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Effects of three-body interaction on collective excitation and stability of Bose–Einstein condensate*

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This paper investigates the collective excitation and stability of low-dimensional Bose–Einstein condensates with two- and three-body interactions by the variational analysis of the time-dependent Gross–Pitaevskii–Ginzburg equation. The spectrum of the low-energy excitation and the effective potential for the width of the condensate are obtained. The results show that: (i) the repulsive two-body interaction among atoms makes the frequency red-shifted for the internal excitation and the repulsive or attractive three-body interaction always makes it blue-shifted; (ii) the region for the existence of the stable bound states is obtained by identifying the critical value of the two- and three-body interactions.

Keywords: Bose–Einstein condensate, three-body interaction, collective excitation, stable bound states

PACC: 0365, 0530J, 0420F

1. Introduction

In recent years, the collective behavior of the low-dimensional Bose systems has been paid more and more attention.[1−3] The experimental studies about the collective dynamics of low-dimensional Bose–Einstein condensates (BECs) confined in microtraps have pointed out the occurrence of characteristic one dimensional (1D) features at low temperature.[4,5] In particular, the progress with BECs in atomic waveguides[6] and on the surface of atomic chips[7] involves strong compression of the traps and it can significantly increase the density of BECs. Then it poses the problem of taking into account the effects of the three-body interaction due to the high density on the behavior of BECs.[8,9]

As is well known, the existence of three-body dynamics can play an important role in the stability of a trapped Bose condensed gas. For example, the addition of three-body interaction can extend considerably the region of stability for a condensate, and allow the number of condensed atoms to increase even when the strength of the three-body force is very small.[10] On the other hand, besides changes in the density profile of the condensate,[11] the excitation spectrum of the collective oscillations can also be modified,[12,13] as the compressibility of such a system is significantly affected by the three-body contribution to the ground state energy. It is reported that a sufficiently dilute and cold Bose gas exhibits similar three-body dynamics for both signs of the s-wave atom-atom scattering length, and the long-range three-body interaction between neutral atoms is efficiently repulsive for either sign of the scattering length.[14] This interaction is interesting also for the understanding of the fundamental limits for the functioning of devices using BEC.[15] Clearly, it is important to consider the stability of the BEC with the two- and three-body interactions for future applications of the ultracold gases.

In comparison with the case of the same sign for the effective many-body interactions, the stability of the condensates with different signs of the interactions is even more attractive. In Ref.[13], the authors mainly studied the stability of the trapped condensates for two different cases: one is the case of the attractive two-body and repulsive three-body interactions, and another is the repulsive two-body and attractive three-body interactions. Besides the existence of up to two distinct phases, there is no collapse for the former case. For the latter one, the occurrence of collapse is possible, but the critical value of the interactions for the existence of the stable bound states is still lacking.

In the present work, we continue to investigate the collective excitation and stability of BEC in one-

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dimensional trapping geometry for the case of a repulsive or attractive three-body force together with a repulsive two-body interaction by a standard variational approach.[16−19] On the one hand, the dynamical mechanism of the condensate is demonstrated by performing a coordinate of a classical particle moving in an effective potential field. We calculate the spectrum of the low-lying excitation, which shows that the repulsive two-body interaction makes the frequency for the internal excitations red-shifted and the three-body interaction makes the frequency blue-shifted, no matter whether the three-body one is repulsive or attractive. This result has not been shown clearly in previous works. On the other hand, the stationary states as physical solutions are associated with the existence of the minimum of the effective potential, which depends on the two- and three-body interactions. In particular, we obtain the region of the stability for the ground state in the parameter space by identifying the critical value of the two- and three-body interactions for the existence of the minimum. It is noted that the criteria for stability usually involves the number density of the atoms. Here, we introduce the width of the condensate, which is another useful parameter to identify the instability of BEC.[20]

The paper is organized as follows. In Section 2, we introduce the theoretical model. In Section 3, we derive the governing equations for the center-of-mass coordinate of the condensate and its width by the variational approach. In Section 4, we calculate the low-energy excitation spectrum in a low-dimensional harmonic trap, and discuss the effect of the three-body interaction on the internal dynamics of BEC. The existence of the bound states are discussed by the effective potential for the condensate width in Section 4. The final section is our conclusion.

