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Periodic folded waves for a (2+1)-dimensional modified dispersive water wave equation*

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A general solution, including three arbitrary functions, is obtained for a (2+1)-dimensional modified dispersive water-wave (MDWW) equation by means of the WTC truncation method. Introducing proper multiple valued functions and Jacobi elliptic functions in the seed solution, special types of periodic folded waves are derived. In the long wave limit these periodic folded wave patterns may degenerate into single localized folded solitary wave excitations. The interactions of the periodic folded waves and the degenerated single folded solitary waves are investigated graphically and found to be completely elastic.

Keywords: modified dispersive water-wave equation, WTC truncation method, periodic folded wave
PACC: 0340, 0200, 0365

1. Introduction

In nature, there exist very complicated folded phenomena such as the folded protein,[1−3] folded brain, skin surface and many other kinds of folded biologic systems.[4,5] The simplest folded waves may be the bubbles on (or under) a fluid surface. Various ocean waves are really folded waves, too. Since folded solitary wave and foldons were first found by Tang and Lou in the (2+1)-dimensional dispersive long wave equation,[6] which are multivalued localized and folded in all directions, seeking and investigating the folded solitary waves in high-dimensional nonlinear models has aroused increasing interest.[7−13] In addition, recently some new folded localized excitations called semifolded solitary waves which may be folded in one direction and localized in another direction in a usual single-valued way are reported in (2+1)-dimensional nonlinear systems.[14−19] Meanwhile, in the study of nonlinear wave equations, periodic propagating wave patterns, especially in terms of Jacobi elliptic functions, also attract considerable interest.[20−26] In long wave limit, some periodic waves may degenerate into solitary waves. In this paper, we will search for new special types of semifolded wave patterns called periodic folded waves[27] for the (2+1)-dimensional modified dispersive water-wave (MDWW) equation[28,29]

$$u_{yt} + u_{xxy} - 2v_{xx} - (u^2)_{xy} = 0,$$

$$v_t - v_{xx} - 2(uv)_x = 0. \tag{1}$$

The system (1) was used to model nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters of uniform depth. It also can be derived from the well-known Kadomtsev–Petviashvili (KP) equation using the symmetry constraint.[30] Abundant propagating localized excitations were derived by Tang *et al* with the help of Painlevé–Bäcklund transformation and a multilinear variable separation approach.[31,32] Recently, nonpropagating and propagating solitons are also considered by Zheng *et al* by means of the extended mapping approach.[33] The goal of the present work is to conduct further investigations of Eq.(1) using the Weiss–Tabor–Carnevale (WTC) truncate expansion.[34] New periodic folded wave patterns are obtained. The properties of the interactions for the periodic folded waves are investigated graphically.

2. A variable separation solution to the MDWW equation

According to the idea of WTC, we truncate the Painlevé expansion of the MDWW equation (1) in the constant term

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$$u = u_0\phi^{-1} + u_1,$$
$$v = v_0\phi^{-2} + v_1\phi^{-1} + v_2, \tag{2}$$

where ϕ is the singular manifold variable, and $\{u_1, v_2\}$ is an arbitrary known solution. For simplicity, we take the trivial seed solutions as

$$u_1 = u_1(x, y, t), \quad v_2 = 0. \tag{3}$$

Substituting Eq.(2) with Eq.(3) into Eq.(1) and equating the coefficients of like powers of ϕ, we obtain

$$u_0 = \phi_x, \ \ v_0 = -\phi_x\phi_y,$$
$$u_1 = \frac{\phi_t - \phi_{xx}}{2\phi_x}, \ \ v_1 = \phi_{xy}, \tag{4}$$

