Vol 18 No 8, August 2009
1674-1056/2009/18(08)/3247-04

Chinese Physics B

© 2009 Chin. Phys. Soc.
and IOP Publishing Ltd

Transferring an N-atom state between two distant cavities via an optical fiber*

Ma Song-She(马宋设)† and Chen Mei-Feng(陈美锋)

College of Physics and Information Engineering, Fuzhou University, Fuzhou 350108, China

(Received 23 September 2008; revised manuscript received 20 November 2008)

This paper proposes a scheme for transferring an N-atom state between two distant cavities via an optical fiber. The scheme is based on adiabatic passage along a dark state. In the scheme, all the atoms are always in ground state, the field mode of the fiber remains in vacuum state, and the field mode of the cavities being excited can be negligible under certain conditions. Therefore, the scheme is very robust against decoherence. The successful probability of implementing the quantum state transfer increases with increasing number of atoms. Furthermore, the interaction time does not need to be accurately adjusted as long as the adiabaticity condition is fulfilled.

Keywords: quantum state transfer, atom state, optical fiber
PACC: 0367, 4250

1. Introduction

In quantum information theory, transferring information in the form of a coherently prepared quantum state is essential. Usually, there are two methods to transfer quantum states. The first method of transferring a quantum state is quantum teleportation,[1] which was first proposed by Bennett *et al.* In the quantum teleportation process, unknown quantum information can be transmitted from a sender to a receiver without the transmission of the carrier of the quantum information. Since the first quantum teleporting protocol was suggested, there have been numerous schemes proposed to implement teleportation.[2−4] Experimental realizations of quantum teleportation have been reported, in which optical systems[5] and nuclear magnetic resonance were used.[6] Recently, Riebe *et al* and Barrett *et al* have experimentally implemented the teleportation of the states of Ca^+[7] and Be^+.[8]

The second method of transferring a quantum state is through a quantum network, which is based on the idea of transferring a quantum state from one node to another node with the help of a career (a quantum channel). Cirac *et al* have proposed a protocol to implement quantum transfer of an unknown atomic state using photons for ideal quantum transmission between two cavities.[9] Biswas *et al* have also presented a scheme for transferring an unknown quantum state,[10] which is based on the interaction of a two-mode cavity field with a Λ-type atom. In the scheme, the cavities serve as nodes and the atoms serve as qubit carriers. There are also many other schemes for transferring quantum states through a quantum network,[11−13] in these schemes, the qubit carriers are all atoms. As is well known, atoms with long-lived internal states are well suited for storing qubits, and photons are the best qubit carriers for fast and reliable communication over long distances. Recently, Chen *et al* proposed a scheme to implement the quantum transfer of an unknown atomic state utilizing photons as qubit carriers.[14]

In order to implement quantum information processing in two remote cavities, Pellizzari proposed a scheme for transferring quantum information reliably between two atoms in two distant cavities coupled by an optical fiber.[15] By using the same system, there have been several schemes proposed to generate entangled states, to realize quantum gates or to implement quantum state transfer.[16−20] Wilk *et al* have reported an experimental scheme for deterministically generating an atom-photon entangled state.[21] In the scheme of Ref.[20], Li *et al* proposed a scheme for transferring an N-atom state, in which the atoms resonantly and collectively interact with the local cavity fields but there is no direct interaction between the atoms. Inspired by the work of Li *et al*,[20] in this pa-

*Project supported by the Science Foundation of Educational Committee of Fujian Province (Grant No JB06042).
†Corresponding author. E-mail: masshe@126.com

per, we propose an alternative scheme for transferring an N-atom state. Our scheme is based on adiabatic passage along a dark state. Compared with the work of Li *et al*[20] our scheme has the following advantages: (i) all the atoms are always in ground state; (ii) the field mode of the fiber remains in vacuum state; (iii) the field mode of the cavities being excited can be negligible under certain conditions; (iv) the successful probability of implementing the quantum state transfer increases with the increasing number of atoms, (v) the interaction time does not need to be accurately adjusted as long as the adiabaticity condition is fulfilled.

