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Thermal entanglement and teleportation in a three-qubit Heisenberg XXZ model with Dzyaloshinski–Moriya anisotropic antisymmetric interaction*

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This paper investigates the thermal pairwise entanglement of a three-qubit Heisenberg XXZ chain in the presence of the Dzyaloshinski–Moriya (DM) anisotropic antisymmetric interaction and quantum teleportation when using the Heisenberg chain as a channel. The entanglement dependences on the DM interaction and temperature are given in detail. It obtains the relation between the concurrence and average fidelity, and shows that the same concurrence can lead different average fidelities. Moreover, it finds the thermally entangled states which do not violate the Bell inequalities, and can still be used for quantum teleportation.

Keywords: quantum teleportation, concurrence, fidelity, Bell inequality violation
PACC: 0365, 0367

1. Introduction

Entanglement is one of the most fascinating features of quantum mechanics and plays a central role in quantum information processing. An entanglement composite system has nonlocal correlation between its subsystems. This nonlocal character enables the uses of local quantum operations and classical communication to teleport an unknown quantum state via a shared pair of entangled particles, with fidelity better than any classical communication protocol.[1−3]

In recent years, there has been an ongoing effort to characterize qualitatively and quantitatively the entanglement properties of condensed matter systems at finite temperatures.[4−8] The state of a typical condensed matter system at thermal equilibrium (temperature T) is $\rho = \mathrm{e}^{-\beta H}/Z$ where H is the Hamiltonian, $Z = \mathrm{tr}\ \mathrm{e}^{-\beta H}$ is the partition function, and $\beta = 1/kT$ where k is Boltzmann's constant. The entanglement associated with the thermal state ρ is referred to as thermal entanglement.[9] Also quantum teleportation via the thermally entangled state of a two-qubit Heisenberg XX chain has been theoretically demonstrated in Ref.[10]. Furthermore, quantum teleportation of an unknown state using the thermally entangled state of a three-qubit Heisenberg chain has been reported.[11,12] But only the spin-spin interaction was considered in Refs.[11] and [12]; the effects of spin-orbit coupling on the entanglement and teleportation in a three-qubit Heisenberg chain are rarely included. In addition, only a few studies are directed to the relation between the teleportation fidelity and the violation of Bell inequality in the quantum channel. These are the motivations of this paper.

Here, we investigate the thermal pairwise entanglement in a three-qubit Heisenberg XXZ model with DM interaction, and use the thermal equilibrium state as a quantum channel to teleport an unknown quantum state. The relations between the teleportation fidelities and thermally entangled states are discussed.

2. XXZ Heisenberg model with DM interaction

This paper considers the three-qubit Heisenberg XXZ model with DM interaction, which can be described by

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$$H=\frac{1}{2}\sum_{i=1}^{3}[J(\sigma_i^x\sigma_{i+1}^x+\sigma_i^y\sigma_{i+1}^y)+J\Delta\sigma_i^z\sigma_{i+1}^z+\boldsymbol{D}\cdot(\boldsymbol{\sigma}_i\times\boldsymbol{\sigma}_{i+1})], \tag{1}$$

where J is the coupling strength and $\boldsymbol{D}$ is the DM vector coupling. The DM anisotropic antisymmetric interaction arises from spin-orbit coupling.[13,14] The parameter Δ quantifies the anisotropy in spin space. The value of positive and negative J corresponds to the antiferromagnetic and ferromagnetic cases, respectively. For simplicity, we choose $\boldsymbol{D}=D\boldsymbol{z}$. Then the Hamiltonian H becomes

$$H=\sum_{i=1}^{3}\left[(J+\mathrm{i}D)\sigma_i^+\sigma_{i+1}^-+(J-\mathrm{i}D)\sigma_i^-\sigma_{i+1}^++\frac{J\Delta}{2}\sigma_i^z\sigma_{i+1}^z\right], \tag{2}$$

where $\sigma_i^{\pm}$ are the raising and lowering operators. Note that we are working in units so that D, T, J, Δ are dimensionless. The eigenvalues and eigenstates of the Hamiltonian in Eq.(2) can be calculated analytically. In a three-qubit Heisenberg chain with periodic boundary conditions, the eigenvalues are given by