2. Basic model

Our starting point is the three-dimensional Gross–Pitaevskii equation (GPE) for the wave function $\Psi(\boldsymbol{r},t)$ of a trapped condensate, generalized to include three-body interaction,

$$\mathrm{i}\hbar\frac{\partial\Psi}{\partial t}=\Big[-\frac{\hbar^2}{2m}\nabla^2+\frac{m}{2}(\omega_r^2r^2+\omega_x^2x^2)+u_1N|\Psi|^2+u_2N^2|\Psi|^4\Big]\Psi, \tag{1}$$

which is also called the Gross–Pitaevskii–Ginzburg equation (GPGE) in the field of ultracold quantum gases. Here $\Psi(\boldsymbol{r},t)$ is normalized to unity and N is the number of atoms in the condensate. The coefficient $u_1=4\pi\hbar^2a_s/m$, which related to the mass of the atoms m and the two-body s-wave scattering length a_s, u_2 measures the strength of the three-body interaction. It is important to note that, in general, when we consider the three-body interaction, the three-body recombination should be taken into account, i.e., u_2 should be complex. However, recent theoretical studies estimate that, for Rb atoms, the real part of u_2 is the order of $\hbar\times10^{-26}\sim\hbar\times10^{-27}$ cm^6/s,[21−24] and the experiment[25] shows that the imaginary part of u_2 for Rb has the order of $\hbar\times10^{-30}$ cm^6/s. That is, $\mathrm{Re}[u_2]/\mathrm{Im}[u_2]\simeq10^3\sim10^4$. Hence, the three-body recombination expressed by the imaginary part of u_2 is negligible on time scales which are comparable to three-body elastic collisional duration.

For the low-dimensional trapping system governed by Eq.(1), ω_r and ω_x are the radial and longitudinal frequencies of the anisotropic harmonic trap, respectively and $\varepsilon\equiv\omega_r/\omega_x\gg1$. Consider $\Psi(\boldsymbol{r},t)=\phi(r)\psi(x,t)\exp(-\mathrm{i}\omega_rt)$, where $\phi(r)=\dfrac{1}{\sqrt{\pi}l_r}\exp\left(\dfrac{-r^2}{2l_r^2}\right)$ is the wave function of the ground state along the radial direction, and $l_r=\sqrt{\hbar/m\omega_r}$. Multiplying both sides of the GPGE (1) by $\phi(r)^*$ and integrating over the transverse variable r, we obtain the following quasi 1D GPGE

$$\mathrm{i}\hbar\frac{\partial\psi}{\partial t}=\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}+\frac{m}{2}\omega_x^2x^2+v_1|\psi|^2+v_2|\psi|^4\right)\psi, \tag{2}$$

where the parameters of the nonlinearity $v_1=\dfrac{u_1N}{2\pi l_r^2}$ and $v_2=\dfrac{u_2N^2}{3\pi^2l_r^4}$. All variables can be re-scaled to be dimensionless by the following transformations: $t\to\omega_xt$, $x\to x/l_x$, and $\psi\to\sqrt{l_x}\psi$, where $l_x=\sqrt{\hbar/m\omega_x}$. Then, we obtain the normalized 1D-GPGE in a harmonic trap with cubic and quintic nonlinearities:

$$\mathrm{i}\frac{\partial\psi}{\partial t}=-\frac{1}{2}\frac{\partial^2\psi}{\partial x^2}+\frac{x^2}{2}\psi+g_1|\psi|^2\psi+g_2|\psi|^4\psi, \tag{3}$$

where $g_1=\dfrac{v_1}{\hbar\omega_xl_x}$ and $g_2=\dfrac{v_2}{\hbar\omega_xl_x^2}$ represent the parameters of the two- and three-body interactions, respectively. Experimentally, by changing two-body interaction via the Feschbach resonance techniques, we can also come to the BEC with dominating two-body or three-body interactions. As studied in Ref.[26], in principle, both signs for the three-body interaction are allowed. So our interaction parameters are easily implemented in experiments.

3. Variational analysis

The problem of solving Eq.(3) can be restated as a variational problem corresponding to the minimization of the action related to the Lagrangian density

$$\ell = \frac{i}{2}\left(\psi\frac{\partial\psi^*}{\partial t} - \psi^*\frac{\partial\psi}{\partial t}\right) + \frac{1}{2}\left|\frac{\partial\psi}{\partial x}\right|^2 + \frac{x^2}{2}|\psi|^2 + \frac{g_1}{2}|\psi|^4 + \frac{g_2}{3}|\psi|^6, \quad (4)$$

where the asterisk denotes a complex conjugate. In order to obtain the dynamics of the condensate in the trapping potential we will find the extremum of Eq.(4) with a set of trial functions. Because the Gaussian is the ground state of the linear Schrödinger equation, there is a natural choice of the trial function. We take

$$\psi(x,t) = a(t)\mathrm{e}^{-[x-\xi(t)]^2/2w(t)^2 + \mathrm{i}\,x\alpha(t) + \mathrm{i}\,x^2\beta(t)}. \quad (5)$$

At a given time t, this function defines a Gaussian distribution centered at the position ξ with width w. The other variational parameters a, α and β are all real variables. Without dissipative terms, the normalization of the wave function is conserved, i.e., $\int |\psi(x,t)|^2 \mathrm{d}x = 1$. From this condition, we can obtain $\sqrt{\pi}|a(t)|^2 w(t) = 1$.