2. Transferring of an N-atom state

We consider that N atoms are trapped in each of two cavities, and the two cavities are linked by an optical fiber. Each atom has three levels: an excited state $|r\rangle$, and two ground states $|f\rangle$ and $|g\rangle$. The transition $|g\rangle \rightarrow |r\rangle$ of each atom is resonantly coupled to the cavity mode and the transition $|f\rangle \rightarrow |r\rangle$ is resonantly driven by a classical field. We assume that the size of the space occupied by the N atoms in each of the cavities is much smaller than the wavelength of the fields, then all the atoms in the cavity "see" the same fields. On the other hand, the atoms in each cavity are so separated that they have no direct interaction with each other.[20] In the short fiber limit $(2L\bar{\nu})/(2\pi C) \leq 1$, only one resonant mode of the fiber will interact with the cavity modes, where L is the length of the fiber and $\bar{\nu}$ is the decay rate of the cavity fields into a continuum of the fiber modes. Then in the interaction picture, the Hamiltonian of the whole system can be written as

$$\begin{aligned} H = \Bigg[\sum_{i=1}^{N} & (g_1 a_1 |r\rangle_{i,1} \langle g|_{i,1} + \Omega_1 |r\rangle_{i,1} \langle f|_{i,1} \\ & + g_2 a_2 |r\rangle_{i,2} \langle g|_{i,2} + \Omega_2 |r\rangle_{i,2} \langle f|_{i,2}) \\ & + \nu b (a_1^{\dagger} + a_2^{\dagger}) \Bigg] + \text{H.c.}, \end{aligned} \tag{1}$$

where $a_1^{\dagger}$ (a_1) and $a_2^{\dagger}$ (a_2) are the creation (annihilation) operators for cavity 1 and cavity 2 respectively, b is the annihilation operator of the resonant mode of the fiber, ν is the coupling strength between the cavity mode and the fiber mode, Ω_1 (Ω_2) is the Rabi frequency of the classical field applied to cavity 1 (2), and g_1 (g_2) is the coupling strength between the atom and the field in cavity 1 (2). For simplicity, we assume $g_1 = g_2 = g$.

We assume that the initial state of the system is $|f, N-1\rangle_1 |0, N\rangle_2 |000\rangle_f$, where $|0, N\rangle$ is the state having N atoms in the state $|g\rangle$, $|f, N-1\rangle$ is the state having $N-1$ atoms in the state $|g\rangle$ and one atom in the state $|f\rangle$, and $|n_1 n_2 n_f\rangle_f$ is the state having n_1, n_2, and n_f photons in cavity 1, cavity 2 and the fiber respectively. The state evolves in the subspace spanned by the basis state vectors:

$$|f, N-1\rangle_1 |0, N\rangle_2 |000\rangle_f, \quad |r, N-1\rangle_1 |0, N\rangle_2 |000\rangle_f,$$
$$|0, N\rangle_1 |0, N\rangle_2 |100\rangle_f, \quad |0, N\rangle_1 |0, N\rangle_2 |001\rangle_f,$$
$$|0, N\rangle_1 |0, N\rangle_2 |010\rangle_f, \quad |0, N\rangle_1 |r, N-1\rangle_2 |000\rangle_f,$$

and

$$|0, N\rangle_1 |f, N-1\rangle_2 |000\rangle_f,$$

where $|r, N-1\rangle$ has $N-1$ atoms in the state $|g\rangle$ and one atom in the state $|r\rangle$. There is a dark state with null eigenvalue, which is

$$\begin{aligned} |\psi\rangle = & \cos\theta \cos\alpha |f, N-1\rangle_1 |0, N\rangle_2 |000\rangle_f \\ & + \cos\theta \sin\alpha |0, N\rangle_1 |f, N-1\rangle_2 |000\rangle_f \\ & + \sin\theta |0, N\rangle_1 |0, N\rangle_2 (|100\rangle_f - |010\rangle_f), \end{aligned} \tag{2}$$

where

$$\sin\theta = \frac{\sqrt{2}\Omega_1 \Omega_2}{\sqrt{Ng^2\Omega_2^2 + Ng^2\Omega_1^2 + 2\Omega_1^2\Omega_2^2}}, \tag{3}$$

$$\cos\theta = \frac{\sqrt{Ng^2\Omega_2^2 + Ng^2\Omega_1^2}}{\sqrt{Ng^2\Omega_2^2 + Ng^2\Omega_1^2 + 2\Omega_1^2\Omega_2^2}}, \tag{4}$$

$$\sin\alpha = \frac{\sqrt{N} g \Omega_1}{\sqrt{Ng^2\Omega_2^2 + Ng^2\Omega_1^2}}, \tag{5}$$

$$\cos\alpha = \frac{\sqrt{N} g \Omega_2}{\sqrt{Ng^2\Omega_2^2 + Ng^2\Omega_1^2}}. \tag{6}$$

