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Controlling nonclassical properties of the two-photon process by a time-varying field*

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The interactions between a two-level atom and a field via two-photon transition without rotating wave approximation have been investigated. We emphasize the dynamic behaviors of the atomic population inversion, the field squeezing, and the atomic dipole squeezing numerically when the field frequency varies with time in the forms of sine and rectangle. Some interesting phenomena are discovered and discussed. The good periodic character of the atomic population inversion in the standard two-photon Jaynes–Cummings model is weakened by the influence of the sine field frequency modulation. The rectangular field frequency modulation can change the correlation among different oscillations suddenly and induce new collapse-revival processes of the atomic population inversion. The field squeezing increases at the beginning of time, but then decreases and loses as the time increases after it reaches the maximum due to the sine modulation. The effects of the rectangular modulation on the field squeezing depend mostly on the appearance time of the modulation. The atomic dipole squeezing is weakened under the influence of the sine or rectangular modulation. Our results indicate that it is possible to perform the dynamic controlling of the system properties by changing the parameters of the system with time. This implies that one can dynamically control a quantum information process by choosing the system modulation properly.

Keywords: two-photon transition, quantum controlling, field squeezing
PACC: 0365, 4250

1. Introduction

In recent years, entanglement and decoherence have been two subjects of current interest because they lie at the heart of proposals for quantum information processing and quantum computation. One of the challenges of realizing a quantum information process is control of the evolution of qubits in the presence of environmental noises and manipulation inaccuracies. Since the 1970s many techniques for controlling single quantum systems have been developed. By obtaining complete control over single quantum systems, people are exploring untouched regimes of Nature in the hope of discovering new and unexpected phenomena.[1] For example, methods have been developed for trapping a single atom in an "atom trap", isolating it from the rest of the world and allowing us to probe many different aspects of its behavior with incredible precision.[2] The two-qubit states with the desired degree of entanglement can be generated by using time-dependent atom-cavity coupling.[3] Recently, some methods have been posed to realize the quantum controlling.[4] There are a few works on the interaction between an atom and a field with time-varying frequency[5] or time-dependent amplitude.[6] In fact, the time variation of the frequency will influence the interaction and cause the coupling between an atom and a field to vary with time. The time-dependent coupling can change the dynamic properties of the atom and field with one-photon transition greatly.[7]

In this paper, we study how to control the quantum properties by introducing the modulation of the field frequency and discuss the influence of the field frequency modulation on the dynamic properties of a two-level atom and an electromagnetic field via two-photon transition with the virtual-photon processes, which are the source of the Lamb shift[8] and also lead to the quantum noise of the system.[9] Here we mainly investigate three dynamic properties: the atomic population inversion, the field squeezing, and the atomic dipole squeezing. And our attention is restricted to two typical cases, the frequency of the field varying with time in the forms of sine and rectangle. These two modulation forms are typical and easier to implement in experiments. Based on this model, we find

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that the dynamic properties are very different from the previous static field case and show striking new features. The good periodic character of the atomic population inversion in the standard two-photon Jaynes–Cummings model (TPJCM) is weakened by the influence of the sine field frequency modulation. The rectangular field frequency modulation can change the correlation among different oscillations suddenly and induce new collapse-revival processes of the atomic population inversion. Owing to the influence of the sine modulation, the field squeezing increases at the beginning, but then decreases and vanishes as the time increases after it reaches the maximum. Some quantum correlation is enhanced first and then lost as the time increases due to the sine modulation. The effects of the rectangular modulation on the field squeezing depend mostly on the appearance of the modulation. Thus atomic dipole squeezing is weakened under the influence of the sine or rectangular modulation. These results indicate that we can complete the dynamic controlling of the simple JCM by changing the field frequency. In the future, we can extend the research to more complex systems. And our conclusions may also be an important reference for the experimental implementation.

This paper is organized as follows. In Section 2, the model and the basic theory to study the near-resonant interaction between a single atom and an electromagnetic field with a two-photon process are given. In Sections 3, 4 and 5, we discuss the influence of the field frequency modulation on the atomic population, the field squeezing and the atomic dipole squeezing, respectively. Furthermore, we study the relationship between the atomic population inversion and the squeezing properties. Finally, we summarize our results in Section 6.

2. The model and the equations of motion

We consider a system in which a two-level atom interacts with an electromagnetic field via a two-photon process. The frequency of the field varies with time. When the variation of frequency is very small, the Hamiltonian for the system without the rotating wave approximation (RWA) can be written as

$$H = \hbar\nu a^{\dagger}a + \frac{1}{2}\hbar\omega\sigma_z + \hbar g(\sigma_+ a^2 + \sigma_- a^{\dagger 2} + \sigma_+ a^{\dagger 2} + \sigma_- a^2). \quad (1)$$

Here $a^{\dagger}$ (a) is the creation (annihilation) operator of the field with time-varying frequency $\nu(t)$. $\sigma_{\pm}$ and σ_z denote the usual pseudospin operators acting in the space of atomic states, and satisfy the relations $[\sigma_+, \sigma_-] = \sigma_z$ and $[\sigma_z, \sigma_{\pm}] = \pm\sigma_{\pm}$, and ω is the atomic transition frequency, and g represents the atom-field coupling coefficient. The frequency $\nu(t)$ of the field can be rewritten as $\nu(t) = \nu_0 + f(t)$, and $f(t)$ is a time-dependent function and represents the modulation of the radiation field frequency. From Ref.[10], we know that the coupling coefficient g is proportional to $\sqrt{\nu(t)/V(t)}$, where the quantization volume $V(t)$ is dependent on time t, and takes the form $V(t) = V_0/[1 + f(t)/\nu_0]$. The coupling coefficient g can be rewritten as $g{=}g_0[1 + f(t)/\nu_0]$, where g_0 is the coupling constant as the field frequency is constant. The last two terms are the counter-rotating terms (or energy-nonconserving terms) which exhibit a rapid oscillatory behavior and represent the virtual-photon processes.

