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An experimental scheme to verify the dynamics of the Aharonov–Bohm effect*

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There are two different viewpoints on the Aharonov–Bohm (A–B) effect. One asserts that the A–B effect is due to the existence of the vector potential $\boldsymbol{A}$. The other asserts that the A–B effect is due to the interaction energy between the magnetic field produced by the moving charges and the magnetic field in the solenoid. The difference of these two viewpoints is analyzed in this paper. To judge which viewpoint is right, this paper suggests a new experimental method.

Keywords: Aharonov–Bohm effect, superconductivity, superconducting quantum interference device

PACC: 0365, 7400, 7450

1. Introduction

In 1959, Y. Aharonov and D. Bohm[1] predicted that a phase difference between two electron beams is produced, which is proportional to the magnetic flux enclosed by their paths, even though they always move in the region where the magnetic field is zero. This effect is difficult to be understood in classical physics and has led to extensive debate. Some authors asserted that the Aharonov–Bohm (A–B) effect does not exist.[2] Some authors[3] explained this effect from the standpoint that electrons were affected by inevitable leakage magnetic fields from the finite solenoid in the experiments.[4] Both views were removed by the toroidal experiments of Tonomura[5] which allowed very little stray magnetic flux and indicated that the Aharonov–Bohm (A–B) effect does exist. Other authors suggested that the Aharonov–Bohm shift might be based upon a classical lag effect.[6−8] In their view, the passing charge experiences a net force, which is a non-Lorentzian force arising from the gradient of the interaction energy between the magnetic field produced by the electron and the magnetic field in the solenoid. But whether the net force acting on the passing charge exists or not was questioned by other authors.[9] This problem has been well reviewed in the literature,[10] which asserted that such a net force on the moving charge does not exist.

Since the experiments by Tonomura,[5] no one questions the existence of the Aharonov–Bohm effect, and this effect has become a standard part of the quantum mechanics textbook. The current view is: the Aharonov–Bohm effect represents a new quantum topological effect, and the vector potential $\boldsymbol{A}$ can result in some observable phenomena in quantum mechanics, though it is only a mathematical field in classical physics.[1] But in our view, this view does not reveal the physical nature of the Aharonov–Bohm phase shift.

In our view, the concept of 'force' plays a key role in classical mechanics. All the phenomena can be explained by analyzing the forces acting on the objects. But in quantum mechanics, the concept of 'energy' becomes most important; in contrast, the concept of 'force' disappears. All the phenomena in quantum mechanics are determined by the Hamiltonian, which represents the energy of the system. As the A–B effect being a typical quantum phenomenon, we should explain this effect with the standpoint of energy, instead of the standpoint of topology or classical electromagnetism.

In this paper, we state that the interaction energy between the magnetic field produced by the moving charges and the magnetic field in the solenoid can affect the phase of the moving charge in quantum mechanics, though this interaction energy does not affect the velocity of the moving charge in classical physics.[10] The A–B effect is due to this interaction energy but not the vector potential $\boldsymbol{A}$. (Some similar proposals have been suggested by several authors.[11−14] They proposed that the A–B effect could be explained in terms of this interaction

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energy between the incident electron and solenoid). In Ref.[15], we have provided a strict quantum mechanical proof for such a conclusion. If the superposition between the magnetic fields can be eliminated, the A–B effect will disappear, even though the vector potential still exists in the space. (For reference [15] is a Chinese paper, to help the reader to understand this conclusion, the theoretical proof for this conclusion was translated from Chinese to English in the Appendix of this paper).

Certainly, such a conclusion needs to be verified by experiments. To verify this proposal, an experimental method similar to that used by Tonomura was suggested by Erlichson.[11] But as shown in this paper, such an experimental method cannot judge whether the A–B effect results from vector potential $\boldsymbol{A}$ or from the interaction energy between the magnetic fields. To verify that the A–B effect is due to the interaction energy but not the vector potential $\boldsymbol{A}$, we will provide a feasible experimental method to test it in the third part of this paper.

2. Dynamical analysis of the A–B effect

Suppose that the static magnetic field created by the solenoid is denoted by $\boldsymbol{B}_0$, which only exists in the region enclosed by the solenoid, and denote the magnetic field produced by a moving charge q by $\boldsymbol{B}_1$, which is determined by the following formula:[16]

$$\boldsymbol{B}_1(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{q\boldsymbol{v}\times(\boldsymbol{r}-\boldsymbol{x})}{|\boldsymbol{r}-\boldsymbol{x}|^3}, \tag{1}$$

where $\boldsymbol{B}_1(\boldsymbol{r})$ denotes the magnetic induction at the point $\boldsymbol{r}$, $\boldsymbol{v}$ and $\boldsymbol{x}$ are the velocity and position of the charge q respectively.

The magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$ can be superposed with the field $\boldsymbol{B}_0$ and form a total magnetic field $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1$. The energy of the total magnetic field is given by:

$$\begin{aligned} W &= \int \frac{1}{2\mu_0}\boldsymbol{B}^2 \mathrm{d}r^3 \\ &= \int \frac{1}{2\mu_0}\boldsymbol{B}_0^2 \mathrm{d}r^3 + \int \frac{1}{2\mu_0}\boldsymbol{B}_1^2 \mathrm{d}r^3 \\ &\quad + \int \frac{1}{\mu_0}\boldsymbol{B}_0\cdot\boldsymbol{B}_1 \mathrm{d}r^3. \end{aligned} \tag{2}$$

Clearly, the first term is the self-energy of the magnetic field $\boldsymbol{B}_0$, the second term is the self-energy of the magnetic field $\boldsymbol{B}_1(\boldsymbol{r})$, and the third term is the interaction energy W' between these two magnetic fields. If Ω denotes the region where the magnetic field $\boldsymbol{B}_0$ exists, then the interaction energy W' is

$$\begin{aligned} W' &= \int \frac{1}{\mu_0}\boldsymbol{B}_0\cdot\boldsymbol{B}_1 \mathrm{d}r^3 \\ &= \frac{1}{4\pi}\int_\Omega \boldsymbol{B}_0(\boldsymbol{r})\cdot\frac{q\boldsymbol{v}\times(\boldsymbol{r}-\boldsymbol{x})}{|\boldsymbol{r}-\boldsymbol{x}|^3}\mathrm{d}r^3 \\ &= \frac{1}{4\pi}\int_\Omega \frac{[\boldsymbol{B}_0(\boldsymbol{r})\times(\boldsymbol{x}-\boldsymbol{r})]\cdot q\boldsymbol{v}}{|\boldsymbol{x}-\boldsymbol{r}|^3}\mathrm{d}r^3 \\ &= \boldsymbol{A}(\boldsymbol{x})\cdot q\boldsymbol{v}, \end{aligned} \tag{3}$$

where $\boldsymbol{x}$ still denotes the position of the moving charge, and

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi}\int_\Omega \frac{[\boldsymbol{B}_0(\boldsymbol{r})\times(\boldsymbol{x}-\boldsymbol{r})]}{|\boldsymbol{x}-\boldsymbol{r}|^3}\mathrm{d}r^3. \tag{4}$$

Using the same method as that in subsection 5.3 of the Ref.[16], we can show $\boldsymbol{\nabla}\times\boldsymbol{A} = \boldsymbol{B}_0$ and $\boldsymbol{\nabla}\cdot\boldsymbol{A} = 0$, i.e. $\boldsymbol{A}(\boldsymbol{x})$ is a vector potential field describing the magnetic field $\boldsymbol{B}_0(\boldsymbol{r})$ and satisfies the Coulomb gauge. Here, the vector potential $\boldsymbol{A}(\boldsymbol{x})$ is determined uniquely and any gauge transformation is forbidden, for if we add to $\boldsymbol{A}(\boldsymbol{x})$ the gradient of a scalar function $\chi(\boldsymbol{x})$ and form $\boldsymbol{A}'(\boldsymbol{x}) = \boldsymbol{A}(\boldsymbol{x}) + \boldsymbol{\nabla}\chi(\boldsymbol{x})$, then $\boldsymbol{A}'(\boldsymbol{x})\cdot q\boldsymbol{v}$ does not equal the interaction energy W'. For this reason, all the vector potentials in this paper are determined by Eq.(4).

As shown in Fig.1, a coherent electron beam splits into two beams at F. One beam bypasses the solenoid through the side C, and the other through the side D. Then, they combine to interfere at E. The direction of the magnetic field in the solenoid is perpendicular to the paper and points upward. $\boldsymbol{A}_C$ and $\boldsymbol{A}_D$ are the vector potentials at the points C and D. According to Eq.(4), $\boldsymbol{A}_C = -\boldsymbol{A}_D$. Suppose that $\boldsymbol{v}_C$ and $\boldsymbol{v}_D$ are the velocities of the electrons at the points C and D, obviously, $\boldsymbol{v}_C = \boldsymbol{v}_D$. So, when an electron passes by the point C, the interaction energy W'_C between the magnetic field produced by this electron and the static magnetic field in the solenoid is equal to $\boldsymbol{A}_C\cdot(-e)\boldsymbol{v}_C$ where $-e$ is the charge of an electron. Similarly, when an electron passes by the point D, the interaction energy W'_D equals $\boldsymbol{A}_D\cdot(-e)\boldsymbol{v}_D$, obviously, $W'_C = -W'_D$. For the wave function $\psi(\boldsymbol{r},t) = \psi(\boldsymbol{r})\exp(-\mathrm{i}Et/\hbar)$, where E is the energy of the particle, this difference in interaction energy inevitably results in the different phases of the wave function at the points C and

[1)] Here, the velocities V_C and V_D are classical quantities. In quantum mechanics, they should be replaced by the current densities J_C and J_D at the points C and D, then, the interaction energies W'_C and W'_D can be obtained by integration over the space. But the analysis above remains applicable.

