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A New type of conserved quantity deduced from Mei symmetry of nonholonomic systems in terms of quasi-coordinates

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This paper studies the new type of conserved quantity which is directly induced by Mei symmetry of nonholonomic systems in terms of quasi-coordinates. A coordination function is introduced, and the conditions for the existence of the new conserved quantities as well as their forms are proposed. Some special cases are given to illustrate the generalized significance of the new type conserved quantity. Finally, an illustrated example is given to show the application of the nonholonomic system's results.

Keywords: Mei symmetry, conserved quantity, quasi-coordinate, nonholonomic system
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1. Introduction

The research on symmetries and conserved quantities of a mechanical system has very great significance in modern mathematics, mechanics and physics. The modern methods of finding conserved quantities of a mechanical system are mainly the Noether symmetry, the Lie symmetry and the Mei symmetry.[1−5] The conserved quantities induced directly or indirectly from the three symmetries mentioned above are mainly the Noether conserved quantity, the Hojman conserved quantity, and the Mei conserved quantity.[6−10] In the past 20 years, there have been many achievements in this popular research field,[11−18] and seeking for other new types of conserved quantities is a developing trend in symmetry theories. References [19] and [20] have discussed the new type of conserved quantities induced directly from the Mei symmetry of the Lagrange and Hamilton systems respectively. As we know, introducing the concept and mark of quasi-coordinates is of great significance in mechanical systems, especially in nonholonomic mechanical systems, and is closely related to quasi-velocity. The quasi-coordinate is superior to the generalized coordinate, and quasi-velocity is more general than the generalized velocity.

We study the new types of conserved quantities deduced directly from the Mei symmetry for nonholonomic systems in terms of quasi-coordinates in this article. The conditions for the existence of the new type of conserved quantities are proposed and the forms of the new type of conserved quantities are given. Finally an illustrated example is given to show the application of the results.

2. Dynamical equations of motion of nonholonomic system in terms of quasi-coordinates

Suppose that the configuration of the mechanical system is determined by n generalized coordinates q_s $(s = 1, \cdots, n)$, and the system is subjected to g ideal nonholonomic constraints

$$a_{\varepsilon+\beta,k}(\boldsymbol{q})\dot{q}_k = 0, \quad (\beta = 1, \ldots g; \quad \varepsilon = n - g; \quad k = 1, \ldots n). \tag{1}$$

Choose n independent and compatible quasi-velocities

$$\omega_s = a_{sk}(\boldsymbol{q})\dot{q}_k, \quad (s = 1, \ldots, n), \tag{2}$$

and their reverse transformations

$$\dot{q}_s = b_{sk}(\boldsymbol{q})\omega_k, \quad (s = 1, \ldots, n), \tag{3}$$

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the nonholonomic constraints Eq.(1) become

$$\omega_{\varepsilon+\beta}=0,\quad(\beta=1,\dots g;\quad\varepsilon=n-g). \tag{4}$$

Limitations of the nonholonomic constraints Eq.(4) to the virtual displacements of quasi-coordinates are shown as

$$\delta\pi_{\varepsilon+\beta}=0,(\beta=1,\dots g). \tag{5}$$

From the D'Alembert–Lagrange principle in terms of quasi-coordinates and the equations of virtual displacements Eq.(5), we can derive the Boltzmann–Hamel equations of the nonholonomic system[4]

$$\begin{aligned}&\tilde{E}_\sigma(L^*)+\frac{\partial L^*}{\partial\omega_r}\gamma^r_{\nu\sigma}\omega_\nu=P^*_\sigma,\\&(\sigma,\nu=1,\dots\varepsilon,r=1,\dots,n),\end{aligned} \tag{6}$$

where

$$\tilde{E}_\sigma=\frac{\mathrm{d}}{\mathrm{d}t}\frac{\partial}{\partial\omega_\sigma}-\frac{\partial}{\partial\pi_\sigma}$$