We assume the atom initially in the upper level $|1\rangle$. The wave function of the system at an arbitrary time t can be written as

$$|\psi(t)\rangle = \sum_n [C_{1,n}(t)|1,n\rangle + C_{0,n}(t)|0,n\rangle] \quad (2)$$

with $\sum_n |C_{1,n}(0)|^2 = 1$, and $C_{0,n}(0) = 0$. Here $|1,n\rangle$ ($|0,n\rangle$) represents the atom in its excited state $|1\rangle$ (ground state $|0\rangle$) with n photons. Suppose the field is initially in a coherent state, which is described by $\rho_{nn}(0) = \langle n\rangle^n \mathrm{e}^{-\langle n\rangle}/n!$. Here $\langle n\rangle$ is the mean number of photons and $\rho_{nn}(0)$ is the probability of finding n photons in the field initially. For a coherent state the probability distribution $\rho_{nn}(0)$ obeys the Poisson distribution.

Using the time-dependent Schrödinger equation with the Hamiltonian (1), we can obtain the infinite set of equations of motion for the probability amplitudes $C_{1,n}(t)$ and $C_{0,n}(t)$:

$$\frac{\mathrm{d}C_{1,n}(t)}{\mathrm{d}t} = -\mathrm{i}\, n\nu(t)C_{1,n}(t) - \frac{\mathrm{i}}{2}\omega C_{1,n}(t) - \mathrm{i}\, g\sqrt{(n+1)(n+2)}C_{0,n+2}(t) - \mathrm{i}\, g\sqrt{(n-1)n}C_{0,n-2}(t), \quad (3a)$$

$$\frac{\mathrm{d}C_{0,n}(t)}{\mathrm{d}t} = -\mathrm{i}\,n\nu(t)C_{0,n}(t) + \frac{\mathrm{i}}{2}\omega C_{0,n}(t) - \mathrm{i}\,g\sqrt{(n+1)(n+2)}C_{1,n+2}(t) - \mathrm{i}\,g\sqrt{(n-1)n}C_{1,n-2}(t). \tag{3b}$$

Setting

$$C_{1,n}(t) = \left[\exp -\mathrm{i}\left(n\nu_0 + \frac{1}{2}\omega\right)t\right]\exp\left[-\mathrm{i}\,n\int_0^t f(t)\mathrm{d}t\right]M_{1,n}(t)$$

and

$$C_{0,n}(t) = \exp\left[-\mathrm{i}\left(n\nu_0 - \frac{1}{2}\omega\right)t\right]\exp -\mathrm{i}\,n\int_0^t f(t)\mathrm{d}t\Big]M_{0,n}(t),$$

we obtain the equations of motion for the slowly varying amplitudes $M_{1,n}(t)$ and $M_{0,n}(t)$.

$$\frac{\mathrm{d}M_{1,n}(t)}{\mathrm{d}t} = -\mathrm{i}\,g\sqrt{(n+1)(n+2)}\,\mathrm{e}^{\mathrm{i}(-2\nu_0+\omega)t}\mathrm{e}^{-2\,\mathrm{i}\int_0^t f(t)\mathrm{d}t}M_{0,n+2}(t) - \mathrm{i}\,g\sqrt{n(n-1)}\,\mathrm{e}^{\mathrm{i}(2\nu_0+\omega)t}\mathrm{e}^{2\,\mathrm{i}\int_0^t f(t)\mathrm{d}t}M_{0,n-2}(t), \tag{4a}$$

$$\frac{\mathrm{d}M_{0,n}(t)}{\mathrm{d}t} = -\mathrm{i}\,g\sqrt{(n-1)n}\,\mathrm{e}^{\mathrm{i}(2\nu_0-\omega)t}\mathrm{e}^{2\,\mathrm{i}\int_0^t f(t)\mathrm{d}t}M_{1,n-2}(t) - \mathrm{i}\,g\sqrt{(n+1)(n+2)}\,\mathrm{e}^{-\,\mathrm{i}(2\nu_0+\omega)t}\mathrm{e}^{-2\,\mathrm{i}\int_0^t f(t)\mathrm{d}t}M_{1,n+2}(t). \tag{4b}$$

It is obvious that the terms including $M_{0,n+2}(t)$ and $M_{1,n-2}(t)$ in Eqs.(4) are the contribution of real-photon processes to the probability amplitude, and the terms including $M_{0,n-2}(t)$ and $M_{1,n+2}(t)$ are the contribution of virtual-photon processes.