D. Therefore, the pattern of interference at E should change with the magnetic flux in the solenoid,[1)] i.e. the A–B effect. So, the interaction energy can be considered as the dynamical cause for the A–B effect. If we have a way to eliminate this interaction energy, the A–B effect should disappear (The theoretical proof for this conclusion is provided in the Appendix of this paper).

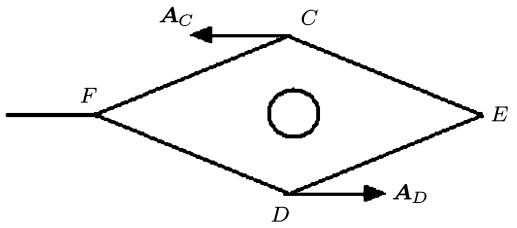


Fig.1. Dynamical analysis of the A–B effect. The circle represents the solenoid creating a magnetic field $\boldsymbol{B_0}$. Whereas $\boldsymbol{A}_C$ and $\boldsymbol{A}_D$ represent the vector potentials at the points C and D respectively, which are due to the magnetic field $\boldsymbol{B_0}$ in the solenoid.

To verify this deduction, a simple method is to coat a superconductor cylinder outside the solenoid. When the superconductor cylinder comes into the superconducting state, due to the Messiner effect, it can completely shield the magnetic field due to the moving electron out of the cylinder. Consequently, the magnetic field produced by the moving electron cannot superimpose with the static magnetic field in the solenoid, and the interaction energy between them becomes zero. According to the analysis above, the A–B effect should not be observed anymore. But this conclusion is contradictory to the experiments by A. Tonomura *et al.*[5] In their experiments, a tiny toroidal magnet covered entirely with a superconducting layer and further with a copper layer is fabricated. The thickness of the superconducting layer (about 250 nm) is much larger than its magnetic penetration depth. When the temperature is below the critical temperature of the superconducting layer, the magnetic flux ϕ_S enclosed within the superconducting layer is quantized, i.e. $\phi_S = nh/2e$.[17] The A–B effect asserts that the relative phase shift φ between two electron beams is proportional to the magnetic flux ϕ enclosed by their paths and equals

$$\Delta\varphi = 2\pi\frac{\phi}{h/e}. \tag{5}$$

Here, $\phi = \phi_S$. If $\phi_S = (2n+1)\left(\dfrac{h}{2e}\right)$, i.e. the phase shift $\Delta\varphi$ between two electron beams equals $(2n+1)\pi$, each interference fringe inside the ring should lie just in the middle of two fringes outside the ring. If $\phi_S = 2n\left(\dfrac{h}{2e}\right)$, i.e. the phase shift $\Delta\varphi$ equals $2n\pi$, the interference fringes inside the ring should align with the fringes outside the ring. Both cases have been observed in their experiments. That is to say, though the solenoid is coated by a superconducting cylinder, the magnetic flux enclosed by the solenoid can still affect the quantum phase of the electron passing outside the cylinder. Obviously, these experimental results are contradictory to our analysis about the A–B effect. But further analysis can show that our viewpoint is not wrong. The cause for this contradiction is that the superconducting layer in their experiments can only confine the magnetic flux within it, but cannot shield the magnetic field produced by the electron beams.

In their experiments, an electron beam can be regarded as a system of successive wave packets, and the coherent length of each wave packet is about 3–5 μm.[18] The wave packets are separated with each other and the quantum interference that occurs in forming an electron hologram involves one electron at a time. Each wave packet produces a magnetic field at the tiny toroidal magnet. According to Eq.(1), the magnitude of this magnetic field is inversely proportional to the square of the distance. So, the magnetic field at the tiny magnet forms a pulse, and its maximum is produced when the wave packet passes through the plane where the tiny magnet lies. The time width of this pulse equals the length of the wave packet Δl divided by its velocity v, i.e. $\Delta t \approx \Delta l/v$. In their experiments, the velocity of the electron wave packets is about 2×10^8 m/s so, $\Delta t \approx 2\times10^{-14}$ s. If the Fourier transform is introduced, the main frequency of this pulse is about $\nu \approx 5\times10^{13}$ Hz, i.e. $h\nu \approx 2\times10^{-2}$ eV which is much larger than the energy gap in the Nb film (about 3×10^{-3} eV). Obviously, the Nb film cannot shield the magnetic field variation with such high frequency. Therefore, the magnetic field produced by the moving electron can still penetrate the superconducting film and be superposed with the static magnetic field in the tiny magnet. Just for this reason, the A–B effect was observed in their experiments.