is the Euler operator in terms of quasi-coordinates, π_σ are quasi-coordinates, and $L^*=L^*(t,\boldsymbol{q},\boldsymbol{\omega})$ is the Lagrangian in terms of quasi-coordinates, $P^*_\sigma=P^*_\sigma(t,\boldsymbol{q},\boldsymbol{\omega})$ are the generalized non-potential forces in terms of quasi-coordinates,

$$\gamma^r_{\nu\sigma}=\left(\frac{\partial a_{rm}}{\partial q_l}-\frac{\partial a_{rl}}{\partial q_m}\right)b_{l\upsilon}b_{m\sigma}$$

are Boltzmann marks.

Suppose that equation (6) is nonsingular, i.e.

$$\det\left[\frac{\partial^2 L^*}{\partial\omega_\upsilon\partial\omega_\sigma}\right]\neq0,$$

then all the ω_σ can be solved from Eq.(6) as follows:

$$\dot{\omega}_\sigma=\alpha_\sigma(t,\boldsymbol{q},\boldsymbol{\omega}),\quad(\sigma=1,\dots,\varepsilon). \tag{7}$$

3. Mei symmetry and the new type of conserved quantity for nonholonomic system in terms of quasi-coordinates

Introducing the infinitesimal transformations of groups of time and quasi-coordinates

$$\begin{aligned}&t^*=t+\varepsilon\xi_0(t,\boldsymbol{q},\boldsymbol{\omega}),\\&\pi^*_s(t^*)=\pi_s(t)+\varepsilon\xi_s(t,\boldsymbol{q},\boldsymbol{\omega}),\quad(s=1,\dots n)\end{aligned} \tag{8}$$

under the infinitesimal transformations Eq.(8), we get the Lagrange function L^* and the generalized force P^*_σ in terms of quasi-coordinates

$$\begin{aligned}\tilde{L}^*&=L^*(t^*,\boldsymbol{q}^*,\boldsymbol{\omega}^*)\\&=L^*(t,\boldsymbol{q},\boldsymbol{\omega})+\varepsilon\tilde{X}^{(1)}(L^*)+O(\varepsilon^2),\\\tilde{P}^*_\sigma&=P^*_\sigma(t^*,\boldsymbol{q}^*,\boldsymbol{\omega}^*)\\&=P^*_\sigma(t,\boldsymbol{q},\boldsymbol{\omega})+\varepsilon\tilde{X}^{(1)}(P^*_\sigma)+O(\varepsilon^2),\end{aligned} \tag{9}$$

where

$$\begin{aligned}\tilde{X}^{(1)}=&\xi_0\frac{\partial}{\partial t}+\xi_s\frac{\partial}{\partial\pi_s}\\&+\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_s-\omega_s\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0-\gamma^s_{r\alpha}\omega_r\xi_\alpha\right)\frac{\partial}{\partial\omega_s},\end{aligned} \tag{10}$$

and

$$\frac{\bar{\mathrm{d}}}{\mathrm{d}t}=\frac{\partial}{\partial t}+\omega_k\frac{\partial}{\partial\pi_k}+\alpha_k\frac{\partial}{\partial\omega_k}. \tag{11}$$

Definition 1 Under the infinitesimal transformations Eq.(8), if the Lagrange function $\tilde{L}^*$ and the generalized forces $\tilde{P}^*_\sigma$ in terms of quasi-coordinates still satisfy Eq.(6), i.e.

$$\tilde{E}_\sigma(\tilde{L}^*)+\frac{\partial\tilde{L}^*}{\partial\omega_r}\gamma^r_{\upsilon\sigma}\omega_\upsilon=\tilde{P}^*_\sigma, \tag{12}$$

the invariance is called Mei symmetry of the system.