Now we want to implement the quantum state transfer as the following:

$$\begin{aligned} & (\alpha |0, N\rangle_1 + \beta |f, N-1\rangle_1) \otimes |0, N\rangle_2 \\ & \rightarrow (\alpha |0, N\rangle_2 + \beta |f, N-1\rangle_2) \otimes |0, N\rangle_1, \end{aligned} \tag{7}$$

where $|\alpha|^2 + |\beta|^2 = 1$. Initially, we choose that Ω_2, $\sqrt{N}g \gg \Omega_1$, and let the system in the state $(\alpha |0, N\rangle_1 + \beta |f, N-1\rangle_1) |0, N\rangle_2 |000\rangle_f$. Under the adiabatic condition, the state $|f, N-1\rangle_1 |0, N\rangle_2 |000\rangle_f$ undergoes the evolution decided by Eq.(2). If we adiabatically increase Ω_1 and decrease Ω_2 until $\Omega_2 \ll \sqrt{N}g$ and Ω_1, then the state $|f, N-1\rangle_1 |0, N\rangle_2 |000\rangle_f$ evolves

to $|0,N\rangle_1|f,N-1\rangle_2|000\rangle_f$, leaving all the field modes in the vacuum state. Meanwhile, the state $|0,N\rangle_1|0,N\rangle_2|000\rangle_f$ remains unchanged during this stage. Therefore, after the above procedure we can implement the quantum state transfer decided by Eq.(7).

Now we discuss the experimental feasibility of the present scheme. We can see from Eq.(2) that during the adiabatic transferring process

$$|f,N-1\rangle_1|0,N\rangle_2|000\rangle_f \rightarrow |0,N\rangle_1|f,N-1\rangle_2|000\rangle_f,$$

the atoms in the two cavities are always in ground states, the mode of the fiber remains in vacuum state, and the field mode of the cavities being excited can be negligible under the condition that $\sqrt{N}g \gg \Omega_1$ and Ω_2. However, if the condition $\sqrt{N}g \gg \Omega_1$ and Ω_2 is not ideally satisfied, there is a probability of the cavity modes being excited, which will affect the successful probability of the scheme. Now we discuss the probability of the cavity modes being excited. We suppose that initially $\Omega_{2\max} = g/4$, $\Omega_{1\min} \approx 0$ and the increasing rate of Ω_1 is equal to the decreasing rate of Ω_2, then we have the relation $\Omega_1 + \Omega_2 = g/4$ (in experiments, g more than four times larger than $\Omega_{2\max}$ or $\Omega_{1\max}$ has been achieved[22]). The probability of the cavity modes being excited can be estimated as

$$P = \frac{1}{\theta_{\max}}\int_0^{\theta_{\max}} \sin^2\theta \mathrm{d}\theta. \tag{8}$$

Because of the cavity decay, the probability for the state $|f,N-1\rangle_1|0,N\rangle_2|000\rangle_f$ evolving to $|0,N\rangle_1|0,N\rangle_2|000\rangle_f$ through the intermediate state $|0,N\rangle_1|0,N\rangle_2(|100\rangle_f - |010\rangle_f)/\sqrt{2}$ is approximately $\varepsilon = \kappa t P$, where κ is the cavity decay rate and t is the interaction time.

$$\begin{aligned} &|f,N-1\rangle_1|0,N\rangle_2|000\rangle_f \\ \rightarrow &\sqrt{1-\varepsilon}|0,N\rangle_1|f,N-1\rangle_2|000\rangle_f \\ &+\sqrt{\varepsilon}|0,N\rangle_1|0,N\rangle_2|000\rangle_f. \end{aligned} \tag{9}$$

