{aligned} \tag{16}$$

It follows that the probability amplitudes satisfy the following equations of motion:

$$\mathrm{i}\dot{C}_a' = g_c C_b', \tag{17}$$

$$\mathrm{i}\dot{C}_b' = g_c C_a' + g_a C_c' + g_b C_d' + \Delta C_b', \tag{18}$$

$$\mathrm{i}\dot{C}_c' = g_a C_b' + \Omega C_e', \tag{19}$$

$$\mathrm{i}\dot{C}_d' = g_b C_b' + \Omega C_f', \tag{20}$$

$$\mathrm{i}\dot{C}_e' = \sqrt{2}g_b C_g' + \Omega C_c', \tag{21}$$

$$\mathrm{i}\dot{C}_f' = \sqrt{2}g_a C_h' + \Omega C_d', \tag{22}$$

$$\mathrm{i}\dot{C}_g' = \sqrt{2}g_b C_e' + g_c C_i' + \Delta C_g', \tag{23}$$

$$\mathrm{i}\dot{C}_h' = \sqrt{2}g_a C_f' + g_c C_j' + \Delta C_h', \tag{24}$$

$$\mathrm{i}\dot{C}_i' = g_c C_g', \tag{25}$$

$$\mathrm{i}\dot{C}_j' = g_c C_h'. \tag{26}$$

We set

$$\dot{C}_b' = \dot{C}_c' = \dot{C}_d' = \dot{C}_e' = \dot{C}_f' = \dot{C}_g' = \dot{C}_h' = 0, \tag{27}$$

in order to guarantee that there is no transition to the states $|e,1_a,1_b,0_c\rangle$, $|r,0_a,1_b,0_c\rangle$, $|f,1_a,0_b,0_c\rangle$, $|f,0_a,1_b,0_c\rangle$, $|r,1_a,0_b,0_c\rangle$, $|e,0_a,2_b,0_c\rangle$ and $|e,2_a,0_b,0_c\rangle$. We can then solve for C_b', C_c', C_d', C_e', C_f', C_g' and C_h' from Eqs.(18)–(24) and substitute the resulting solutions into Eqs.(17), (25) and (26), we obtain

$$\mathrm{i}\dot{C}_a' = -\frac{\sqrt{2}g_a g_b g_c^2(C_i'+C_j')}{\Delta^2\Omega\left(1-\dfrac{4g_a^2 g_b^2}{\Delta^2\Omega^2}\right)} - \frac{g_c^2 C_a'}{\Delta\left(1-\dfrac{4g_a^2 g_b^2}{\Delta^2\Omega^2}\right)},$$

$$\mathrm{i}\,\dot{C}'_i = -\frac{\sqrt{2}g_a g_b g_c^2 C'_a}{\Delta^2\Omega\left(1-\frac{4g_a^2g_b^2}{\Delta^2\Omega^2}\right)} - \frac{2g_a^2g_b^2g_c^2(C'_j-C'_i)}{\Delta^3\Omega^2\left(1-\frac{4g_a^2g_b^2}{\Delta^2\Omega^2}\right)} - \frac{g_c^2C'_i}{\Delta\left(1-\frac{4g_a^2g_b^2}{\Delta^2\Omega^2}\right)},$$

$$\mathrm{i}\,\dot{C}'_j = -\frac{\sqrt{2}g_a g_b g_c^2 C'_a}{\Delta^2\Omega\left(1-\frac{4g_a^2g_b^2}{\Delta^2\Omega^2}\right)} - \frac{2g_a^2g_b^2g_c^2(C'_i-C'_j)}{\Delta^3\Omega^2\left(1-\frac{4g_a^2g_b^2}{\Delta^2\Omega^2}\right)} - \frac{g_c^2C'_j}{\Delta\left(1-\frac{4g_a^2g_b^2}{\Delta^2\Omega^2}\right)}. \quad (28)$$

Then, with the same limit $\Delta \gg \{g_a, g_b, g_c, \Omega\}$, the subsystem evolution is governed by the following Hamiltonian in the subspace $\{|g,1_a,1_b,1_c\rangle, |g,2_a,0_b,1_c\rangle, |g,0_a,2_b,1_c\rangle\}$:

$$\begin{aligned} H'_{\mathrm{eff}} = &-\frac{\sqrt{2}g_a g_b g_c^2}{\Delta^2\Omega}\big[|g,1_a,1_b,1_c\rangle(\langle g,0_a,2_b,1_c| \\ &+\langle g,2_a,0_b,1_c|) + \mathrm{H.c.}\big] \\ &-\frac{g_c^2}{\Delta}(|g,1_a,1_b,1_c\rangle\langle g,1_a,1_b,1_c| \\ &+|g,0_a,2_b,1_c\rangle\langle g,0_a,2_b,1_c| \\ &+|g,2_a,0_b,1_c\rangle\langle g,2_a,0_b,1_c|). \end{aligned} \quad (29)$$