Substituting Eq.(9) into Eq.(12), using Eq.(6), ignoring ε^2 and the higher-order infinitesimal terms, we obtain

$$\tilde{E}_\sigma[\tilde{X}^{(1)}(L^*)]+\frac{\partial\tilde{X}^{(1)}(L^*)}{\partial\omega_r}\gamma^r_{\upsilon\sigma}\omega_\upsilon=\tilde{X}^{(1)}(P^*_\sigma), \tag{13}$$

Equations in Eq.(13) are called the Mei symmetry criterion equations of nonholonomic system in terms of quasi-coordinates.

The invariance of Eq.(4) under the infinitesimal transformations Eq.(8) leads to the following equations:

$$\tilde{X}^{(1)}(\omega_{\varepsilon+\beta})=0, \tag{14}$$

because of $\omega_{\varepsilon+\beta}=0$ $(\beta=1,\dots g)$, equations (14) lead to the following restriction equations by using Eq.(10):

$$\dot{\xi}_{\varepsilon+\beta}=0,\quad(\beta=1,\dots g), \tag{15}$$

so we have

$$\xi_{\varepsilon+\beta}=c_\beta=\text{const.} \tag{16}$$

Criterion 1 For the nonholonomic mechanical system in terms of quasi-coordinates Eqs.(4) and (6), if the infinitesimal generators ξ_0, ξ_s satisfy Eqs.(13) and (16), the corresponding invariance is Mei symmetry of the nonholonomic system in terms of quasi-coordinates.

Proposition 1 For the infinitesimal generators ξ_0,ξ_s satisfying the criterion equations (13) and the additive restriction equations (16), if there exists such

a gauge function $G_M = G_M(t, \boldsymbol{q}, \boldsymbol{\omega})$ satisfying the following structure equation

$$\begin{aligned}&\frac{\partial \tilde{X}^{(1)}(L^*)}{\partial t} f + \xi_\sigma \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \pi_\sigma} \\ &+ \left(\frac{\bar{\mathrm{d}}\xi_\sigma}{\mathrm{d}t} - \omega_\sigma \frac{\bar{\mathrm{d}} f}{\mathrm{d}t} - \gamma^\sigma_{rv}\omega_r \xi_v \right) \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} \\ &+ \tilde{X}^{(1)}(L^*) \frac{\bar{\mathrm{d}}}{\mathrm{d}t} f \\ &+ \tilde{X}^{(1)}(P^*_\sigma)(\xi_\sigma - \omega_\sigma f) + \frac{\bar{\mathrm{d}}}{\mathrm{d}t} G_M = 0, \end{aligned} \tag{17}$$

then the Mei symmetry of the nonholonomic system in terms of quasi-coordinates can lead to the following new conserved quantity:

$$\begin{aligned} I_M &= \tilde{X}^{(1)}(L^*) f + \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma}(\xi_\sigma - \omega_\sigma f) + G_M \\ &= \text{const.}, \end{aligned} \tag{18}$$

where $f = f(t, \boldsymbol{q}, \boldsymbol{\omega})$ is any function and we call it the coordination function.

Proof Using Eqs.(11) to (18), we have

$$\begin{aligned} \frac{\bar{\mathrm{d}} I_M}{\mathrm{d}t} &= f\omega_\sigma \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \pi_\sigma} + f\alpha_\sigma \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} + \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} \frac{\bar{\mathrm{d}}}{\mathrm{d}t}(\xi_\sigma - \omega_\sigma f) \\ &\quad + (\xi_\sigma - \omega_\sigma f) \frac{\bar{\mathrm{d}}}{\mathrm{d}t} \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} - \left(\frac{\bar{\mathrm{d}}\xi_\sigma}{\mathrm{d}t} - \omega_\sigma \frac{\mathrm{d} f}{\mathrm{d}t} - \gamma^\sigma_{rv}\omega_r \xi_v \right) \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} \\ &\quad - \xi_\sigma \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \pi_\sigma} - \tilde{X}^{(1)}(P^*_\sigma)(\xi_\sigma - \omega_\sigma f), \end{aligned} \tag{19}$$

substituting Eq.(13) into Eq.(19), and noticing that $\gamma^r_{v\sigma}\omega_v\omega_\sigma = 0$, we get