According to Ref.[23], the time-dependent Rabi frequencies Ω_1 and Ω_2 can be expressed as $\Omega_2 = \frac{g}{4}\mathrm{e}^{-(gt)^2/32}$ and $\Omega_1 = \frac{g}{4}\left(1-\mathrm{e}^{-(gt)^2/32}\right)$. If we set the interaction time $t = 20/g$, the condition that Ω_2, $\sqrt{N}g \gg \Omega_1$ in the initial of the interaction and $\Omega_2 \ll \sqrt{N}\ g$, Ω_1 in the final stage of the interaction can be well fulfilled. Then we can calculate the successful probability P' to implement the quantum state transfer decided by Eq.(7). In Fig.1, we show the relation of P' versus atom number N for $\kappa = 0.1g$. In Ref.[24], the cavity QED parameters describing the cavity-atom interaction system were obtained as $(g,\,\kappa)/2\pi = (16,\,1.4)$ MHz, which satisfies the condition $\kappa = 0.1g$. It is obvious that the successful probability increases with increasing atom number. If we use two high-finesse optical cavities,[25] $(g,\,\kappa)/2\pi = (16,\,0.14)$ MHz can be obtained, which satisfies $\kappa = 0.01g$. Then our scheme can be completed with a higher success probability.

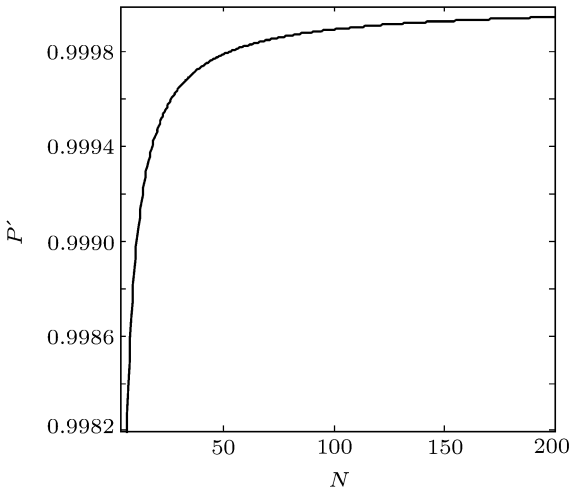


Fig.1. The probability P' of transferring N-atom states versus atom number N, taking $\kappa = 0.1g$.

For the atomic level structure, we can take Cs as our choice.[19] The states $|f\rangle$ and $|g\rangle$ correspond to the hyperfine levels $|F=4,\,m=-1\rangle$ and $|F=4,\,m=1\rangle$ of $6S_{1/2}$ respectively, while the state $|r\rangle$ corresponds to $|F=4,\,m=0\rangle$ of $6P_{3/2}$. The transition $|f\rangle \rightarrow |r\rangle$ is driven by a right-circular polarized laser pulse, while the transition $|g\rangle \rightarrow |r\rangle$ is coupled to a left-circular polarized cavity mode. In the above analyses, we assume that: (i) the size of the space occupied by the N atoms in each of the cavities is much smaller than the wavelength of the fields, then all the atoms in the cavity "see" the same fields; (ii) the atoms in the cavity are so separated that they have no direct interaction with each other. Can these assumptions be fulfilled in current experiments? Suppose that all the atoms are trapped in a flat-disk-shaped trap of transverse radius δr and axis thickness δz in the cavity. This flat-disk trap with radius $\delta r = 2.3$ μm and thickness $\delta z = 33$ nm has been realized experimentally.[26] According to the current experimental parameters,[26] we know that the nodirect interaction condition can be satisfied very well when $N < 200$, and all the atoms in the trap have nearly the same coupling strength with the cavity, as the spatial variation in the trap

results in a very small change of the coupling strength $\delta g/g_0 < 10^{-2}$.[26] As for the classical fields, we employ the idea of Ref.[27]: all the applied laser pulses are incident from one mirror of their corresponding cavities and collinear with axes of their respective cavities. Thus, the lasers share the same spatial mode structure with their respective cavity, all the atoms in each cavity "see" the same classical field.