If one wants to verify what is the dynamical cause for the A–B effect, the vector potential $\boldsymbol{A}$ or the interaction energy between the magnetic fields, a new experiment has to be designed, in which the superconducting film must be able to completely shield the magnetic field produced by the moving charges.

3. A new experimental scheme

We have proposed an experimental method to test this proposal, but this method is very difficult.[15] This experimental method is to judge the origin of the A–B effect by measuring the resistance of a metal ring, through which a solenoid coated by a superconducting cylinder passes. The diameter of the metal ring in this method should be less than 1 μm. Moreover, a superconducting cylinder and a solenoid must pass through this metal ring, which is extremely difficult. In addition, the amplitude of the h/e oscillations is about 0.1% of the resistance at $\boldsymbol{B} = 0$. Even though this h/e oscillation disappears in the experiment, it is still difficult to assert that the A–B effect is due to the interaction energy between the magnetic fields but not the existence of the vector potential $\boldsymbol{A}$, for many other unexpected factors can result in the disappearance of the h/e oscillations. So this experimental method is not a good one. To help the physicist to study this problem with experiments, here, we will design a much easier experimental scheme.

We know that the physical essence of the SQUID (Superconducting Quantum Interference Device[19,20]) is similar to that of the A–B effect. The critical current $I_{\rm C}$ passing through the SQUID depends on the flux Φ enclosed by the superconducting loop,

$$I_{\rm C} = I_0 \left|\cos\left(\frac{\pi\Phi}{\Phi_0}\right)\right|. \tag{6}$$

Where $\Phi_0 = h/2e$, for the current carriers in the SQUID are Cooper pairs. Our experimental scheme is shown in Fig.2. A SQUID with two Josephson junctions is placed in the plane of $z = 0$. A magnetic field $\boldsymbol{B}_0(\boldsymbol{x})$ confined by a long and straight solenoid is perpendicular to the plane and passes through the region enclosed by the superconducting loop. The vector potential due to this confined flux is denoted by $\boldsymbol{A}_0$. The solenoid is divided into two parts by the plane of $z = 0$. The part above the plane is coated by a superconducting cylinder and the part below the plane is naked. Therefore, the magnetic field produced by the currents in the SQUID can only be superposed with the magnetic field $\boldsymbol{B}_0(\boldsymbol{x})$ below the plane, with the interaction energy being equal to $\int \frac{1}{2}\boldsymbol{A}_0(\boldsymbol{x}) \cdot \boldsymbol{J}'(\boldsymbol{x})\mathrm{d}x^3$ instead of $\int \boldsymbol{A}_0(\boldsymbol{x}) \cdot \boldsymbol{J}'(\boldsymbol{x})\mathrm{d}x^3$, where $\boldsymbol{J}'(\boldsymbol{x})$ denotes the current density in the SQUID.

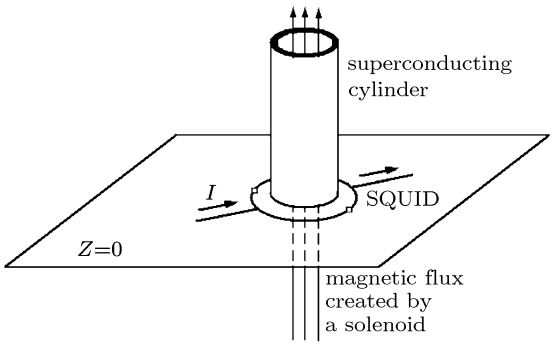


Fig.2. A new experimental scheme which can be used to test the dynamical cause of the A–B effect.

So, if the A–B effect results from the interaction energy between the magnetic fields, the critical current $I_{\rm C}$ passing through the SQUID should be determined by the following equation:

$$I_{\rm C} = I_0 \left|\cos\left[\frac{\pi(\Phi/2)}{\Phi_0}\right]\right|. \tag{7}$$

When $\Phi = 2n\Phi_0$, $I_{\rm C} = I_0$; when $\Phi = (2n+1)\Phi_0$, $I_{\rm C} = 0$.

If the A–B effect results from the vector potential $\boldsymbol{A}_0$, but not the interaction energy between the fields, the critical current $I_{\rm C}$ should still be determined by Eq.(6). No matter for $\Phi = 2n\Phi_0$ or for $\Phi = (2n+1)\Phi_0$ both the critical current $I_{\rm C}$ should be I_0.