and ϕ satisfies the system of equations

$$\begin{aligned}
&\phi_t^2\phi_x\phi_{xxy} - 3\phi_x\phi_{xxy}\phi_{xx}^2 + 2\phi_{xxy}\phi_{xxx}\phi_x^2\\
&+ \phi_{xy}\phi_{xxxx}\phi_x^2 - \phi_{tt}\phi_{xy}\phi_x^2 - \phi_{xxxxy}\phi_x^3\\
&- 2\phi_t\phi_{xyt}\phi_x^2 + 3\phi_{xy}\phi_{xx}^3 + 2\phi_{xx}\phi_{xxxy}\phi_x^2\\
&- 2\phi_{yt}\phi_{xt}\phi_x^2 - 4\phi_x\phi_{xx}\phi_{xxx}\phi_{xy}\\
&- 3\phi_{xy}\phi_{xx}\phi_t^2 + 2\phi_t\phi_x\phi_{yt}\phi_{xx}\\
&+ 4\phi_t\phi_x\phi_{xt}\phi_{xy} + \phi_{ytt}\phi_x^3 = 0,\\
&\phi\phi_x\phi_{xy}\phi_{xxx} - \phi\phi_{xy}\phi_{xx}^2 + \phi\phi_t\phi_{xy}\phi_{xx} - \phi\phi_x\phi_{xy}\phi_{xt}\\
&+ \phi\phi_{xyt}\phi_x^2 + \phi\phi_x\phi_{xx}\phi_{xxy} - \phi\phi_t\phi_x\phi_{xxy}\\
&- \phi\phi_{xxxy}\phi_x^2 - \phi_{xy}\phi_{xx}\phi_x^2 - \phi_{yt}\phi_x^3\\
&+ \phi_t\phi_{xy}\phi_x^2 + \phi_{xxy}\phi_x^3 = 0.
\end{aligned} \tag{5}$$

From Eqs.(5), after detailed and complex computations, one can find a special solution

$$\phi = f(x, t) + g(y) + \tau(t), \tag{6}$$

where $f(x, t)$, $g(y)$ and $\tau(t)$ are arbitrary functions of indicated variables. The substitution of Eqs.(4) and (6) into Eqs.(2) yields a general solution to Eq.(1)

$$u = \frac{f_x}{f + g + \tau} + \frac{f_t + \tau_t - f_{xx}}{2f_x}, \tag{7}$$
$$v = -\frac{f_x g_y}{(f + g + \tau)^2}. \tag{8}$$

Thanks to the arbitrariness of the functions f, g and τ, when functions f, g and τ are selected appropriately and avoid the singularities of Eqs.(7) and (8), the general solution of the (2+1)-dimensional MDWW equation reveal a diversity of exact propagating wave solutions and coherent localized soliton excitations. In the following sections, we will focus our attention on the possible periodic folded wave patterns.

3. Periodic folded waves for the MDWW system

We consider the field v of the MDWW system. For simplicity, we fix $\tau(t) = A =$ constant in Eq.(8), then the quantity v is rewritten as

$$v = -\frac{f_x g_y}{(A + f + g)^2}. \tag{9}$$

If introducing proper single-valued functions in the arbitrary functions f and g, the field v (9) may present many localized coherent structures such as dromions, lumps, breathers, instantons, ring solitons, peakons, compactons, and fractal localized structures, etc. One can see similar discussions in Refs.[31] and [32]. If the functions f and g are fixed as multivalued and single-valued functions appropriately, folded solitary waves and semifolded localized excitations can also be constructed easily, just like that in Refs.[6], [13], [16] and [19]. Here, we only seek possible periodic folded wave patterns for the field v expressed by Eq.(9).

In order to obtain different kinds of interesting periodic folded waves, we should introduce some suitable multi-valued functions in the function $f(x, t)$, with single-valued periodic functions in the function $g(y)$, respectively. For the function f, we set

$$\begin{aligned}
f_x &= \sum_{i=1}^{M} V_i(\xi + c_i t),\\
x &= \xi + \sum_{i=1}^{M} X_i(\xi + c_i t),\\
f &= \int^{\xi} f_x x_\xi \mathrm{d}\xi,
\end{aligned} \tag{10}$$

where V_i and X_j are localized excitations with properties $V_i(\pm\infty) = 0$, $X_i(\pm\infty) =$ const. From Eq.(10), one knows that ξ may be a multi-valued function in some suitable regions of x by selecting the functions X_i appropriately. Therefore, the function f_x, which is obviously an interaction solution of M localized excitations, since the property $\xi|_{x\to\infty} \to \infty$, may be a multi-valued function of x in these areas, though it is a single-valued function of ξ. Actually, most of the known (1+1)-dimensional multi-loop solutions are the special case of Eq.(10). For the other arbitrary function g, we introduce the Jacobian elliptic function in the simple form

$$g = \alpha + \sum_{j=1}^{N} \beta_j \mathrm{qn}(\gamma_j y, m), \tag{11}$$

where $\alpha, \beta_j, \gamma_j$ are arbitrary constants, qn denotes the Jacobian elliptic function and m is the modulus parameter ($0 \le m \le 1$). For briefness, we only consider the Jacobian elliptic function qn as the Jacobi elliptic sine function sn and the Jacobi elliptic cosine function cn in this paper. When the modulus $m \to 1$, sn and cn degenerate into tanh, sech, respectively, while for $m \to 0$, sn and cn degenerate into sin, cos, respectively.