In the case of $f(t) = 0$, the Hamiltonian (1) is reduced to that of the standard TPJCM. Equation (4) can be rewritten as

$$\frac{\mathrm{d}M_{1,n}(t)}{\mathrm{d}t} = -\mathrm{i}\,g_0\sqrt{(n+1)(n+2)}\,\mathrm{e}^{\mathrm{i}(\omega-2\nu_0)t}M_{0,n+2}(t) - \mathrm{i}\,g\sqrt{n(n-1)}\,\mathrm{e}^{\mathrm{i}(2\nu_0+\omega)t}M_{0,n-2}(t), \tag{5a}$$

$$\frac{\mathrm{d}M_{0,n}(t)}{\mathrm{d}t} = -\mathrm{i}\,g_0\sqrt{(n-1)n}\,\mathrm{e}^{-\,\mathrm{i}(\omega-2\nu_0)t}M_{1,n-2}(t) - \mathrm{i}\,g\sqrt{(n+1)(n+2)}\,\mathrm{e}^{-\,\mathrm{i}(2\nu_0+\omega)t}M_{1,n+2}(t). \tag{5b}$$

In the following discussion, we focus on two typical cases: the frequency of the field varies with time in the forms of sine and rectangle. For the sine-varying case, the time-dependent function $f(t)$ can be defined as $f(t) = \alpha\sin(\beta t)$, where α and β are the parameters for the frequency modulation. We shall limit our discussion to the regime where the modulation amplitude $\alpha \ll \omega$ (and ν_0) and the modulation frequency β are also small. The field frequency varies with time in the form of $\nu(t) = \nu_0 + \alpha\sin(\beta t)$. For the rectangular-varying case, the time function of the frequency modulation, $f(t)$, is a form of pulse, $f(t) = \alpha$ for $t \in [mT + t_0, mT + \tau + t_0]$ and $(m = 0, 1, 2, \ldots)$, otherwise $f(t) = 0$. Here T, τ, α and t_0 are the period, the width, the amplitude and the initial time of the rectangular frequency modulation, respectively.

The above equations can be solved numerically. After obtaining $M_{1,n}(t)$ and $M_{0,n}(t)$, we can calculate and study the quantum dynamic properties of the atom and the field, such as the atomic population inversion, the field squeezing and the atomic dipole squeezing. In the following discussion, the mean number of photons for the coherent state is assumed as $\langle n\rangle = 25$ in the initial field. We will firstly give the numerical results of the standard TPJCM. It will be useful for the investigation of the influences of the field frequency modulation on the evolution of the properties of the field and the atom by comparing these results with those of the standard TPJCM.

3. Atomic population inversion

The atomic population inversion, which is defined as the difference between the excited-state and the ground-state probabilities, can be written as

$$W(t) = \sum_n \left[|C_{1.n}(t)|^2 - |C_{0.n}(t)|^2\right] = \sum_n \left[|M_{1.n}(t)|^2 - |M_{0.n}(t)|^2\right]. \tag{6}$$

For the standard TPJCM ($f(t) = 0$) case with RWA and the field being initially in a coherent state, we

find that the typical periodic collapse-revival phenomena occur in the time-evolution process of the atomic population inversion (as shown in Fig.1(a)). In addition, the duration of collapse is much wider than that of revival in a period. The shape of every revival envelope is very sharp and exhibits good symmetry with respect to its own maximum amplitude. In the single-photon process, the revivals of the inversion become wider as the time t increases, so the first two successive revivals exhibit a head-tail overlap.[11] Eventually, owing to the overlap, several revivals interfere with each other, displaying an irregular character. Unlike the single-photon case, the revivals in the two-photon case are perfect with no spreading as t increases. The reason for this periodic phenomena of Rabi oscillations is well understood now. In the case of the average photons $\langle n\rangle \gg 1$, equation (6) can be written as $W(t) \approx \exp[-2\langle n\rangle \sin^2(g_0t)]\cos[\langle n\rangle \sin(2g_0t) - g_0t]$ approximately. The exponential function $\exp[-2\langle n\rangle \sin^2(g_0t)]$ will rapidly decay to zero and then grow to unity in each period, so it is called the collapse-revival function and governs the short behavior of the Rabi oscillation envelope. The cosine function $\cos[\langle n\rangle \sin(2g_0t) - g_0t]$ shows that the quantum Rabi oscillations of the inversion have combined into a collective oscillation, which can be called the combination (or collectivity) Rabi oscillations.

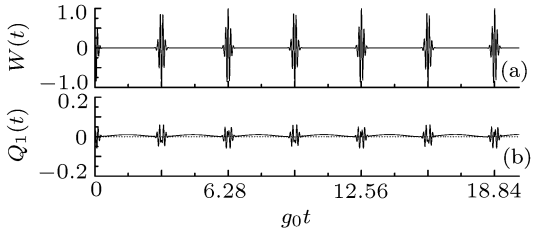


Fig.1. The time evolution of the quantum properties (solid line) in the standard TPJCM with RWA for $\langle n\rangle = 25$, $\omega_0 = 1000g_0$, $\nu_0 = 500g_0$. (a) The time evolution of the atomic population inversion. (b) The time evolution of the field squeezing parameter $Q_1(t)$. The dotted line at zero represents the reference line.

In Fig.2(a), we have plotted the time evolution of the atomic population inversion $W(t)$ without RWA in the case of on-resonance, i.e. $\omega = 2\nu_0 = 1000g_0$. Comparison of the atomic population inversion with RWA (Fig.1(a)) with that without RWA (Fig.2(a)) shows the effect of the virtual-photon processes on the inversion. The envelope does not collapse altogether to zero but there are rapid oscillations during the otherwise relaxed period for the RWA case. From Eqs.(3a) and (3b), it is clear that the coefficient $C_{0,n}(t)$ can be obtained not only on the transition from the state $|1,n\rangle$ to the state $|0,n+2\rangle$, but also on the transition from $|1,n\rangle$ to the state $|0,n-2\rangle$. The transition from the state $|1,n\rangle$ to the state $|0,n+2\rangle$ makes contribution to $C_{0,n}(t)$ due to the real-photon processes, which is pronounced for $\omega = 2\nu_0$. The transition from the state $|1,n\rangle$ to the state $|0,n-2\rangle$ makes a contribution to $C_{0,n}(t)$ due to the virtual-photon processes, which can become as prominent as the real-photon processes for $\omega \neq 2\nu_0$. The mini periodic distinct collapse-revival phenomena also occur in the time evolution process of the small amplitude rapid oscillations resulting from the virtual-photon processes. The physical essence of the mini oscillations is that the virtual-photon processes are non-energy-conserving processes, and the rapidly emitted and absorbed virtual photons cause energy to change in the form of two-photon energy and create a virtual-photon cloud around the field source. However, the main Rabi oscillations display sharp revivals and the dead oscillations (where the atomic population inversion collapses to zero) still exist