Therefore, this experimental scheme can provide us with a direct method to judge that the dynamical mechanism of the A–B effect is due to the interaction energy between the magnetic fields, or due to the existence of the vector potential $\boldsymbol{A}_0$.

The technology of SQUID has been developed for many years and has been an ordinary experimental method. Now, it is not difficult to shield the geomagnetic field to less than 10 nT by permalloy. In this case, provided that the diameter of the superconducting loop of the SQUID is less than 300 μm, the geomagnetic flux enclosed by the superconducting loop will be less than Φ_0, then, the influence of the geomagnetic field on the experiment can be neglected. Moreover, it is not difficult to put a solenoid in the superconducting loop. So, this experimental scheme is much easier than before. In addition, as analyzed above, if the A–B effect results from the vector potential $\boldsymbol{A}_0$, the critical current $I_{\rm C}$ of the SQUID should be I_0, no matter the magnetic flux Φ enclosed by the superconducting loop equals $2n\Phi_0$ or $(2n+1)\Phi_0$; But if the A–B effect results from the interaction energy between the magnetic fields, the critical current $I_{\rm C}$ of

the SQUID should be I_0 when $\Phi = 2n\Phi_0$, or should be zero when $\Phi = (2n+1)\Phi_0$. These two experimental results are quite different, which will help us to draw a more conformable conclusion on the origin of the A–B effect.

4. Conclusion

The current viewpoint about the Aharonov–Bohm effect asserts that this effect results from the vector potential $\boldsymbol{A}$. However, it is only a mathematical description on the A–B effect, not a physical explanation. We tried to explain the Aharonov–Bohm effect as a result of the interaction energy between the magnetic field due to the moving charge and that in the solenoid. If the solenoid is not shielded by superconductors, the same conclusions can always be drawn from these two different viewpoints. But if the solenoid is shielded by a superconducting cylinder and this superconducting cylinder can completely shield the magnetic field produced by the moving charge out of the solenoid, these two viewpoints will result in different conclusions. Based upon this difference, we have designed a new experimental scheme to distinguish which one is correct.

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Appendix A

Here, to help the reader to understand the experimental scheme proposed in this paper, it is necessary to show our viewpoint with the quantum mechanical method. In Ref.[15], we have shown such a conclusion: suppose there is a straight and infinitely long solenoid, outside which an infinitely long superconducting cylinder coats. The depth of this cylinder is much larger than its magnetic penetration depth. A charge system G lies outside the cylinder. If the superconducting cylinder can completely shield the magnetic field produced by the system G out of the solenoid, then the magnetic field enclosed by the solenoid cannot have any effect on the charge system G.

For simplicity, we assume that the moving charges in the system G are only electrons. Obviously, the current distribution in the system G will produce a magnetic field around itself, and induce a shielding current distribution on the outer surface of the superconductor cylinder ensuring that the magnetic field arising from the system G cannot penetrate the superconducting cylinder. When the current distribution in the system G changes, the shielding current distribution on the outer surface of the superconducting cylinder will change with it. So, the electrons in the system G and the electrons in the superconducting cylinder should be taken as one system, denoted as Σ. We assume that the wave function of the system Σ is $|\varphi_n\rangle$, which includes the electrons in the system G and the electrons in the superconducting cylinder. The Hamiltonian $\hat{H}$ of the system Σ can be divided into two terms: $\hat{H} = \hat{H}_1 + \hat{H}_2$, where

$$\hat{H}_1 = \int \mathrm{d}x^3 \hat{\psi}_1^{\dagger}(\boldsymbol{x}) \frac{1}{2m} \left[-\mathrm{i}\,\hbar\nabla + e\boldsymbol{A}(\boldsymbol{x})\right]^2 \hat{\psi}_1(\boldsymbol{x}) + \hat{V}_1, \tag{A1}$$

$$\hat{H}_2 = \int \mathrm{d}x^3 \hat{\psi}_2^{\dagger}(\boldsymbol{x}) \frac{1}{2m} \left[-\mathrm{i}\,\hbar\nabla + e\boldsymbol{A}(\boldsymbol{x})\right]^2 \hat{\psi}_2(\boldsymbol{x}) + \hat{V}_2. \tag{A2}$$

Here, $\hat{H}_1$ represents the Hamiltonian of the electrons in the system G, $\hat{\psi}_1^{\dagger}(\boldsymbol{x})$ and $\hat{\psi}_1(\boldsymbol{x})$ represent the creation and annihilation operators of these electrons respectively, and $\hat{V}_1$ denotes all the other potential terms in G which is not related to the magnetic field. $\hat{H}_2$ represents the Hamiltonian of the electrons in the superconducting cylinder, $\hat{\psi}_2^{\dagger}(\boldsymbol{x})$ and $\hat{\psi}_2(\boldsymbol{x})$ represent the creation and annihilation operators of these electrons respectively, and $\hat{V}_2$ represents all the other potential terms in the cylinder, which is not related to the magnetic field either, including the effective interaction between the superconducting electrons due to phonon exchange.