$$\begin{aligned}&E_1=E_8=\frac{3J\Delta}{2},\quad E_2=E_5=2J-\frac{J\Delta}{2},\quad E_3=E_6=-\left(\frac{J\Delta}{2}+J\right)+\sqrt{3}D,\\&E_4=E_7=-\left(\frac{J\Delta}{2}+J\right)-\sqrt{3}D,\end{aligned} \tag{3}$$

and the corresponding eigenstates can be explicitly expressed as

$$\begin{aligned}|\psi_1\rangle&=|111\rangle,\quad |\psi_2\rangle=\frac{1}{\sqrt{3}}\left(|110\rangle+|011\rangle+|101\rangle\right),\\|\psi_3\rangle&=\frac{1}{\sqrt{3}}\left(|110\rangle+\exp\left(\frac{\mathrm{i}2\pi}{3}\right)|011\rangle+\exp\left(\frac{\mathrm{i}4\pi}{3}\right)|101\rangle\right),\\|\psi_4\rangle&=\frac{1}{\sqrt{3}}\left(|110\rangle+\exp\left(\frac{\mathrm{i}4\pi}{3}\right)|011\rangle+\exp\left(\frac{2\mathrm{i}\pi}{3}\right)|101\rangle\right),\\|\psi_5\rangle&=\frac{1}{\sqrt{3}}\left(|100\rangle+|010\rangle+|001\rangle\right),\\|\psi_6\rangle&=\frac{1}{\sqrt{3}}\left(|100\rangle+\exp\left(\frac{\mathrm{i}4\pi}{3}\right)|010\rangle+\exp\left(\frac{\mathrm{i}2\pi}{3}\right)|001\rangle\right),\\|\psi_7\rangle&=\frac{1}{\sqrt{3}}\left(|100\rangle+\exp\left(\frac{\mathrm{i}2\pi}{3}\right)|010\rangle+\exp\left(\frac{\mathrm{i}4\pi}{3}\right)|001\rangle\right),\\|\psi_8\rangle&=|000\rangle.\end{aligned} \tag{4}$$

For the composite system in equilibrium at temperature T, the density operator is

$$\begin{aligned}\rho_{\mathrm{ABC}}=\frac{1}{Z}\Bigg\{&\exp\left(-\frac{3J\Delta}{2}\beta\right)(|\psi_1\rangle\langle\psi_1|+|\psi_8\rangle\langle\psi_8|)+\exp\left[-\left(2J-\frac{J\Delta}{2}\right)\beta\right](|\psi_2\rangle\langle\psi_2|+|\psi_5\rangle\langle\psi_5|)\\&+\exp\left[\left(\frac{J\Delta}{2}+J-\sqrt{3}D\right)\beta\right](|\psi_3\rangle\langle\psi_3|+|\psi_6\rangle\langle\psi_6|)\\&+\exp\left[\left(\frac{J\Delta}{2}+J+\sqrt{3}D\right)\beta\right](|\psi_4\rangle\langle\psi_4|+|\psi_7\rangle\langle\psi_7|)\Bigg\},\end{aligned} \tag{5}$$

where

$$Z=2\exp\left(-\frac{3J\Delta}{2}\beta\right)+2\exp\left[-\left(2J-\frac{J\Delta}{2}\right)\beta\right]+4\exp\left[\left(\frac{J\Delta}{2}+J\right)\beta\right]\cosh\sqrt{3}D\beta. \tag{6}$$

In the following calculation, we will write the Boltzmann constant $k=1.0$.