Now, we give a concrete simple choice of Eqs.(10) and (11) as follows:

$$\begin{aligned} f_x &= -\frac{1}{2}\mathrm{sech}^2(\xi - 0.5t), \\ x &= \xi - 2\tanh(\xi - 0.5t), \\ f &= \int^{\xi} f_x x_\xi \mathrm{d}\xi, \end{aligned} \tag{12}$$

$$g = 8 + \frac{2}{3}\mathrm{sn}(y, m), \tag{13}$$

then a special type of periodic folded propagating wave pattern is constructed successfully and depicted in Fig.1. The field v shows a "worm" shape periodic folded wave with novel properties. From Fig.1(a)–1(c), one can see that it oscillates periodically in the way of a single-valued Jacobian elliptic sine function in the space of the y direction and is a mulitivalued loop soliton folded in the x-direction. Thus, this wave pattern can be viewed as a special kind of semifolded wave excitation actually. Interestingly, in the long wave limit $m \to 1$ in Eq.(13), the periodic folded wave pattern may degenerate into a single "worm" shape folded solitary wave excitation, which is plotted in Fig.2. From Fig.2(a)–(c), one can see that it is a loop soliton folded in the x-direction and localized like a bell in the y-direction, i.e, it is a bell-like semi-foldon excitation.

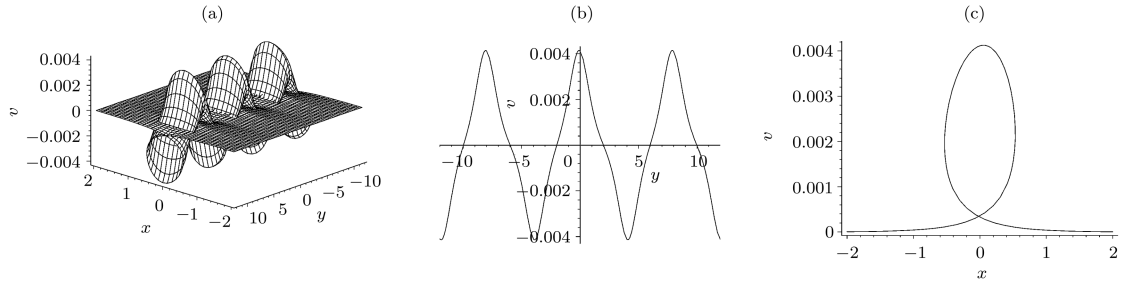


Fig.1. The "worm" shape periodic folded wave for the field v determined by Eqs.(12),(13) with $A = 1$ and $m = 0.8$. (a) At time $t = 0$; (b) a sectional view related to (a) at $x = 0$, (c) a sectional view related to (a) at $y = 0$.

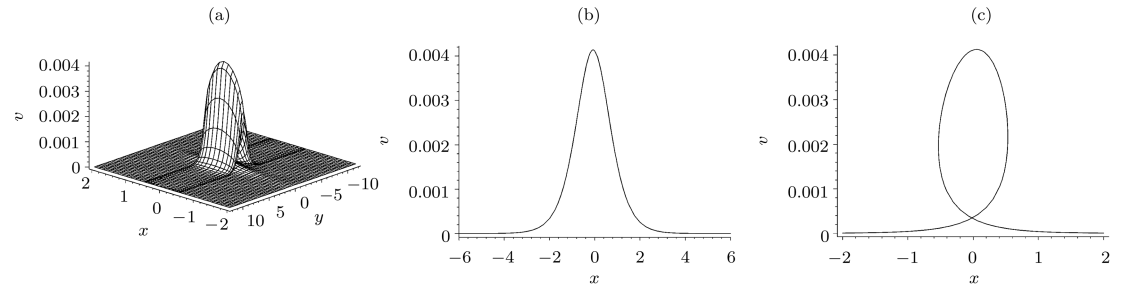


Fig.2. The "worm" shape folded solitary wave for the field v determined by Eqs.(12),(13) with $A = 1$ and $m = 1$; (a) at time $t = 0$; (b) a sectional view related to (a) at $x = 0$, (c) a sectional view related to (a) at $y = 0$.