Finally, we assume that the state for the system at time t is

$$\begin{aligned} |\Psi''(t)\rangle = &C''_a(t)|g,0_a,0_b,1_c\rangle \\ &+C''_b(t)|e,0_a,0_b,0_c\rangle. \end{aligned} \quad (30)$$

Similar to the above processes, the probability amplitudes satisfy the following equations of motion:

$$\mathrm{i}\,\dot{C}''_a = g_c C''_b, \quad (31)$$

$$\mathrm{i}\,\dot{C}''_b = g_c C''_a + \Delta C''_b. \quad (32)$$

Setting $\dot{C}''_b = 0$, we acquire

$$\mathrm{i}\,\dot{C}''_a = -\frac{g_c^2}{\Delta}C''_a. \quad (33)$$

Under the condition $\Delta \gg \{g_a, g_b, g_c, \Omega\}$, the evolution of the subsystem is equivalent to the Hamiltonian in the subspace $\{|g,0_a,0_b,1_c\rangle\}$:

$$H''_{\mathrm{eff}} = -\frac{g_c^2}{\Delta}|g,0_a,0_b,1_c\rangle\langle g,0_a,0_b,1_c|. \quad (34)$$

Now, we can conclude that if the atom stays in the state $|g\rangle$, an effective Hamiltonian through Eqs.(15), (29), and (34) is obtained as follows:

$$H_{\mathrm{fnl}} = -\frac{g_c^2}{\Delta}c^\dagger c - \frac{g_a g_b g_c^2}{\Delta^2\Omega}c^\dagger c(a^\dagger b + a b^\dagger), \quad (35)$$

through the operator $\exp(\mathrm{i}\,g_c^2 c^\dagger c t/\Delta)$, we obtain

$$H_{\mathrm{Fredkin}} = -\frac{g_a g_b g_c^2}{\Delta^2\Omega}c^\dagger c(a^\dagger b + a b^\dagger). \quad (36)$$

After selecting the interaction time $\frac{g_a g_b g_c^2}{\Delta^2\Omega}t = \pi/2$, we can realize the Fredkin gate on the three modes of the cavity:

$$F = \begin{pmatrix} 1&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0 \\ 0&0&0&0&\alpha&0&0&0 \\ 0&0&0&0&0&0&\beta&0 \\ 0&0&0&0&0&\beta&0&0 \\ 0&0&0&0&0&0&0&\theta \end{pmatrix} \quad (37)$$

with $\alpha = 1$, β=i and $\theta = -1$. The form of Eq.(37) is similar to Ref.[23]. Note that in comparison with the genuine Fredkin gate, our scheme is different by some phase factors. Nevertheless, the gate in Eq.(37) can be considered as a Fredkin gate operation because the phase factors can be eliminated via setting the phase of subsequent logic operations appropriately. In order to characterize the availability of Eq.(36), we plot the corresponding evolution of states vs. time t by numerical calculation with Eq.(4) in Figs.2–7. The coefficients obtained through the effective Hamiltonian H_{Fredkin} in comparison with the ones obtained from the interaction Hamiltonian H'_{I} after a $\pi/2$ pulse are given in Table 1. The result shows that in the large-detuning condition $\Delta \gg \{g_a, g_b, g_c, \Omega\}$, the effective Hamiltonian in Eq.(36) is correct.

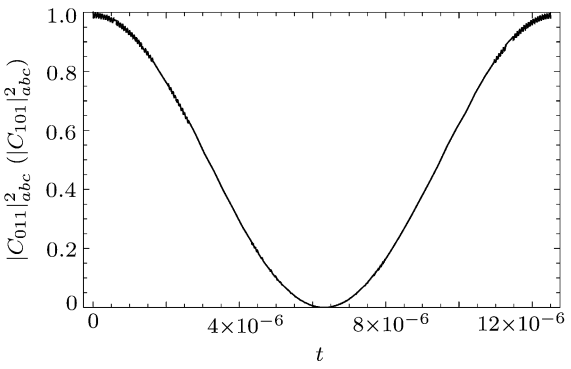


Fig.2. The probability of the initial state $|0_a,1_b,1_c\rangle$ ($|1_a,0_b,1_c\rangle$) vs. t.