3. Thermal pairwise entanglement

In order to study the entanglement between two spins, we require their reduced density matrices. The density matrix for the different pair is obtained by tracing over the third spin, that is, $\rho_{AB} = \mathrm{Tr}_C\rho_{ABC}$, $\rho_{AC} = \mathrm{Tr}_B\rho_{ABC}$, $\rho_{BC} = \mathrm{Tr}_A\rho_{ABC}$. By symmetry under cyclic shift, the three reduced density matrices are equal, and are labelled as ρ. In the standard bases $\{|11\rangle, |10\rangle, |01\rangle, |00\rangle\}$, ρ is given by

$$\rho = \frac{1}{3Z}\begin{pmatrix} \mu & 0 & 0 & 0 \\ 0 & \varsigma & \xi & 0 \\ 0 & \xi^* & \varsigma & 0 \\ 0 & 0 & 0 & \mu \end{pmatrix}, \tag{7}$$

where

$$\begin{aligned}
\mu &= 3\exp\left(\frac{-3J\Delta}{2}\beta\right) + \exp\left[-\left(2J - \frac{J\Delta}{2}\right)\beta\right] \\
&\quad + 2\exp\left[\left(\frac{J\Delta}{2} + J\right)\beta\right]\cosh(\sqrt{3}D\beta), \\
\varsigma &= 2\exp\left[-\left(2J - \frac{J\Delta}{2}\right)\beta\right] \\
&\quad + 4\exp\left[\left(\frac{J\Delta}{2} + J\right)\beta\right]\cosh(\sqrt{3}D\beta), \\
\xi &= 2\exp\left[-\left(2J - \frac{J\Delta}{2}\right)\beta\right] \\
&\quad - 2\exp\left[\left(\frac{J\Delta}{2} + J\right)\beta\right]\cosh(\sqrt{3}D\beta) \\
&\quad - 2\sqrt{3}\mathrm{i}\exp\left[\left(\frac{J\Delta}{2} + J\right)\beta\right]\sinh(\sqrt{3}D\beta).
\end{aligned} \tag{8}$$

The entanglement of two qubits can be measured by the concurrence C, which is defined as $C = \max\{\lambda_1 - \lambda_2 - \lambda_3 - \lambda_4, 0\}$,[15] where λ_i $(i = 1, 2, 3, 4)$ are the square roots of the eigenvalues in the decreasing order of magnitude of the "spin-flipped" density operator $R = \rho_{12}(\hat{\sigma}_{1y} \otimes \hat{\sigma}_{2y})\rho_{12}^*(\hat{\sigma}_{1y} \otimes \hat{\sigma}_{2y})$, and the asterisk stands for the complex conjugate. Based on the definition of the concurrence, we can obtain the concurrence as

$$C = \frac{2}{3Z}\max\{0,\ |\xi| - \mu\}. \tag{9}$$

Figure 1 demonstrates the dependence of thermal entanglement on T and D both for the ferromagnetic and the antiferromagnetic cases. From Fig.1(a), for $J = 1$, it is easy to find that the critical temperature increases with the increase of the DM interaction D. When the temperature is higher than the critical temperature, the entanglement disappears. As is shown in Fig.1(b), for $J = -1$, the critical temperature first decreases, and then quickly increases with the increase of the DM interaction D.

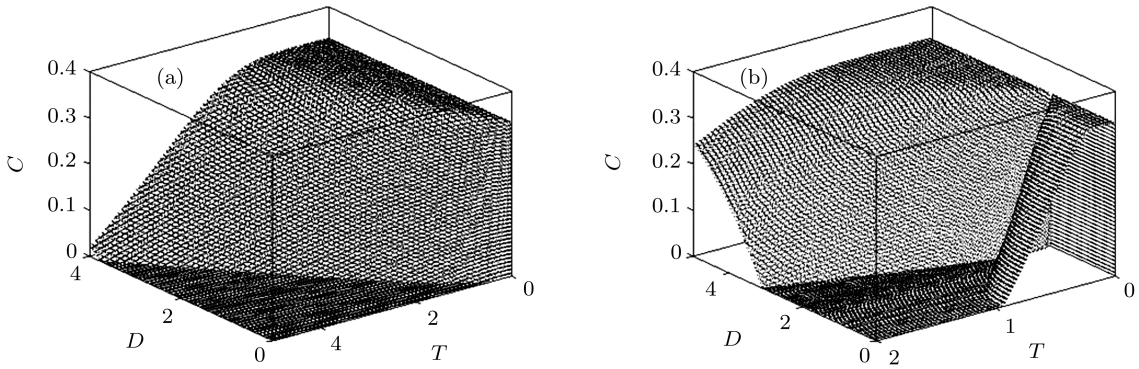


Fig.1. The concurrence C as a function of temperature T and DM interaction D both for the antiferromagnetic and the ferromagnetic cases: (a) $J = 1$, $\Delta = 0.4$; (b) $J = -1$, $\Delta = 0.4$.