If replacing the Jacobi elliptic function sn with cn in Eq.(13), i.e, the function g is rewritten as

$$g = 8 + \frac{2}{3}\mathrm{cn}(y, m), \tag{14}$$

then the field v determined by Eqs.(12) and (14) shows another type of periodic folded wave pattern. A detailed picture is shown in Fig.3. The periodic folded wave now oscillates periodically in the way of a Jacobian elliptic cosine function in the y direction, and is loop soliton folded in the x direction as well. When the modulus $m \to 1$ in Eq.(14), the field v reduces to a single lump-like folded solitary wave structure with one pit and one peak, which is plotted in Fig.4(a)–4(c).

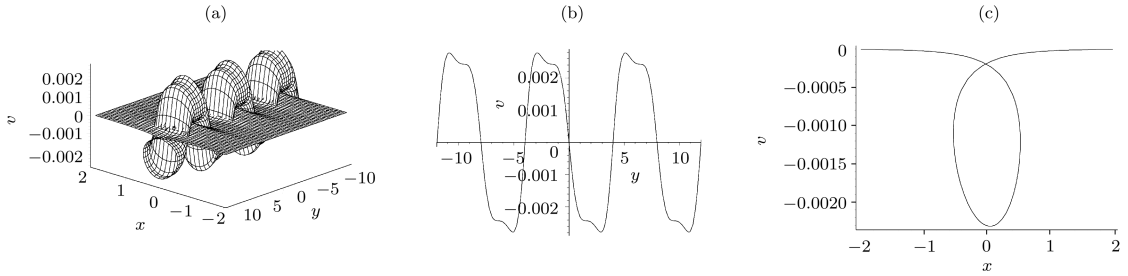


Fig.3. The periodic folded wave for the field v determined by Eqs.(12), (14) with $A = 1$ and $m = 0.8$. (a) At time $t = 0$; (b) a sectional view related to (a) at $x = 0$, (c) a sectional view related to (a) at $y = 0$.

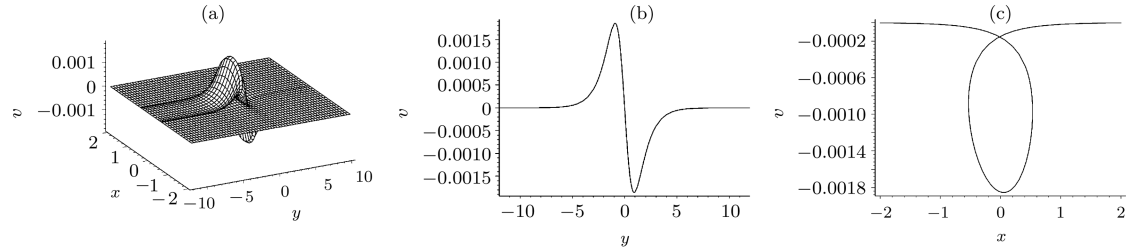


Fig.4. The folded solitary wave for the field v determined by Eqs.(12), (14) with $A = 1$ and $m = 1$; (a) at time $t = 0$; (b) a sectional view related to (a) at $x = 0$, (c) a sectional view related to (a) at $y = 0$.

Though two special types of periodic folded wave patterns are derived above, we know little about the interaction properties of the periodic folded wave patterns. For the MDWW system, we set the functions f and g in the forms

$$\begin{aligned} f_x &= \frac{2}{5}\mathrm{sech}^2(\xi) + \frac{1}{4}\mathrm{sech}^2(\xi - 0.3t), \\ x &= \xi - 1.5\tanh(\xi) - 1.5\tanh(\xi - 0.3t), \\ f &= \int^{\xi} f_x x_\xi \mathrm{d}\xi, \qquad (15) \\ g &= 10 + \mathrm{sn}(y, m), \qquad (16) \end{aligned}$$