Inserting Eq.(5) into Eq.(4) we can calculate an effective Lagrangian by averaging the Lagrangian density, $L = \langle\ell\rangle = \int_{-\infty}^{+\infty}\ell\mathrm{d}x$. Then, from the Lagrange equations, we obtain the evolution equations of all variational parameters. The dynamical equations for the center-of-mass and width of the condensate are derived as follows:

$$\frac{\mathrm{d}^2\xi}{\mathrm{d}t^2} = -\xi, \quad (6)$$

$$\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = -w + \frac{\lambda_1}{w^2} + \frac{1+\lambda_2}{w^3}, \quad (7)$$

where $\lambda_1 \equiv g_1/\sqrt{2\pi}$ and $\lambda_2 \equiv 4g_2/(3\sqrt{3}\pi)$, which denote the two- and three-body interactions between the particles, respectively. The other variational parameters can be obtained from the center-of-mass coordinate and the width through the equations $\alpha = \dot{\xi} - \xi\dot{w}/w$ and $\beta = \dot{w}/2w$. Therefore, once we know the behavior of the center and width of the condensate, we can calculate the evolution of the rest of the parameters, and then completely characterize the dynamics of the condensate.

4. Low-energy excitation spectrum of the condensate

Obviously, equation (6) shows that the dipole oscillation for the center-of-mass is independent of the two- and three-body interactions, and its oscillation frequency is determined by the external harmonic trapping potential. The internal dynamics described by Eq.(7) are not only dominated by the two-body force, but also affected by the three-body force between atoms. From Eqs.(6) and (7), we can obtain the equilibrium points as follows:

$$\xi_0 = 0, \quad w_0 = \frac{\lambda_1}{w_0^2} + \frac{1+\lambda_2}{w_0^3}, \quad (8)$$

which correspond to the stationary states of the condensate trapped in harmonic potential. Figure 1(a) gives the variation of the equilibrium width with two-body interaction for different three-body interactions. It shows that the width becomes large with increasing the two- and three-body interactions. When the two-body interaction is very small, the three-body interaction makes a great effect on the width, but when the

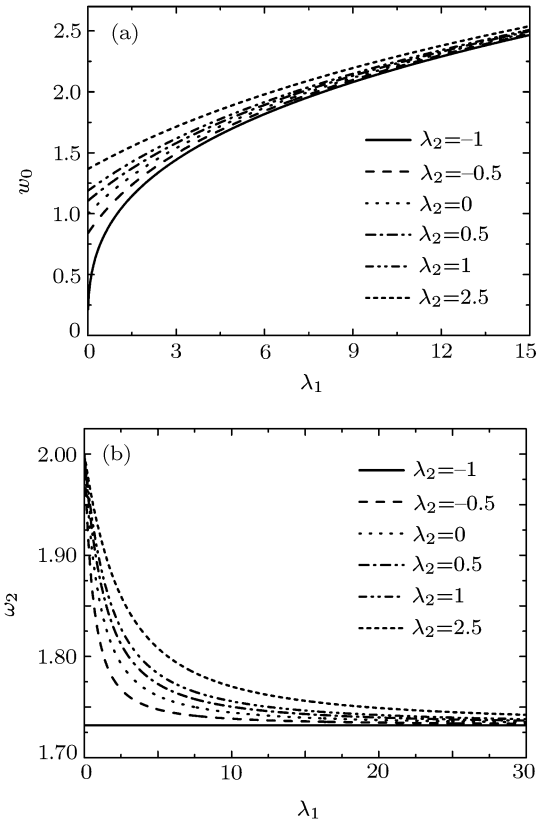


Fig.1. (a) Variation of the equilibrium width with λ_1 for different values of λ_2. (b) Frequency ω_2 as a function of λ_1 for different λ_2.

two-body interaction is much larger than the three-body interaction, the width is almost determined by the two-body interaction.