4. Relations between fidelities and thermally entangled states

Now, we discuss the characters of teleportation by using the chain given by Eq.(7) in equilibrium state at any temperature T as a quantum channel. Our teleportation involves a sender, Alice (at site A), and two receivers, Bob (at site B) and Cindy (at site C). Alice possesses one-qubit S to be teleported and the spin A of the chain. The spins B and C belong to Bob and

Cindy, respectively. Following the protocol given by Refs.[16] and [17], we describe the quantum teleportation by using the three-qubit mixed state ρ_{ABC} as a resource. The total system $SABC$ is initially in the state expressed as

$$\rho_{SABC} = \pi_S \otimes \rho_{ABC},$$

where

$$\begin{aligned} \pi_S &= |\psi\rangle_S\langle\psi|, \\ |\psi\rangle_S &= \cos\frac{\theta}{2}|0\rangle_S + \mathrm{e}^{\mathrm{i}\phi}\sin\frac{\theta}{2}|1\rangle_S, \end{aligned} \tag{10}$$

$0 \le \theta \le \pi, \quad 0 \le \phi \le 2\pi$ are the polar and azimuthal angles, respectively, and ρ_{ABC} is as given in Eq.(5). To teleport the input state π_S to Bob and Cindy, Alice performs a joint Bell basis measurement on systems S and A, described by operators $\Pi^j_{SA} \otimes I_{BC}$, where I_{BC} is the identity operator on the composite subsystem BC, j labels the outcome of the measurement, and

$$\begin{aligned} \Pi^1_{SA} &= |\Phi^+\rangle_{SA}\langle\Phi^+|, \\ \Pi^2_{SA} &= |\Phi^-\rangle_{SA}\langle\Phi^-|, \\ \Pi^3_{SA} &= |\Psi^+\rangle_{SA}\langle\Psi^+|, \\ \Pi^4_{SA} &= |\Psi^-\rangle_{SA}\langle\Psi^-|. \end{aligned} \tag{11}$$

Where $|\Phi^\pm\rangle_{SA} = \frac{1}{\sqrt{2}}(|00\rangle_{SA} \pm |11\rangle_{SA})$, $|\Psi^\pm\rangle_{SA} = \frac{1}{\sqrt{2}}(|01\rangle_{SA} \pm |10\rangle_{SA})$ are the Bell states. If Alice's measurement has outcome j, she broadcasts her measurement result (two bits) to Bob and Cindy in a classical way. The joint state of Bob's target system B and Cindy's target system C conditioned on Alice's measurement result j is given by

$$\rho^j_{BC} = \frac{1}{P_j}\mathrm{tr}_{SA}\left[\left(\Pi^j_{SA} \otimes I_{BC}\right)(\pi_S \otimes \rho_{ABC})\right], \tag{12}$$

where

$$P_j = \mathrm{tr}_{SABC}\left[\left(\Pi^j_{SA} \otimes I_{BC}\right)(\pi_S \otimes \rho_{ABC})\right]. \tag{13}$$

Substituting Eqs.(5), (10), and (11) into Eq.(13) we get

$$P_1 = P_2 = P_3 = P_4 = \frac{1}{4}. \tag{14}$$

It is noticed that $\rho^j_B = \mathrm{tr}_C\rho^j_{BC} = \mathrm{tr}_B\rho^j_{BC} = \rho^j_C = \rho^j$. To successfully complete the teleportation protocol, Bob and Cindy perform j-dependent unitary operations $U^j_B = U^j_C = U^j$ on the systems B and C, respectively. After that the output state is

$$\tau^j_B = \tau^j_C = \rho^j_{\mathrm{out}} = U^j\rho^j U^{j+}, \tag{15}$$

where U^j could either be the identity matrix or one of the Pauli matrices (see Table 1).