$$\begin{aligned} \frac{\bar{\mathrm{d}} I_M}{\mathrm{d}t} &= f\alpha_\sigma \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} + \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} \left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_\sigma - \omega_\sigma \frac{\bar{\mathrm{d}}}{\mathrm{d}t} f - f\alpha_\sigma \right) \\ &\quad - \left(\frac{\bar{\mathrm{d}}\xi_\sigma}{\mathrm{d}t} - \omega_\sigma \frac{\bar{\mathrm{d}} f}{\mathrm{d}t} - \gamma^r_{v\sigma}\omega_{vr}\xi_\sigma \right) \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma} - \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_r} \gamma^r_{v\sigma}\omega_v(\xi_\sigma - \omega_\sigma f) \\ &= 0. \end{aligned} \tag{20}$$

The new types of conserved quantities obtained above, deduced from Mei symmetry of nonholonomic system in terms of quasi-coordinates as well as the conditions for their existence, have more general significance, and, under certain conditions, we can obtain the following special cases.

Special case 1 When taking $f = \xi_0$, equations (17) and (18) respectively become

$$\begin{aligned} &\tilde{X}^{(1)}\{\tilde{X}^{(1)}(L^*)\} + \tilde{X}^{(1)}(L^*) \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0 \\ &+ \tilde{X}^{(1)}(P^*_\sigma)(\xi_\sigma - \omega_\sigma \xi_0) + \frac{\bar{\mathrm{d}}}{\mathrm{d}t} G_M = 0, \end{aligned} \tag{21}$$

$$\begin{aligned} I_M &= \tilde{X}^{(1)}(L^*)\xi_0 + \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \omega_\sigma}(\xi_\sigma - \omega_\sigma \xi_0) + G_M \\ &= \text{const.} \end{aligned} \tag{22}$$

Equation (21) is just the structure equation of Mei symmetry for nonholonomic systems in terms of quasi-coordinates, and equation (22) is the Mei conserved quantity of nonholonomic system in terms of quasi-coordinates.

Special case 2 When taking $f = \xi_0$, $\omega_s = \dot{q}_s$, equations (17) and (18) respectively become

$$\begin{aligned} &\tilde{X}^{(1)}\{\tilde{X}^{(1)}(L)\} + \tilde{X}^{(1)}(L) \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0 \\ &+ \tilde{X}^{(1)}(Q_s)(\xi_s - \dot{q}_s \xi_0) + \frac{\bar{\mathrm{d}}}{\mathrm{d}t} G_F = 0, \end{aligned} \tag{23}$$

$$\begin{aligned} I_F &= \tilde{X}^{(1)}(L^*)\xi_0 + \frac{\partial \tilde{X}^{(1)}(L^*)}{\partial \dot{q}_s}(\xi_s - \dot{q}_s \xi_0) + G_F \\ &= \text{const.}, \end{aligned} \tag{24}$$

where $\tilde{X}^{(1)}$ turns to

$$\tilde{X}^{(1)} = \xi_0 \frac{\partial}{\partial t} + \xi_s \frac{\partial}{\partial q_s} + \left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_s - \dot{q}_s \frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0 \right) \frac{\partial}{\partial \dot{q}_s}. \tag{25}$$

Equation (23) is the structure equation of Mei symmetry for the nonholonomic system in generalized coordi-

nates, and equation (24) is the Mei conserved quantity of nonholonomic system in generalized coordinates.

Moreover, since the holonomic system can be regarded as a special case to the nonholonomic system in quasi-coordinates, so when the system is not subjected to the nonholonomic constraints Eq.(1), the above obtained results will turn to the results of holonomic system in terms of quasi-coordinates.