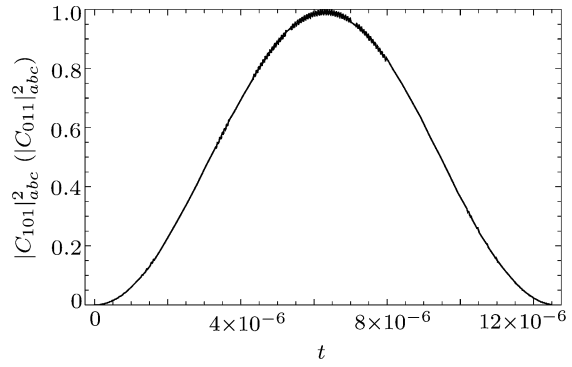


Fig.3. The probability of the state $|1_a, 0_b, 1_c\rangle$ ($|0_a, 1_b, 1_c\rangle$) vs. t.

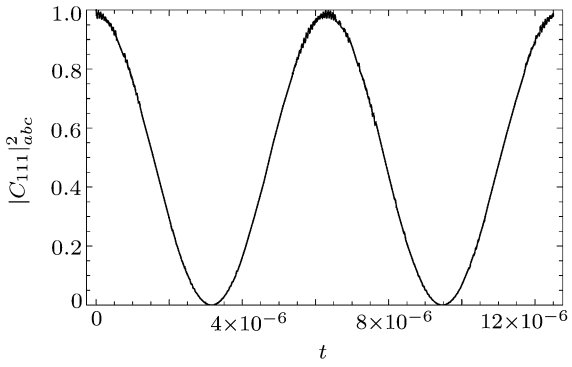


Fig.4. The probability of the initial state $|1_a, 1_b, 1_c\rangle$ vs. t.

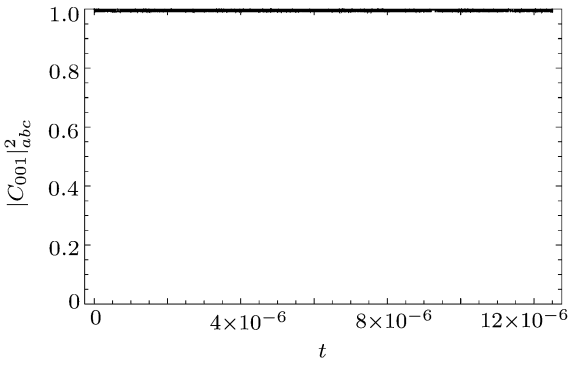


Fig.5. The probability of the initial state $|0_a, 0_b, 1_c\rangle$ vs. t.

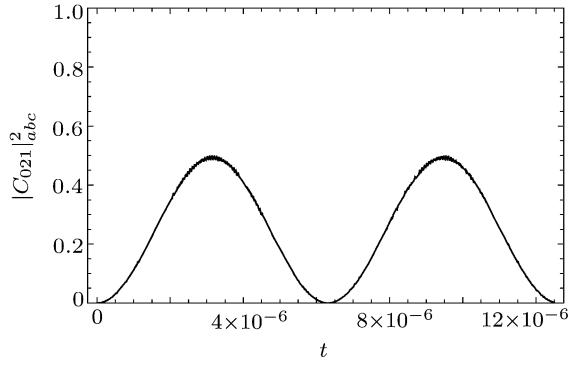


Fig.6. The probability of the state $|0_a, 2_b, 1_c\rangle$ vs. t.

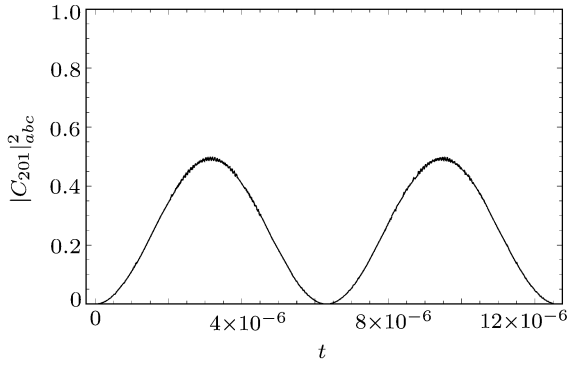


Fig.7. The probability of the state $|2_a, 0_b, 1_c\rangle$ vs. t.

Table 1. The coefficients obtained through the effective Hamiltonian $H_{\rm Fredkin}$ in comparison with the ones obtained from the interaction Hamiltonian $H'_{\rm I}$ after a $\pi/2$ pulse.