Table 1. Bob's and Cindy's unitary operations conditioned only on Alice's measurement results.

Alice's measurement result j	Bob's unitary operation U^j	Cindy's unitary operation U^j
1	σ_x	σ_x
2	σ_y	σ_y
3	I	I
4	σ_z	σ_z

To characterize the quality of the teleported state ρ^j_{out}, it is often quite useful to look at the fidelity between the output state ρ^j_{out} and input state π_S defined by[18]

$$\begin{aligned} F^j &= \mathrm{tr}(\rho^j_{\mathrm{out}}, \pi_S) \\ &= \left\{\mathrm{tr}\left[\sqrt{(\pi_S)^{1/2}\rho^j_{\mathrm{out}}(\pi_S)^{1/2}}\right]\right\}^2, \end{aligned} \tag{16}$$

the concept of fidelity has been a useful indicator of the teleportation performance of a quantum channel when the input state is a pure state. Here we are interested in the average fidelity, which is given by averaging over all Alice's possible measurement outcomes j and over an isotropic distribution of input state π_S,

$$\bar{F} = \frac{1}{4\pi}\int_0^\pi\int_0^{2\pi}\sin\theta\mathrm{d}\theta\mathrm{d}\phi\sum_{j=1}^4 P_jF^j. \tag{17}$$

It follows from Eq.(10) and results from Eq.(12) that

$$F^1 = F^2 = F^3 = F^4 = 4\Big\{h_1\left(1 - \frac{\sin^2\theta}{2}\right) + \frac{\sin^2\theta}{2}h_2 + \sin^2\theta h_3\Big\}, \tag{18}$$

where

$$h_1 = \frac{1}{3Z}\left\{\exp\left[-\left(2J - \frac{J\Delta}{2}\right)\beta\right] + 2\exp\left[\left(\frac{J\Delta}{2} + J\right)\beta\right]\cosh\sqrt{3}D\beta\right\},$$

$$h_2 = \frac{1}{6Z}\left\{3\exp\left(-\frac{3J\Delta}{2}\beta\right)+\exp\left[-\left(2J-\frac{J\Delta}{2}\right)\beta\right]+2\exp\left[\left(\frac{J\Delta}{2}+J\right)\beta\right]\cosh\sqrt{3}D\beta\right\},$$
$$h_3 = \frac{1}{6Z}\left\{\exp\left(-\frac{3J\Delta}{2}\beta\right)-\exp\left[\left(\frac{J\Delta}{2}+J\right)\beta\right]\cosh\sqrt{3}D\beta\right\}, \quad (19)$$

substituting Eqs.(14) and (18) into Eq.(17), we get

$$\bar{F} = \frac{1}{3}+\frac{4}{9Z}\left\{2\exp\left[-\left(2J-\frac{J\Delta}{2}\right)\beta\right]+\exp\left[\left(\frac{J\Delta}{2}+J\right)\beta\right]\cosh(\sqrt{3}D\beta)\right\}. \quad (20)$$

In order to transmit π_S with fidelity better than any classical communication protocol, we require Eq.(18) to be strictly greater than $\frac{2}{3}$ (≈ 0.67), then the inequality can be written as

$$\exp\left[-\left(2J-\frac{J\Delta}{2}\right)\beta\right]-4\exp\left[\left(\frac{J\Delta}{2}+J\right)\beta\right]\cosh\sqrt{3}D\beta-3\exp\left(-\frac{3J\Delta}{2}\beta\right)>0. \quad (21)$$

It is not difficult to testify that only $J<0$ can satisfy the inequality (21), which means that quantum teleportation can be realized via a ferromagnetic chain, while the antiferromagnetic chain is not suitable for quantum teleportation.