The vector potential $\boldsymbol{A}$ in Eqs.(A1) and (A2) includes two parts $\boldsymbol{A} = \boldsymbol{A}_0 + \boldsymbol{A}_1$. The first vector potential $\boldsymbol{A}_0$ is defined by the static magnetic field $\boldsymbol{B}_0$, i.e. $\boldsymbol{\nabla} \times \boldsymbol{A}_0 = \boldsymbol{B}_0$, where $\boldsymbol{B}_0$ is created by the solenoid and only exists in the region enclosed by the solenoid. The second vector potential $\boldsymbol{A}_1$ is defined by the magnetic field $\boldsymbol{B}_1$, i.e. $\boldsymbol{\nabla} \times \boldsymbol{A}_1 = \boldsymbol{B}_1$, where $\boldsymbol{B}_1$ is produced by both the currents in the system G and the currents on the outer surface of the superconducting cylinder, obviously, $\boldsymbol{B_1}$ only exists outside the cylinder.

When the superconducting cylinder comes into the superconducting state, the magnetic flux enclosed by it should be quantized in units of $h/2e$. So, for simplicity, we assume that the magnetic flux Φ created by

the solenoid equals $nh/2e$. In this case, when the superconducting cylinder comes into the superconducting state, no current is formed on its inner surface, and only the currents on its outer surface need to be taken into account. Therefore, the Hamiltonian of the system Σ can be rewritten as:

$$\begin{aligned}\hat{H} &= \hat{H}_1 + \hat{H}_2 \\ &= \int \mathrm{d}x^3 \hat{\psi}_1^\dagger(\boldsymbol{x}) \frac{1}{2m} \left\{-\mathrm{i}\hbar\boldsymbol{\nabla} + e[\boldsymbol{A}_0(\mathbf{x}) + \boldsymbol{A}_1(\boldsymbol{x})]\right\}^2 \hat{\psi}_1(\boldsymbol{x}) + \hat{V}_1 \\ &\quad + \int \mathrm{d}x^3 \hat{\psi}_2^\dagger(\boldsymbol{x}) \frac{1}{2m} \left\{-\mathrm{i}\hbar\boldsymbol{\nabla} + e[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})]\right\}^2 \hat{\psi}_2(\boldsymbol{x}) + \hat{V}_2. \end{aligned} \tag{A3}$$

According to the variational method, the wave function $|\varphi_n\rangle$ satisfies the following variational equation $\delta\bar{H} = \delta \langle\varphi_n| \hat{H} |\varphi_n\rangle = 0$ under the normalized condition $\langle\varphi_n | \varphi_n\rangle = 1$ Besides this boundary condition, the Meissner effect provides another boundary condition. For the magnetic field $\boldsymbol{B}_1(\boldsymbol{x})$ only exists in the region outside the cylinder and cannot be superposed with the magnetic field $\boldsymbol{B}_0(\boldsymbol{x})$ enclosed by the solenoid, the interaction energy W' between $\boldsymbol{B}_1(\boldsymbol{x})$ and $\boldsymbol{B}_0(\boldsymbol{x})$ must be zero, i.e.

$$W' = \int \frac{1}{\mu_0} \boldsymbol{B}_0(\boldsymbol{x}) \cdot \boldsymbol{B}_1(\boldsymbol{x}) \mathrm{d}x^3 = \int \boldsymbol{A}_0(\boldsymbol{x}) \cdot \boldsymbol{J}(\boldsymbol{x}) \mathrm{d}x^3 = 0, \tag{A4}$$

where, $\boldsymbol{J}(\boldsymbol{x})$ is the current density, including the current density $\boldsymbol{J}_1(\boldsymbol{x})$ in the system G and the shielding current density $\boldsymbol{J}_2(\boldsymbol{x})$ on the outer surface of the superconducting cylinder. $\boldsymbol{J}(\boldsymbol{x})$ can be rewritten as