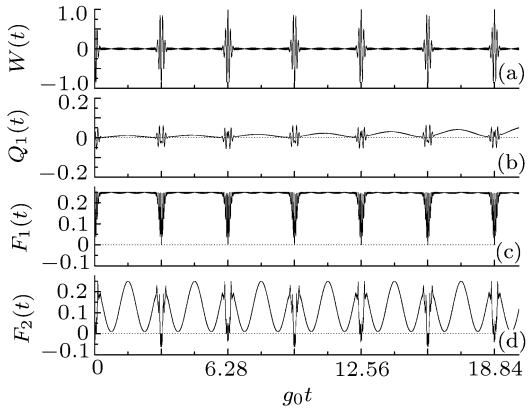


Fig.2. The time evolution of the quantum properties (solid line) in the standard TPJCM without RWA for $\langle n\rangle = 25$. (a) The time evolution of the atomic population inversion $W(t)$. (b) The time evolution of the field squeezing parameter $Q_1(t)$; (c) and (d) The time evolution of the atomic squeezing parameters $F_1(t)$ and $F_2(t)$. The dotted line at zero represents the reference line.

as well as the inversion in the RWA case. The virtual processes do not change the period of the main collapses-revivals. The envelopes of the revivals corresponding to the main Rabi oscillations and the mini rapid oscillations display strict symmetry with respect to the maximum amplitude. In the following discussion, we will investigate the influence of the different field frequency modulation on the evolution of the atomic population inversion $W(t)$, especially focusing on the evolution of the rapid oscillations of

the atomic population inversion $W(t)$ corresponding to the virtual-photon processes.

First, we consider the frequency of the field varying with time in the form of sine, i.e. $f(t) = \alpha \sin(\beta t)$. In Figs.3(a), 4(a), we have plotted the time evolution of the atomic population inversion $W(t)$ for various values of the system parameters in the case of $\omega = 2\nu_0 = 1000g_0$. When both the amplitude α and the angular frequency β of the field frequency modulation are very small, the influence of the frequency modulation on the collapses and revivals is very weak. The time evolution of the atomic population inversion $W(t)$ in this case is similar to that without field frequency modulation (see Fig.2(a)). When the amplitude α increases, the influence of the frequency modulation on the population inversion is enhanced (as shown in Fig.3(a), $\alpha = 10g_0, \beta = 0.5g_0$). The amplitude of the main Rabi oscillations decreases and the collapse time increases as the time t increases as indicated by comparing Fig.3(a) with Fig.2(a). The shape of the revival envelope cannot keep its symmetry with respect to the maximum amplitude any longer. For the Poisson distribution of the initial coherent field, the root-mean-square deviation in the photon number Δn is equal to $\sqrt{\langle n \rangle}$. An estimate of the collapse time t_c can therefore be obtained from the condition $(\Omega_{\langle n \rangle + \sqrt{\langle n \rangle}} - \Omega_{\langle n \rangle - \sqrt{\langle n \rangle}})t_c \sim 1$. Here Ω is the Rabi oscillation frequency, which is equal to $\sqrt{\Delta^2 + 4g^2(n+1)(n+2)}$. We may approximately regard the field frequency variation α as the effective detuning Δ. Since $\langle n \rangle \gg 1$, t_c reads

$$t_c \sim \frac{1}{\Omega_{\langle n \rangle + \sqrt{\langle n \rangle}} - \Omega_{\langle n \rangle - \sqrt{\langle n \rangle}}} \simeq \frac{1}{2g\sqrt{\langle n \rangle}} \left[1 + \frac{\Delta^2}{4g^2 \langle n \rangle^2}\right]^{1/2} . \quad (7)$$

Due to $g = g_0[1 + \Delta/\nu_0] \approx g_0$, t_c can be written as $t_c = 1/(2g_0\sqrt{\langle n \rangle})\left[1 + \Delta^2/(4g_0^2\langle n \rangle^2)\right]^{1/2}$. The collapse time will increase as the detuning value Δ increases. So the collapse time increases when the amplitude of sine modulation increases. On the other hand, the durations of the revivals become wider and wider as the time t increases, so that the two successive revivals tend to exhibit a head-tail overlap similar to that of the single-photon process. The good periodic character of the atomic population inversion $W(t)$ in the standard TPJCM is weakened by the influence of the field frequency modulation. However, when β increases (see Fig.4(a)), the main Rabi oscillation envelopes have not much change as indicated by comparing Fig.4(a) with Fig.3(a). So we conclude that the effect of the amplitude α of field frequency modulation on the atomic population inversion $W(t)$ is stronger than the angular frequency β.