then the field v represents the interaction between two "worm" shape periodic folded waves. Figures 5(a)–5(c) show the detailed evolution of this interaction. The two periodic folded waves may travel in the x-direction, but there is no change of shapes and velocities between them before and after the collision besides phase shifts. To see more carefully, the position located by the large static periodic folded wave is altered from about $x = -1.5$ to $x = 1.5$ and its shape is completely preserved after the interaction. That means there is no exchange of energy and momentum. The interaction between them is completely elastic, which is very similar to the completely elastic collision between two classical particles. In the long wave limit $m \to 1$ in Eq.(16), the field v presents the interaction between two single folded bell-like solitary wave excitations correspondingly. This interaction is illustrated in Figs.6(a)–6(c) and is shown to be completely elastic. Similarly, if we fix the elliptic function sn in Eq.(16) as cn, the interaction of the periodic folded waves determined by $g = 10 + \mathrm{cn}(y, m)$

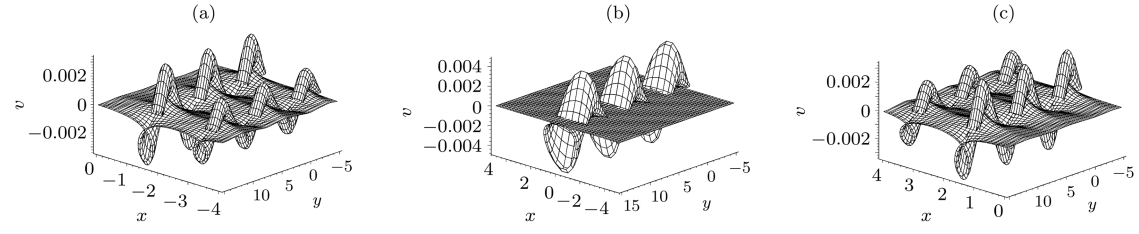


Fig.5. Evolution plots of two periodic folded waves for the field v determined by Eqs.(15), (16) with $A = 1, m = 0.8$ at (a) $t = -16$, (b) $t = -1$, (c) $t = 16$ respectively.

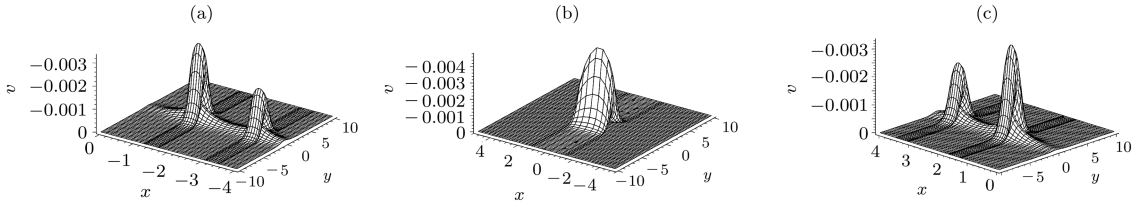


Fig.6. Evolution plots of two single bell-like folded solitary waves for the field v determined by Eqs.(15), (16) with $A = 1, m = 1$ at (a) $t = -16$, (b) $t = -1$, (c) $t = 16$ respectively.

and Eq.(15) is also found to be completely elastic, so is the interaction of their degenerated lump-like folded solitary waves in the long wave limit. The corresponding discussion is omitted here for briefness.

4. Summary and discussion

To summarize, applying the WTC truncation Painlevé expansion method, a general solution with three arbitrary variable separation functions for the MDWW system is obtained. By introducing proper multi-valued functions and Jacobian elliptic functions in the seed solution, we derived two new special types of periodic folded wave patterns. The periodic folded waves may oscillate periodically in the way of the single-valued Jacobian elliptic function in the space of the y direction and be mulitivalued loop soliton folded in the x-direction. In the long wave limit, they will degenerate into single folded solitary waves. The interactions of the periodic folded waves are graphically investigated and found to be completely elastic, as the interaction of the corresponding degenerated single folded solitary waves showed. For the MDWW system, to obtain periodic folded waves, Jacobi elliptic functions sn and cn are only selected in Eq.(11) here. Actually, quite rich periodic folded wave patterns can also be obtained by introducing the other Jacobi elliptic functions such as dn, sd, cd, nd in Eq.(11) similarly. We believe the periodic folded wave patterns may exist in other high-dimensional nonlinear models, and our discussion in this paper may be important and physically significant for further study of the complexity of the periodic folded phenomena in nature. The exploration of the periodic folded waves for the MDWW equation and seeking periodic folded waves in other higher dimensional nonlinear models are worthy of further study.

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