4. An illustrated example

Suppose that the Lagrangian and generalized forces of the mechanical system are

$$L = \dot{q}_1\dot{q}_2 \sin q_1 + \dot{q}_2^2 \cos^2 q_1 + \dot{q}_3^2 + 2\dot{q}_2\dot{q}_3 \cos q_1,$$
$$Q_1 = g, \quad Q_2 = Q_3 = 0, \tag{26}$$

and the system is subjected to the following nonholonomic constraint

$$\dot{q}_3 = -\dot{q}_2 \cos q_1, \tag{27}$$

we try to study the Mei symmetry and the new type of conserved quantity of the system.

Choose quasi-velocities as

$$\omega_1 = \dot{q}_1, \quad \omega_2 = \dot{q}_2 \mathrm{sin} q_1, \quad \omega_3 = \dot{q}_2 \mathrm{cos} q_1 + \dot{q}_3 \tag{28}$$

the conditions of nonholonomic constraint turn to

$$\omega_3 = 0, \tag{29}$$

from Eq.(28), we can obtain

$$\dot{q}_1 = \omega_1, \quad \dot{q}_2 = \frac{\omega_2}{\mathrm{sin} q_1}, \quad \dot{q}_3 = \omega_3 - \omega_2 \operatorname{ctg} q_1, \tag{30}$$

so we have

$$a_{11} = 1, \quad a_{22} = \sin q_1, \quad a_{32} = \cos q_1, \quad a_{33} = 1,$$
$$b_{11} = 1, \quad b_{22} = \frac{1}{\sin q_1}, \quad b_{32} = -\operatorname{ctg} q_1, \quad b_{33} = 1. \tag{31}$$

Boltzmann marks can be solved as

$$\gamma_{12}^2 = -\gamma_{21}^2 = \operatorname{ctg} q_1, \quad \gamma_{21}^3 = -\gamma_{12}^3 = 1, \tag{32}$$

the others are equal to zero.

By using quasi-velocities, the Lagrangian and the nonpotential generalized forces are expressed as

$$L^* = \omega_1\omega_2 + \omega_3^2, \quad P_1^* = g, \quad P_2^* = P_3^* = 0, \tag{33}$$

the Boltzmann–Hamel equations (6) give

$$\dot{\omega}_1 = 2\omega_1\omega_2 - \omega_1^2 \operatorname{ctg} q_1,$$
$$\dot{\omega}_2 = \omega_1\omega_2 \operatorname{ctg} q_1 - 2\omega_2\omega_3, \quad \dot{\omega}_3 = 0. \tag{34}$$

After the calculation, we get

$$\begin{aligned}
&\tilde{X}^{(1)}(L^*) \\
&= \omega_2\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_1 - \omega_1\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0\right) \\
&\quad + \omega_1\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_2 - \omega_2\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0 - \omega_1\xi_2 \operatorname{ctg} q_1 + \omega_2\xi_1 \operatorname{ctg} q_1\right) \\
&\quad + 2\omega_3\left(\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_3 - \omega_3\frac{\bar{\mathrm{d}}}{\mathrm{d}t}\xi_0 + \omega_1\xi_2 - \omega_2\xi_1\right),
\end{aligned}$$

$$\tilde{X}^{(1)}(P_\sigma^*) = 0. \tag{35}$$

Considering the restriction equations (16), we take

$$\xi_0 = t, \quad \xi_1 = \omega_1, \quad \xi_2 = \omega_2, \quad \xi_3 = 0, \tag{36}$$

then we have

$$\begin{aligned}
&\tilde{X}^{(1)}(L^*) = -2\omega_1\omega_2, \\
&\tilde{X}^{(1)}(P_\sigma^*) = 0, \\
&\tilde{E}_1\{\tilde{X}^{(1)}(L^*)\} = -2\omega_1\omega_2 \operatorname{ctg} q_1, \\
&\tilde{E}_2\{\tilde{X}^{(1)}(L^*)\} = 2\omega_1^2 \operatorname{ctg} q_1,
\end{aligned} \tag{37}$$

it is easy to testify that they satisfy Eq.(13).