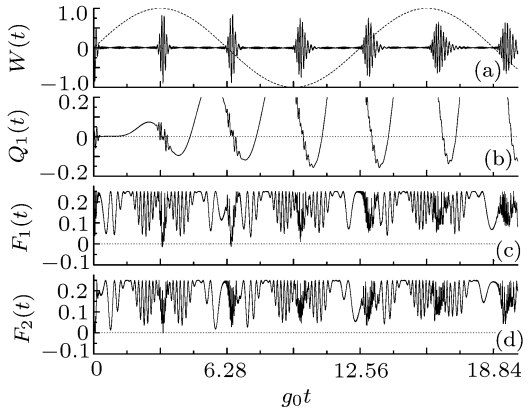


Fig.3. The time evolution of the quantum dynamic properties (solid line) for $\langle n \rangle = 25$ in the TPJCM without RWA. The field frequency varies in the form of sine for $\alpha = 10g_0$, $\beta = 0.5g_0$ (dashed line). (a) The time evolution of the atomic population inversion $W(t)$. (b) The time evolution of the field squeezing parameter $Q_1(t)$; (c) and (d) The time evolution of the atomic squeezing parameters $F_1(t)$ and $F_2(t)$. The dotted line at zero represents the reference line.

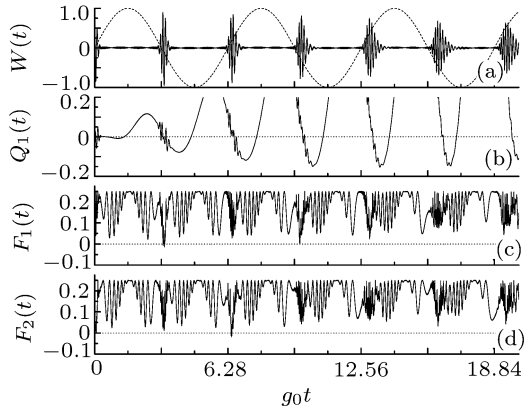


Fig.4. Same as Fig.3, but the field frequency varies in the form of sine for $\alpha = 10g_0$, $\beta = g_0$.

Then, we consider the frequency of the field varying with time in the form of a rectangle. In the following, we investigate the influence of the different pulses on the evolution of the atomic population inversion (as shown in Figs.5(a), 6(a), $\omega = 2\nu_0 = 1000g_0$). When the pulses occur on each revival and the period of the rectangular frequency modulation is exactly equal to that of the collapse-revival for the standard TPJCM, T_0, the sudden change of the field frequency will change the phase of each Rabi oscillation. At the time of the beginning or ending of the pulses, the

sudden jumping of the field frequency will change the correlation among different Rabi oscillations, from being correlated to uncorrelated or from being uncorrelated to correlated and thus induce some new collapse-revival processes of the Rabi oscillations (see Fig.5(a)). If the pulses occur at the time of collapses of the main Rabi oscillation (as shown in Fig.6(a)) and the period is also equal to T_0, the time evolution of the main Rabi oscillations and the mini rapid oscillations is influenced weakly since the Rabi oscillations are uncorrelated to each other at the time of collapse. Due to the build-up effect of pulses, the revival duration becomes longer and longer as time t increases as indicated by comparing Fig.5(a) with Fig.2(a). When the period is equal to $T_0/2$ and pulses occur at every revival and collapse region (as shown in Fig.7), the sudden jumping of the field frequency can transfer the uncorrelated Rabi oscillations to the correlated ones and lead to new revivals in the time evolution of the inversion. So the interference are weakened at revival regions, and enforced at collapse regions. Furthermore, the revivals are not well separated but overlapped, and in fact the inversion behaves in a fairly irregular manner as seen by comparing Fig.7 with Fig.2(a). In the regions without the rectangular frequency modulation, the case is on-resonance ($\Delta = 0$) and the amplitude of the Rabi

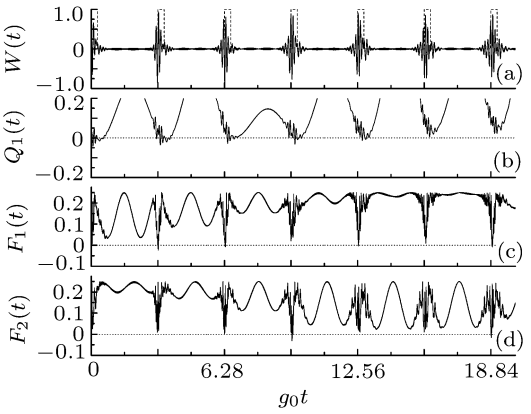


Fig.5. Same as Fig.3, but the field frequency varies in the form of a rectangle for $\alpha = 10g_0$, $g_0T = 3.14$, $g_0t_0 = 0$, $g_0\tau = 0.3$.

oscillations is unchanged. However, during the rectangular frequency modulation, it is equivalent to the off-resonant case. So the amplitude of the Rabi oscillations in these regions is changed greatly and the inversion will exhibit the Rabi oscillations around a new basic value.

The above research indicates that we can affect and control the Rabi oscillations and the rapid virtual-photon processes by the sine and rectangular field frequency modulations.

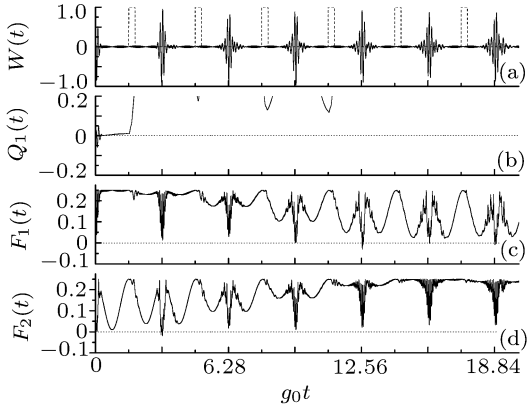


Fig.6. Same as Fig.3, but the field frequency varies in the form of a rectangle for $\alpha = 10g_0$, $g_0T = 3.14$, $g_0t_0 = 1.57$, $g_0\tau = 0.3$.