Firstly, we consider the relation between average fidelity and pairwise entanglement. In Fig.2, we plot average fidelity versus concurrence when the temperature and the DM interaction are changed. Figure 2(a) is a plot of average fidelity versus concurrence when the temperature is allowed to change. Figure 2(b) is a plot of average fidelity versus concurrence when the DM interaction is allowed to change. From Figs.2(a) and 2(b), we can see that the entanglement does not completely reflect average fidelity. In other words, the same entanglement can yield different average teleportation fidelities.

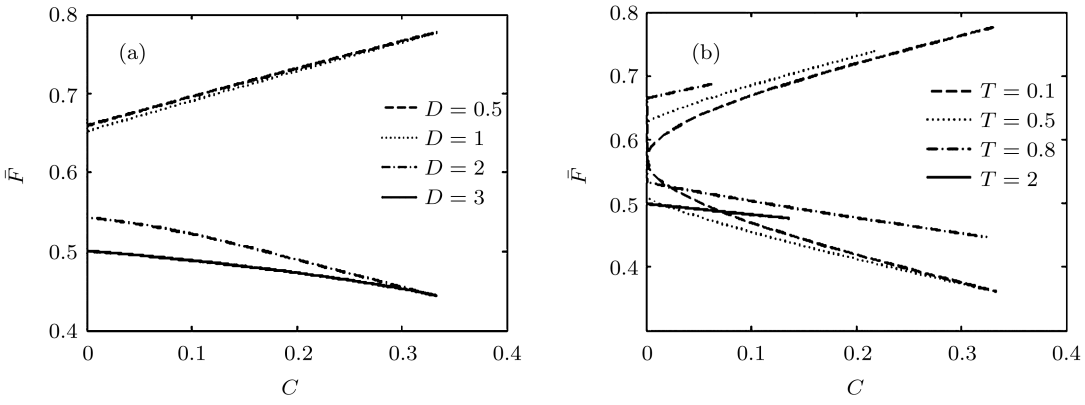


Fig.2. The average fidelity $\bar{F}$ versus concurrence C for ferromagnetic chain $J=-1$, $\Delta=0.4$. Figure 2(a) is plotted for varied DM interactions: $D=0.5$ (dashed line), $D=1$ (dotted line), $D=2$ (dashed-dotted line), $D=3$ (solid line). Figure 2(b) is plotted for varied temperatures $T=0.1$ (dashed line), $T=0.5$ (dotted line), $T=0.8$ (dashed-dotted line), $T=2$ (solid line).

Secondly, we discuss the relation between the average fidelity and the violation of Bell inequality in the quantum channel. The most commonly discussed Bell's inequality is the Clauser–Horne–Shimony–Holt (CHSH) inequality.[19] The CHSH operator is

$$\hat{B} = \boldsymbol{a}\cdot\boldsymbol{\sigma}\otimes(\boldsymbol{b}+\boldsymbol{b}')\otimes\boldsymbol{\sigma} + \boldsymbol{a}'\cdot\boldsymbol{\sigma}\otimes(\boldsymbol{b}-\boldsymbol{b}')\otimes\boldsymbol{\sigma}, \quad (22)$$

where $\boldsymbol{a}$, $\boldsymbol{a}'$, $\boldsymbol{b}$, $\boldsymbol{b}'$ are unit vectors. In the above notation, the Bell inequality is[20]

$$|\langle\hat{B}\rangle = |\mathrm{tr}(\rho\hat{B})| \leq 2, \tag{23}$$

where ρ is an arbitrary two-qubit state. The maximal amount of Bell's violation of a state is given by

$$B = 2\sqrt{\lambda + \tilde{\lambda}}, \tag{24}$$

where λ and $\tilde{\lambda}$ are the two largest eigenvalues of $T_\rho T_\rho^+$. The matrix T_ρ is determined completely by the correlation functions being a 3×3 matrix whose elements are $(T_\rho)_{nm} = \mathrm{tr}(\rho\sigma_n \otimes \sigma_m)$. Here $\sigma_1 \equiv \sigma_x$, $\sigma_2 \equiv \sigma_y$, and $\sigma_3 \equiv \sigma_z$ denote the usual Pauli matrices. We call quantity B the maximal violation measure, which indicates the Bell violation when $B > 2$ and the maximal violation when $B = 2\sqrt{2}$. Based on Ref.[21], the violation measure can be expressed as