3. Summary

In conclusion, a scheme is proposed for transferring an N-atom state between two distant cavities via an optical fiber, which is based on adiabatic passage along a dark state. In the scheme, all the atoms are always in ground state, the field mode of the fiber remains in vacuum state, and the field mode of the cavities being excited can be negligible under certain conditions. Therefore, the scheme is very robust against decoherence. The success probability of implementing the quantum state transfer increases with increasing atom number. Furthermore, the interaction time does not need to be accurately adjusted as long as the adiabaticity condition is fulfilled. According to the current experiments, the assumptions employed in the present scheme are reasonable and realistic. Thus, the scheme can be realized with current optical cavity and atomic trap techniques.

References

[1] Bennett C H, Brassard G, Crepeau C, Jozsa R, Peres A and Wootters W K 1993 *Phys. Rev. Lett.* **70** 1895

[2] Jin L H, Jin X R and Zhang S 2005 *Phys. Rev.* A **72** 024305

[3] Zheng S B 2006 *Chin. Phys. Lett.* **23** 2356

[4] Chen M F and Ma S S 2008 *Chin. Phys.* B **17** 451

[5] Furusawa A, Sørensen J L, Braunstein S L, Fuchs C A, Kimble H J and Polzik E S 1998 *Science* **282** 706
Boschi D, Branca S, De Martini F, Hardy L and Popescu S 1998 *Phys. Rev. Lett.* **80** 1121
Marcikic I, de Riedmatten H, Tittel W, Zbinden H and Gisin N 2003 *Nature* **421** 509

[6] Nielsen M A, Knill E and Laflamme R 1998 *Nature* **396** 52

[7] Riebe M, Häffner H, Roos C F, Hänsel W, Benhelm J, Lancaster G P T, Korber T W, Becher C, Schmidt-Kaler F, James D F V and Blatt R 2004 *Nature* **429** 734

[8] Barrett M D, Chiaverini J, Schaetz T, Britten J, ltano W M, Jost J D, Knill E, Langer C, Leibfried D, Ozeri R and Wineland D J 2004 *Nature* **429** 737

[9] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221

[10] Biswas A and Agarwal G S 2004 *Phys. Rev.* A **70** 022323

[11] Zheng S B 2005 *Phys. Rev.* A **71** 062335

[12] Song K H 2005 *Acta Phys. Sin.* **54** 4730 (in Chinese)

[13] Wu T, Ni Z X and Ye L 2007 *Commun. Theor. Phys.* (Beijing, China) **48** 831

[14] Chen M Y, Lin X M and Chen Z H 2007 *Chin. Phys.* **16** 416

[15] Pellizzari T 1997 *Phys. Rev. Lett.* **79** 5242

[16] Serafini A, Mancini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503

[17] Peng P and Li F L 2007 *Phys. Rev.* A **75** 062320

[18] Song J, Xia Y and Song H S 2007 *J. Phys.* B: *At. Mol. Opt. Phys.* **40** 4503

[19] Ye S Y, Zhong Z R and Zheng S B 2008 *Phys. Rev.* A **77** 014303

[20] Yin Z Q and Li F L 2007 *Phys. Rev.* A **75** 012324

[21] Wilk T, Webster S C, Kuhn A and Rempe G 2007 *Science* **317** 488

[22] Vitanov N V, Fleischhauer M, Shore B W and Bergmann K 2001 *Adv. At. Mol. Opt. Phys.* **46** 55
McKeever J, Boca A, Boozer A D, Buck J R and Kimble H J 2003 *Nature* (London) **425** 268
Miller R, Northup T E, Birnbaum K M, Boca A, Boozer A D and Kimble H J 2005 *J. Phys.* B **38** S551

[23] Deng Z J, Gao K L and Feng M 2006 *Phys. Rev.* A **74** 064303

[24] Hennrich M, Legero T, Kuhn A and Rempe G 2000 *Phys. Rev. Lett.* **85** 4872

[25] Maunz P, Puppe T, Schuster I, Syassen N, Pinkse P W H and Rempe G 2005 *Phys. Rev. Lett.* **94** 033002

[26] Boca A, Miller R, Birnbaum K M, Boozer A D, McKeever J and Kimble H J 2004 *Phys. Rev. Lett.* **93** 233603

[27] Duan L M, Kuzmich A and Kimble H J 2003 *Phys. Rev.* A **67** 032305
Yu B, Zhou Z W, Zhang Y, Xiang G Y and Guo G C 2004 *Phys. Rev.* A **70** 014302