$$\begin{aligned}\boldsymbol{J}(\boldsymbol{x}) &= \boldsymbol{J}_1(\boldsymbol{x}) + \boldsymbol{J}_2(\boldsymbol{x}) \\ &= \langle\varphi_n| \left\{ \frac{\mathrm{i}e\hbar}{2m} \left[\hat{\psi}_1^\dagger(\boldsymbol{x}) \boldsymbol{\nabla} \hat{\psi}_1(\boldsymbol{x}) - \left(\boldsymbol{\nabla}\hat{\psi}_1^\dagger(\boldsymbol{x})\right) \hat{\psi}_1(\boldsymbol{x})\right] - \frac{e^2}{m} \left[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})\right] \hat{\psi}_1^\dagger(\boldsymbol{x}) \hat{\psi}_1(\boldsymbol{x}) \right. \\ &\quad \left. + \frac{\mathrm{i}e\hbar}{2m} \left[\hat{\psi}_2^\dagger(\boldsymbol{x}) \boldsymbol{\nabla} \hat{\psi}_2(\boldsymbol{x}) - \left(\boldsymbol{\nabla}\hat{\psi}_2^\dagger(\boldsymbol{x})\right) \hat{\psi}_2(\boldsymbol{x})\right] - \frac{e^2}{m} \left[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})\right] \hat{\psi}_2^\dagger(\boldsymbol{x}) \hat{\psi}_2(\boldsymbol{x}) \right\} |\varphi_n\rangle . \end{aligned} \tag{A5}$$

Therefore,

$$\begin{aligned} W' &= \int \boldsymbol{A}_0(\boldsymbol{x}) \cdot \boldsymbol{J}(\boldsymbol{x}) \mathrm{d}x^3 \\ &= \int \mathrm{d}x^3 \boldsymbol{A}_0(\boldsymbol{x}) \cdot \langle\varphi_n| \left\{ \frac{\mathrm{i}e\hbar}{2m} \left[\hat{\psi}_1^\dagger(\boldsymbol{x}) \boldsymbol{\nabla} \hat{\psi}_1(\boldsymbol{x}) - \left(\boldsymbol{\nabla}\hat{\psi}_1^\dagger(\boldsymbol{x})\right) \hat{\psi}_1(\boldsymbol{x})\right] - \frac{e^2}{m} \left[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})\right] \hat{\psi}_1^\dagger(\boldsymbol{x}) \hat{\psi}_1(\boldsymbol{x}) \right. \\ &\quad \left. + \frac{\mathrm{i}e\hbar}{2m} \left[\hat{\psi}_2^\dagger(\boldsymbol{x}) \boldsymbol{\nabla} \hat{\psi}_2(\boldsymbol{x}) - \left(\boldsymbol{\nabla}\hat{\psi}_2^\dagger(\boldsymbol{x})\right) \hat{\psi}_2(\boldsymbol{x})\right] - \frac{e^2}{m} \left[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})\right] \hat{\psi}_2^\dagger(\boldsymbol{x}) \hat{\psi}_2(\boldsymbol{x}) \right\} |\varphi_n\rangle . \end{aligned} \tag{A6}$$

Using the integration by parts, and noticing $\hat{\psi}_1^\dagger(\boldsymbol{x}), \hat{\psi}_1(\boldsymbol{x}), \hat{\psi}_2^\dagger(\boldsymbol{x}), \hat{\psi}_2(\boldsymbol{x}) \to 0$ at $x \to \infty$, we have:

$$\begin{aligned} W' = \langle\varphi_n| \int \mathrm{d}x^3 \boldsymbol{A}_0(\boldsymbol{x}) \cdot \left\{ \frac{\mathrm{i}e\hbar}{m} \hat{\psi}_1^\dagger(\boldsymbol{x}) \boldsymbol{\nabla} \hat{\psi}_1(\boldsymbol{x}) - \frac{e^2}{m} \left[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})\right] \hat{\psi}_1^\dagger(\boldsymbol{x}) \hat{\psi}_1(\boldsymbol{x}) \right. \\ \left. + \frac{\mathrm{i}e\hbar}{m} \hat{\psi}_2^\dagger(\boldsymbol{x}) \boldsymbol{\nabla} \hat{\psi}_2(\boldsymbol{x}) - \frac{e^2}{m} \left[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})\right] \hat{\psi}_2^\dagger(\boldsymbol{x}) \hat{\psi}_2(\boldsymbol{x}) \right\} |\varphi_n\rangle = 0. \end{aligned} \tag{A7}$$

This is a boundary condition due to the Meissner effect, called as the second boundary condition.

Now, we deal with the variational equation $\delta\bar{H} = 0$:

$$\begin{aligned}\delta\bar{H} &= \delta \langle\varphi_n| \hat{H} |\varphi_n\rangle \\ &= \delta \langle\varphi_n| \int \mathrm{d}x^3 \hat{\psi}_1^\dagger(\boldsymbol{x}) \frac{1}{2m} \left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\boldsymbol{A}_0(\boldsymbol{x}) + e\boldsymbol{A}_1(\boldsymbol{x})\right]^2 \hat{\psi}_1(\boldsymbol{x}) + \hat{V}_1 \\ &\quad + \int \mathrm{d}x^3 \hat{\psi}_2^\dagger(\boldsymbol{x}) \frac{1}{2m} \left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\boldsymbol{A}_0(\boldsymbol{x}) + e\boldsymbol{A}_1(\boldsymbol{x})\right]^2 \hat{\psi}_2(\boldsymbol{x}) + \hat{V}_2 |\varphi_n\rangle . \end{aligned} \tag{A8}$$