$\pi/2$	α	β	θ
$H_{\rm Fredkin}$	1	i	-1
$H'_{\rm I}$	0.9969–0.0777i	0.9894i+0.0789	–0.9874–0.0775i

Now we consider the effect of decoherence on the scheme. The dissipation channels include the atomic spontaneous emission (at the rate γ) and the cavity decay (at the rate κ). Under the condition that no photon is detected either by the spontaneous emission or by the leakage of a photon by the cavity mirror, the evolution of the system is governed by the conditional Hamiltonian

$$H_{\rm I}^{\rm de} = H'_{\rm I} - \frac{\rm i}{2}(\kappa_a a^\dagger a + \kappa_b b^\dagger b + \kappa_c c^\dagger c) - \frac{\rm i}{2}(\gamma_e |e\rangle\langle e| + \gamma_f |f\rangle\langle f| + \gamma_r |r\rangle\langle r|), \tag{38}$$

where κ_m $(m = a, b, c)$ is the m-th cavity decay rate, and γ_n $(n = e, f, r)$ is the spontaneous emission rate of the n-th level. The quality of the three-qubit Fredkin gate can be described by a fidelity, which is calculated according to the following relation:[31]

$$\mathcal{F} = \overline{\langle \Psi_0 | U^\dagger \rho_t U | \Psi_0 \rangle}, \tag{39}$$

where the overline indicates average over all possible input state $|\Psi_0\rangle$, U is the ideal three-qubit Fredkin gate, and $\rho_t = |\Psi_t\rangle\langle\Psi_t|$, $|\Psi_t\rangle$ is the final state after the Fredkin gate (the phase factors are omitted) operation in our scheme. If the system is initially in the state

$$|\Psi_0\rangle = \frac{1}{2\sqrt{2}}(|0_c\rangle + |1_c\rangle) \otimes (|0_a\rangle + |1_a\rangle) \otimes (|0_b\rangle + |1_b\rangle), \tag{40}$$

which is actually for averaging over all qubit states, and gives a general assessment for our gating implementation. After a $\pi/2$ pulse, we plot the fidelity vs. κ/g in Fig.8. It can be seen that the Fredkin gate is very sensitive to the decoherence because the fidelity rapidly decreases below 0.930 from 1 when κ increases to $0.001g$ from 0. Thus, for the sake of obtaining a high fidelity, we need cavity decay and atomic spontaneousness smaller than the atom-field coherent coupling strength by at least three orders of magnitude, which does not match the current experimental technique and is a formidable problem for the realization of our scheme experimentally. It is promising that in many schemes, the effect of decoherence can be diminished effectively by introducing an atomic-ensemble,[13,15,32−35] and we are trying to find a solution via a similar method.

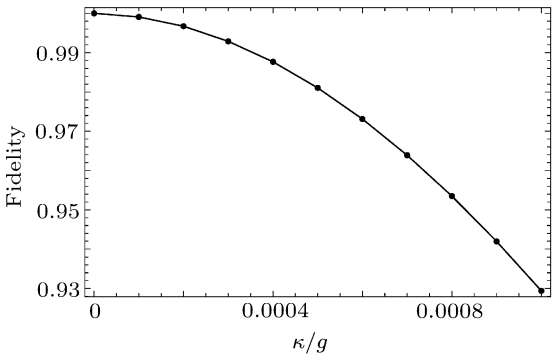


Fig.8. Average fidelity of the Fredkin gate as a function of κ/g. Other common parameters: $g_a = g_b = g_c = g = \Omega = \Delta/20$, $\kappa_a = \kappa_b = \kappa_c = \gamma_e = \gamma_f = \gamma_r$.

Although we have proposed our scheme from a theoretical view point, we still look forward and hope that the conditions for realizing our gate may be reached in the near future. This depends on the advancement of technology for the fabrication industry. Here, we bring forward the basic elements which may be candidates for the intended experiment. The four-level Y-type atomic structure is realistic and has been investigated in the past.[36−38] We may employ the ^{85}Rb atom for our proposal, the lower state $|g\rangle$ corresponds to the F=1 hyperfine state of the $5S_{1/2}$ electronic ground state, the first excited state $|e\rangle$ corresponds to the F=2 hyperfine state of the $5P_{3/2}$ electronic excited state, the higher excited states $|f\rangle$ and $|r\rangle$ correspond to the F=1 hyperfine states of the $7S_{1/2}$ electronic excited states, respectively. The three-mode optical cavity has been widely used in laser physics and industry along with the development of multiwavelength lasers.[39−42] Through this paper, we have adopted the cavity parameter $g = 2\pi \times 16$ MHz,[43,44] and obtain an interaction time about 6.25 μs. Although there has been no report about the strong interaction between the multimode field and a multilevel atom simultaneously inside one cavity experimentally yet, we are inspired with hope since the various cavity techniques have been developed to realize strong coupling.[45−50]

In conclusion, we have proposed a theoretical scheme to implement the Fredkin gate with a Y-type atom and a three-mode cavity. By modulating the cavity frequency and the atomic transition frequency appropriately, we obtain the form of nonlinear interaction between photons in the three-mode cavity. We believe that our scheme is more meaningful for the quantum logic gate both theoretically and experimentally.

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