Substituting Eqs.(36) and (37) into Eq.(17), we get

$$\frac{\bar{\mathrm{d}}G_M}{\mathrm{d}t} = -2\omega_1\omega_2\frac{\bar{\mathrm{d}}}{\mathrm{d}t}f. \tag{38}$$

(i) When we choose $f = \omega_1\omega_2$, we have

$$G_M = \omega_1 \sin q_1, \tag{39}$$

from Eq.(18), we have

$$I_M = 2\omega_1^2\omega_2^2 - 4\omega_1\omega_2 + \omega_1 \sin q_1 = \text{const.}. \tag{40}$$

(ii) choose $f = \xi_0 = t$, we have

$$G_M = -2\omega_1\omega_2 t, \tag{41}$$

$$I_M = -4\omega_1\omega_2 = \text{const.} \tag{42}$$

Equation (42) is a Mei conserved quantity for the nonholonomic system in terms of quasi-coordinates.

5. Conclusion

By introducing the coordination function f, we get a new type of conserved quantity deduced from Mei symmetry of nonholonomic system in terms of quasi-coordinates as well as the conditions for their existence, which are different from the Mei conserved quantities deduced from Mei symmetry and their existent conditions, the latter are the special cases of the former when taking $f = \xi_0$. Because the coordination function can be selected properly according to the demand for finding the gauge function, the gauge function can be found more easily and more conserved quantities of Mei symmetry for the system can be obtained without the restriction of ξ_0, so the conserved quantity given in this paper is a new type of conserved quantity. When taking $f = \xi_0$, $\omega_s = \dot{q}_s$, the new type of conserved quantity will turn into the corresponding Mei conserved quantity in terms of generalized coordinates. Additionally, the results given in this paper are also fitted to the holonomic system in terms of quasi-coordinates.

References

[1] Noether A E 1918 *Nachr. Akad. Wiss. Göttingen Math. Phys.* **KI II 235**
[2] Lyzky M 1979 *J. Phys.* A: *Math. Gen.* **12** 973
[3] Mei F X 2000 *J. Beijing Inst. Technol.* **9** 120
[4] Mei F X 1999 *Application of Lie Groups and Lie Algebras to Constrained Mechanical Systems* (Beijing: Science Press) (in chinese)
[5] Mei F X 2004 *Symmetries and Conserved Quantities of Constrained Mechanical Systems* (Beijing: Beijing Institute of Technology Press) (in Chinese)
[6] Qiao Y F and Zhao S H 2001 *Acta Phys. Sin.* **50** 1 (in Chinese)
[7] Ge W K 2002 *Acta Phys. Sin.* **51** 1156 (in Chinese)
[8] Luo S K 2003 *Acta Phys. Sin.* **52** 2941 (in Chinese)
[9] Fang J H 2003 *Commun. Theor. Phys.* **40** 269
[10] Fu J L and Chen L Q 2003 *Phys. Lett.* A **317** 255
[11] Zhang Y 2004 *Commun. Theor. Phys.* **42** 899
[12] Liu R W and Chen L Q 2004 *Chin. Phys.* **13** 1615
[13] Zhang H B, Chen L Q and Gu S L 2005 *Acta Phys. Sin.* **54** 2489 (in Chinese)
[14] Qiao Y F, Li R J and Ma Y S 2005 *Commun. Theor. Phys* **43** 197
[15] Chen X W, Zhao Y H and Li Y M 2005 *Commum. Theor. Phys.* **44** 773
[16] Zhang Y 2006 *Commun. Theor. Phys* **45** 239
[17] Ding N, Fang J H and Wang P 2006 *Acta Phys. Sin.* **55** 6197 (in Chinese)
[18] Zhang X N, Fang J H, Pang T and Lin P 2008 *Chin. Phys.* B **17** 394
[19] Fang J H, Ding N and Wang P 2007 *Chin. Phys.* **16** 887
[20] Fang J H, Ding N and Wang P 2007 *Acta Phys. Sin.* **56** 3093 (in Chinese)