Expanding the above equation, we obtain

$$\begin{aligned}\delta\bar{H} = {} & \delta\langle\varphi_n| \int \mathrm{d}x^3 \left\{\hat{\psi}_1^\dagger(\boldsymbol{x})\frac{1}{2m}\left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\boldsymbol{A}_1(\boldsymbol{x})\right]^2\hat{\psi}_1(\boldsymbol{x}) + \hat{V}_1\right\} \\ & + \int \mathrm{d}x^3 \left\{\hat{\psi}_2^\dagger(\boldsymbol{x})\frac{1}{2m}\left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\boldsymbol{A}_1(\boldsymbol{x})\right]^2\hat{\psi}_2(\boldsymbol{x}) + \hat{V}_2\right\}|\varphi_n\rangle \\ & - \delta\langle\varphi_n| \int \mathrm{d}x^3 \boldsymbol{A}_0(\boldsymbol{x})\cdot\left\{\frac{\mathrm{i}e\hbar}{m}\hat{\psi}_1^\dagger(\boldsymbol{x})\boldsymbol{\nabla}\hat{\psi}_1(\boldsymbol{x}) - \frac{e^2}{m}[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})]\hat{\psi}_1^\dagger(\boldsymbol{x})\hat{\psi}_1(\boldsymbol{x})\right. \\ & \left. + \frac{\mathrm{i}e\hbar}{m}\hat{\psi}_2^\dagger(\boldsymbol{x})\boldsymbol{\nabla}\hat{\psi}_2(\boldsymbol{x}) - \frac{e^2}{m}[\boldsymbol{A}_0(\boldsymbol{x}) + \boldsymbol{A}_1(\boldsymbol{x})]\hat{\psi}_2^\dagger(\boldsymbol{x})\hat{\psi}_2(\boldsymbol{x})\right\}|\varphi_n\rangle \\ & - \delta\langle\varphi_n| \int \mathrm{d}x^3 \frac{e^2\boldsymbol{A}_0^2(\boldsymbol{x})}{2m}[\hat{\psi}_1^\dagger(\boldsymbol{x})\hat{\psi}_1(\boldsymbol{x}) + \hat{\psi}_2^\dagger(\boldsymbol{x})\hat{\psi}_2(\boldsymbol{x})]\,|\varphi_n\rangle = 0, \end{aligned} \tag{A9}$$

where $\delta\bar{H}$ consists of three terms. The first term does not include the vector potential $\boldsymbol{A}_0(\boldsymbol{x})$. The second term represents the interaction energy between $\boldsymbol{B}_1(\boldsymbol{x})$ and $\boldsymbol{B}_0(\boldsymbol{x})$. According to the second boundary condition, this term must be zero, which is an inevitable result of the Meissner effect; The third term is the square term of $\boldsymbol{A}_0$ and can be neglected, for the vector potential $\boldsymbol{A}_0$ is a small quantity. Thus, we can obtain

$$\begin{aligned}\delta\bar{H} &= \delta\langle\varphi_n|\,\hat{H}\,|\varphi_n\rangle \\ &= \delta\langle\varphi_n| \int \mathrm{d}x^3 \left\{\hat{\psi}_1^\dagger(\boldsymbol{x})\frac{1}{2m}\left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\boldsymbol{A}_1(\boldsymbol{x})\right]^2\hat{\psi}_1(\boldsymbol{x}) + \hat{V}_1\right\} \\ &\quad + \int \mathrm{d}x^3 \left\{\hat{\psi}_2^\dagger(\boldsymbol{x})\frac{1}{2m}\left[-\mathrm{i}\hbar\boldsymbol{\nabla} + e\boldsymbol{A}_1(\boldsymbol{x})\right]^2\hat{\psi}_2(\boldsymbol{x}) + \hat{V}_2\right\}|\varphi_n\rangle = 0. \end{aligned} \tag{A10}$$

In this equation, the vector potential $\boldsymbol{A}_0$ which describes the static magnetic field created by the solenoid has disappeared. It means that the magnetic field enclosed by the superconducting cylinder has no effect on the wave function of the system G which lies outside the cylinder. In fact, this variational equation is the same one which describes the case where the magnetic flux in the solenoid is zero. So, if the magnetic field produced by the charge system G is completely shielded by superconductors, the A–B effect will vanish outside the superconductors.

This appendix is translated from the 4$^{\text{th}}$ part of Ref.[15]

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