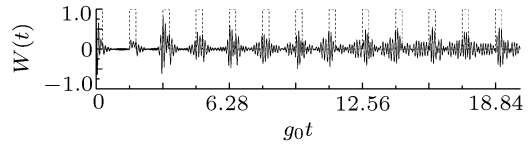


Fig.7. The time evolution of the atomic population inversion (solid line) for $\langle n\rangle = 25$ in the TPJCM without RWA. The field frequency varies in the form of a rectangle for $\alpha = 10g_0$, $g_0T = 1.57$, $g_0t_0 = 0$, $g_0\tau = 0.3$ (dashed line).

4. The field squeezing

In order to investigate the squeezing properties of the radiation field, we introduce two slowly varying Hermitian quadrature operators X_1 and X_2 of the field

$$X_1 = \frac{1}{2}(a\mathrm{e}^{\mathrm{i}\int_0^t \nu dt} + a^\dagger \mathrm{e}^{-\mathrm{i}\int_0^t \nu \mathrm{d}t}), \tag{8a}$$

$$X_2 = \frac{1}{2\mathrm{i}}(a\mathrm{e}^{\mathrm{i}\int_0^t \nu \mathrm{d}t} - a^\dagger \mathrm{e}^{-\mathrm{i}\int_0^t \nu \mathrm{d}t}), \tag{8b}$$

with the communication relation $[X_1,\ X_2] = \mathrm{i}/2$, and the corresponding uncertainty relation

$$(\Delta X_1)^2(\Delta X_2)^2 \geqslant \frac{1}{16}, \tag{9}$$

where $(\Delta X_i)^2$ is the variance of X_i $(i = 1, 2)$, which is defined as $\langle X_i^2\rangle - \langle X_i\rangle^2$. If one of the variances $(\Delta X_1)^2$ or $(\Delta X_2)^2$ of a state satisfies the relation

$$Q_i = (\Delta X_i)^2 - \frac{1}{4} < 0 \quad (i = 1, 2), \tag{10}$$

the state is called the squeezed state.[12]

Using the wave function of the system $|\psi(t)\rangle$ and solving Eqs.(4a) and (4b), we obtain

$$\begin{aligned} Q_1(t) = \frac{1}{2}\Big\{ & \sum_n n \left[M^*_{1,n}(t)M_{1,n}(t) + M^*_{0,n}(t)M_{0,n}(t)\right] \\ & + \sum_n \sqrt{(n+1)(n+2)}\,\mathrm{Re}\left[M^*_{1,n+2}(t)M_{1,n}(t) + M^*_{0,n+2}(t)M_{0,n}(t)\right]\Big\} \\ & - \Big\{ \sum_n \sqrt{n+1}\,\mathrm{Re}\left[M^*_{1,n}(t)M_{1,n+1}(t) + M^*_{0,n}(t)M_{0,n+1}(t)\right]\Big\}^2, \end{aligned} \tag{11a}$$

$$\begin{aligned} Q_2(t) = \frac{1}{2}\Big\{ & \sum_n n[M^*_{1,n}(t)M_{1,n}(t) + M^*_{0,n}(t)M_{0,n}(t)] \\ & - \sum_n \sqrt{(n+1)(n+2)}\,\mathrm{Re}\left[M^*_{1,n+2}(t)M_{1,n}(t) + M^*_{0,n+2}(t)M_{0,n}(t)\right]\Big\} \\ & - \Big\{ \sum_n \sqrt{n+1}\,\mathrm{Im}\left[M^*_{1,n}(t)M_{1,n+1}(t) + M^*_{0,n}(t)M_{0,n+1}(t)\right]\Big\}^2. \end{aligned} \tag{11b}$$

First, we consider the frequency of the field varying with time in the form of sine. In Figs.1(b)–5(b), we have plotted the time evolution of the parameter Q_1 for various system parameters in the case of $\omega = 2\nu_0 = 1000g_0$. In the corresponding figures, a horizontal line at 0 is drawn as a reference. Whenever the curve crosses below this line the field is squeezed. We find that only the component Q_1 may display squeezing in certain time regions. If $\alpha = 0$, the model is equivalent to the standard TPJCM. In the standard TPJCM, the field displays squeezing periodically as the field is initially in a coherent state (as shown in Fig.2(b)). This is different from that of the one-photon JCM, in which the squeezing occurs only at the beginning of time. The squeezing state is the eigenstate of the two-photon operator a^2.[13] The TPJCM is a process for realizing the squeezing state. So the squeezing can be kept for longer time and displays good periodic squeezing in TPJCM. Comparing Fig.2(b) (without RWA) with Fig.1(b) (with RWA), the value of $Q_1(t)$ increases as the time t increases in the non-RWA case. But there is no obvious change of the squeezing of Q_1. The squeezing degree of Q_1 is also small and the squeezing depth can only reach -0.058. We notice that the field squeezing also occurs periodically near $T = n\pi$ $(n = 0, 1, 2\ldots)$, viz., the time regions of the field squeezing correspond to the revival ones of $W(t)$. In the revival regions, the field is in good order due to the constructive correlation of various Rabi oscillations. The quantum chaos of the field is squeezed.