Expanding Eqs.(6) and (7) around the equilibrium points, we can obtain the dynamical equations of the center-of-mass and the width,

$$\begin{aligned} \xi &= A\sin(\omega_1 t + \theta_1), \\ w &= w_0 + B\sin(\omega_2 t + \theta_2), \end{aligned} \tag{9}$$

where A, B, θ_1 and θ_2 are real constants. And the frequencies for low-energy excitation modes are

$$\omega_1 = 1, \quad \omega_2 = \left[1 + \frac{2\lambda_1}{w_0^3} + \frac{3(1+\lambda_2)}{w_0^4}\right]^{1/2}, \tag{10}$$

which correspond to the dipole motion of the center-of-mass for the condensate and the low-lying excitation of its width. It can be seen from Fig.1(b) that the frequency corresponding to the oscillation of width ω_2 changes with the two-body interaction λ_1 and the three-body interaction λ_2. The repulsive two-body interaction among atoms makes the frequency for the internal excitations red-shifted in the regime of the crossover and the three-body interaction makes the frequency blue-shifted, no matter whether the three-body one is repulsive or attractive. In particular, in the regime of the two- or three-body interaction dominating the dynamics, i.e., $\lambda_1 \gg |\lambda_2|$ or $|\lambda_2| \gg \lambda_1$, the value of the frequency gradually tends towards constant $\sqrt{3}$ or 2, which is also obtained for the 1D condensate in the Gross–Pitaeviskii mean-field framework[17] or the Tonks–Girardeau regime[27], respectively.

5. Effective potential for the condensate width and the stability of the ground state

From Eq.(7), the dynamics of the width can be rewritten as

$$\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = -\frac{\mathrm{d}}{\mathrm{d}w}\left[\frac{w^2}{2} + \frac{\lambda_1}{w} + \frac{1+\lambda_2}{2w^2}\right], \tag{11}$$

and the quantity in the square bracket of Eq.(11) can be seen as the effective potential $U(w)$ for the width

$$U(w) = \frac{w^2}{2} + \frac{\lambda_1}{w} + \frac{1+\lambda_2}{2w^2}. \tag{12}$$

The minimum of $U(w)$ represents a shallow potential well that can allow a stable bound state, which corresponds to the ground state of BEC. The stable bound state may exist if the effective potential has the minimum, contrarily, the bound state cannot exist. For the repulsive two- and three-body interactions, the minimum of the effective potential always exists, but for the repulsive two-body interaction and the attractive three-body interaction, it exists unnecessarily. From Fig.2(a), we can see that the minimum of the effective potential disappears with increasing the strengths of the attractive three-body interaction ($\lambda_2 < 0$) for $\lambda_1 = 1.0$. For different strengths of the two-body interaction, we can identify the critical value of the three-body interaction for the existence of the minimum, and obtain the region of the stability and instability for the ground state in parameter space, as shown in Fig.2(b). In the stable region, the ground state may be stationary, but in the unstable region, there is no stationary bound state.

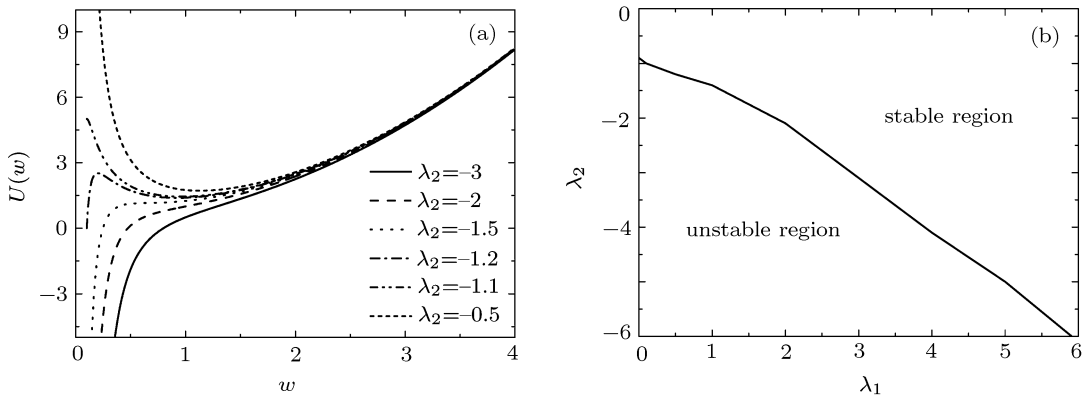


Fig.2. (a) Variation of the effective potential with the value of λ_2 for $\lambda_1 = 1.0$. (b) Regions of the stability and instability for the ground state in parameter space.

6. Conclusion

We have investigated the collective excitation of a BEC in a harmonic trap by solving the time-dependent GPGE, and obtained the analytical expression of the low-energy excitation corresponding to the variation of the center-of-mass and width of the condensate. The frequency-shift for the internal excitation is different for the different combination of the interaction signs, this fact is not shown clearly in previous works. Meanwhile, we discuss the existence of the stable ground state due to the analysis for the effective potential, and give the stable and unstable regions in the parameter space.

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