$$M = \lambda + \tilde{\lambda} - 1, \tag{25}$$

which ranges from -1 to 1. The quantity M, which we call a violation measure of the Bell inequality in this paper, indicates a Bell violation when $M > 0$ and the maximal Bell violation when $M = 1$. From Eq.(7), the violation measure M is obtained as

$$M = \frac{8\,|\xi|^2}{9Z^2} + \frac{4(\mu-\varsigma)^2}{9Z^2} - \min\left\{\frac{4\,|\xi|^2}{9Z^2}, \frac{4(\mu-\varsigma)^2}{9Z^2}\right\} - 1. \tag{26}$$

In Fig.3, we plot average fidelity versus violation measure when the temperature and the DM interaction are changed. Figure 3(a) is a plot of average fidelity versus the violation measure when the temperature is allowed to change. Figure 3(b) is a plot of average fidelity versus the violation measure when the DM interaction is allowed to change. From Figs.3(a) and 3(b), we can see that the average fidelity can exceed $\frac{2}{3}$ while the Bell inequality is not violated, which implies that some thermally entangled states, which do not violate the Bell inequalities, can still be used for quantum teleportation.

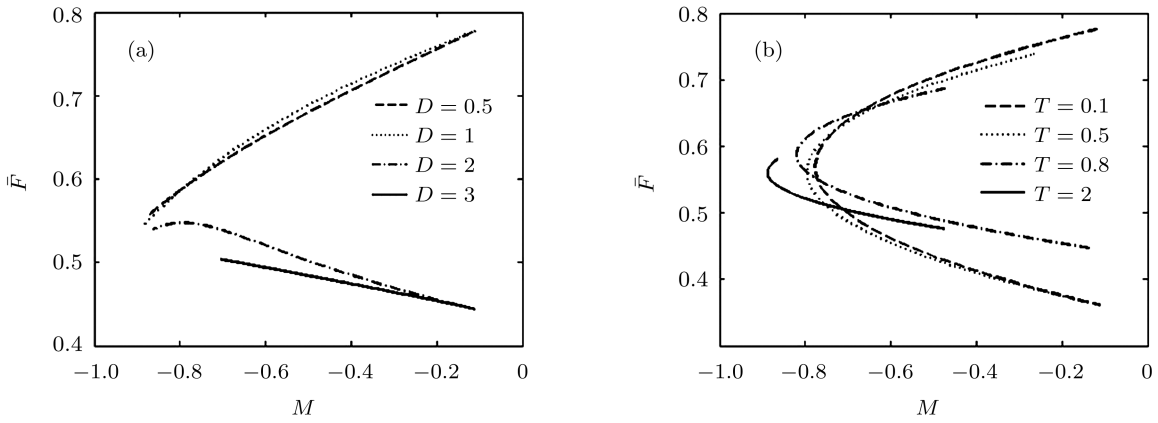


Fig.3. The average fidelity $\bar{F}$ versus violation measure M for ferromagnetic chain $J = -1$, $\Delta = 0.4$. Figure 3(a) is plotted for varied DM interactions: $D = 0.5$ (dashed line), $D = 1$ (dotted line), $D = 2$ (dashed-dotted line), $D = 3$ (solid line). Figure 3(b) is plotted for varied temperatures: $T = 0.1$ (dashed line), $T = 0.5$ (dotted line), $T = 0.8$ (dashed-dotted line), $T = 2$ (solid line).

5. Summary

In conclusion, we have investigated the thermal pairwise entanglement of a three-qubit Heisenberg XXZ chain with DM anisotropic antisymmetric interaction and quantum teleportation via the model. The pairwise entanglement first decreases, and then quickly increases with the increase of the DM interaction for the ferromagnetic chain. Quantum teleportation can be realized via a ferromagnetic chain, while not be realized in an anferromagnetic chain. The entanglement cannot completely reflect the average fidelity, the same concurrence can yield different average fidelities. The thermally entangled states in the model which do not violate the Bell inequalities, can still be used for quantum teleportation.

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