Here we care more about whether we can use the sine and rectangular frequency modulation of the field to yield noticeable squeezing. We consider the sine modulation first. Comparing Fig.3(b) ($\alpha = 10g_0, \beta = 0.5g_0$) with Fig.2(b), there is not much change of the squeezing degree of $Q_1(t)$ at the very beginning time. However, the depth and duration of the squeezing regions can be enhanced as the time t increases. For example, the squeezing depth can be enhanced to -0.156 in the fifth squeezing region in Fig.3(b). So the large amplitude of sine modulation is beneficial to the field squeezing. Now we increase the angular frequency of the field frequency modulation, i.e. $\beta = g_0$. The field squeezing only changes weakly as is seen by comparing Fig.4(b) with Fig.3(b). In conclusion, the field squeezing is more sensitive to the amplitude α than to the angular frequency β. In order to understand the field squeezing property in more detail, its long time evolution status has been constructed in Fig.8. We find that the field squeezing decreases gradually and disappears finally as the time increases after the field squeezing degree reaches the maximum (see Fig.8). Due to the constructive correlation of various Rabi oscillations, the field squeezing can be realized periodically within the time regions of the revival of $W(t)$ within the beginning of time, viz. $g_0t = 9\pi$. However, the quantum correlation tends to be out-of-order as the time increases due to the sine modulation. The quantum correlation property of the TPJCM tends to that of the one-photon JCM and some quantum correlation of the two-photon process is lost, and in turn the squeez-

ing is lost.

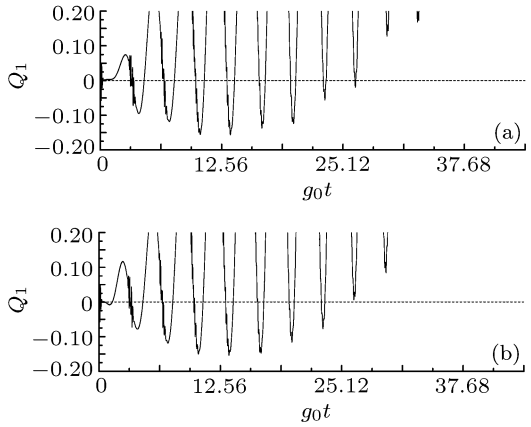


Fig.8. The time evolution of the field squeezing $Q_1(t)$ for $\langle n\rangle = 25$ in the TPJCM without RWA. The field frequency varies in the form of sine for (a) $\alpha = 10g_0$, $\beta = 0.5g_0$; (b) $\alpha = 10g_0$, $\beta = g_0$.

We now turn our attention to the case of the field frequency varying with time in the form of a rectangle. It is well known that the squeezing of the field is mostly dependent on the atomic initial state as well as the initial state of the field. If the field frequency varies in the form of a rectangle, this will lead to the atom interacting with field in a different initial state frequently and the field squeezing effect will be changed. The time evolutions of Q_1 for different initial time of the rectangular frequency modulation are shown in Figs.5(b) and 6(b). When the first pulse occurs at the beginning of time which is the squeezing region (as shown in Fig.5(b)), the squeezing still appears in the same time regions, but the duration and depth of the squeezing decrease and even disappear after $t > 15/g_0$ as seen by comparing Fig.5(b) with Fig.2(b). Here the squeezing regions still correspond to the revival regions of the atomic population inversion. When the first pulse occurs in the non-squeezing regions (see Fig.6(b)), we find that component Q_1 shows squeezing only under the rigid condition that it is squeezed before the first modulation pulse, or it does not squeeze at all permanently. Comparing the influence of the sine frequency modulation with that of the rectangular frequency modulation on the field squeezing, we can understand why the large angular frequency modulation is disadvantaged to the field squeezing. Unlike the small angular frequency β of sine frequency modulation, the large angular frequency of sine modulation and the rectangular frequency modulation are two kinds of quick modulation. The slow modulation is good for field squeezing, while the quick modulation is not useful to keep the field squeezing.

5. Atomic dipole squeezing

A two-level atom is most conveniently described by the Pauli raising and lowering operators σ_+ and σ_- and the inversion operator $\sigma_z = \sigma_+\sigma_- - \sigma_-\sigma_+$. From the Pauli raising and lowering operators σ_+ and σ_-, we define two slowly varying Hermitian quadrature operators

$$S_1 = \frac{1}{2}(\sigma_+ \mathrm{e}^{-\mathrm{i}\,\omega_0 t} + \sigma_- \mathrm{e}^{\mathrm{i}\,\omega_0 t}), \tag{12a}$$

$$S_2 = \frac{1}{2\mathrm{i}}(\sigma_+ \mathrm{e}^{-\mathrm{i}\,\omega_0 t} - \sigma_- \mathrm{e}^{\mathrm{i}\,\omega_0 t}), \tag{12b}$$

$$S_3 = \frac{1}{2}\sigma_z. \tag{12c}$$

In fact, S_1 and S_2 correspond to the in-phase and out-of-phase components for the atomic polarization, respectively. The communication relation

$$[S_1, S_2] = \mathrm{i}\, S_3 \tag{13}$$

leads to the following Heisenberg uncertainty relation

$$(\Delta S_1)^2(\Delta S_2)^2 \geq \frac{1}{4}|\langle S_3\rangle|^2, \tag{14}$$

where $(\Delta S_i)^2 = \langle S_i^2\rangle - \langle S_i\rangle^2$ $(i = 1, 2)$ is the variance in the quadrature component S_i of the atomic dipole. The atomic state is said to be squeezed if the following condition holds.

$$F_i = (\Delta S_i)^2 - \frac{1}{2}|\langle S_3\rangle| < 0, \quad (i = 1, 2). \tag{15}$$

Using the wave function of the system $|\psi(t)\rangle$ and solving Eqs.(4a) and (4b), we obtain

$$F_1 = \frac{1}{4} - \left\{\mathrm{Re}\left[\sum_n M_{1,n}^*(t)M_{0,n}(t)\right]\right\}^2 - \frac{1}{4}\left|\sum_n \left[|M_{1.n}(t)|^2 - |M_{0.n}(t)|^2\right]\right|, \tag{16a}$$

$$F_2 = \frac{1}{4} - \left\{\mathrm{Im}\left[\sum_n M_{1,n}^*(t)M_{0,n}(t)\right]\right\}^2 - \frac{1}{4}\left|\sum_n \left[|M_{1.n}(t)|^2 - |M_{0.n}(t)|^2\right]\right|. \tag{16b}$$

First, we consider the field frequency varying with time in the form of sine. The time evolution of the parameters F_i for various system parameters in the case of on-resonance ($\omega = 2\nu_0 = 1000g_0$) is shown in Figs.2(c), 2(d)–4(c), 4(d). In the corresponding figures, a horizontal line at 0 is drawn as a reference.

Whenever the curve crosses below this line, the atomic operators are squeezed. For the standard TPJCM ($f(t) = 0$) and the field being initially in a coherent state, F_1 does not squeeze at all (see Fig.2(c)). F_2, on the other hand, shows recurrently squeezing (see Fig.2(d)). When the field frequency varies in the form of sine (see Figs.3(c) and 3(d), $\alpha = 10g_0, \beta = 0.5g_0$, and Figs.4(c) and 4(d), $\alpha = 10g_0, \beta = g_0$), F_1 and F_2 display squeezing alternately. The squeezing of F_1 and F_2 disappears permanently after a certain time. Moreover, the larger the angular frequency of the field frequency modulation β is, the earlier the squeezing disappears. After that, the time evolution properties of the atomic dipole operators tend to an out-of-order state. In total, the squeezing is weakened due to the sine modulation of the field. We notice that the squeezing regions correspond to the revival ones of the atomic population inversion whether the field frequency is constant or varies in the form of sine. The Rabi oscillations of various frequencies correlate more strongly with one another at the vicinity of $t = kT_{\mathrm{R}}$ ($k = 0, 1, 2, \ldots$). The interference in these regions is stronger, which results in the chaos of the atomic dipole squeezing being restrained in these vicinities. So the atomic dipole operators recurrently display squeezing at the vicinity of $t = kT_{\mathrm{R}}$. When the field frequency varies in the form of sine, the amplitude of the atomic population inversion decreases (see Figs.3(a) and 4(a)). The interference is weakened in these cases, which leads to the increases of the chaos of $F_i(t)$. So the squeezing effect of the atomic operator is weakened.

Then we consider the field-frequency varying in the form of a rectangle. The time evolution of the atomic dipole squeezing F_i for different initial time of rectangular frequency modulation is shown in Figs.5(c), 5(d) and 6(c), 6(d) for $\omega = 2\nu_0 = 1000g_0$. In the corresponding figures, a horizontal line at 0 is drawn as a reference. When the field frequency varying in the form of a rectangle, the atomic dipole operators F_1 and F_2 still display squeezing alternately (see Figs.5(c), 5(d) and 6(c), 6(d)). Comparing the pulse modulation cases with the no-pulse modulation case, we find that the squeezing is also weakened. The reason is similar to that for the sine modulation cases. However, the amplitude of the Rabi oscillations decreases less in the rectangular modulation cases than in the sine modulation cases. That is to say, a better constructive interference of different Rabi oscillations can hold in the rectangular modulation cases than in the sine modulation cases. So the squeezing of F_1 and F_2 can be kept longer regularly in the rectangular modulation cases than in sine modulation ones.

6. Conclusion

The dynamic properties of the interaction between a two-level atom and a coherent field via two-photon transition in the JCM without RWA have been investigated. In particular, we have studied the time evolution of the atomic population inversion, the field squeezing and the atomic dipole squeezing as the field frequency varying with time in the form of sine and rectangle. The consideration of the virtual-photon processes and the varying field frequency makes our results closer to the actual experimental condition. And the results obtained are found to be completely different from those in the case of the exact resonance. The good periodic character of the atomic population inversion in the standard TPJCM is weakened by the influence of the sine field frequency modulation. The rectangular field frequency modulation can change the correlation among different oscillations suddenly and induce new collapse-revival processes of the atomic population inversion. Based on the influence of the sine modulation, the field squeezing increases at the beginning of time, but then decreases and loses as the time increases after it reaches the maximum. Some quantum correlation is enhanced first and then lost as the time increases due to the sine modulation. The effects of the rectangular modulation on the field squeezing depend mostly on the appearance of time of the modulation. The atomic dipole squeezing is weakened under the influence of the sine or rectangular modulation. Finally we also analyze the relationship between the atomic population inversion and the squeezing properties. We discover the regime of the influence of the field frequency modulation on the properties of the TPJCM system. Our results also indicate that it is possible to realize the dynamic controlling of the system properties by changing the parameters of the system. And the right forms of field frequency modulation can be chosen to advance the degree of squeezing in the presence of environmental noises and manipulation inaccuracies. The conclusions in this paper may offer important reference to put them in force. This also implies the possibility of the application of the dynamic quantum controlling in quantum information processes.

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